UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

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Symmetry and Music

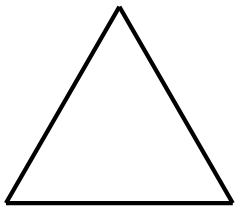
Introduction

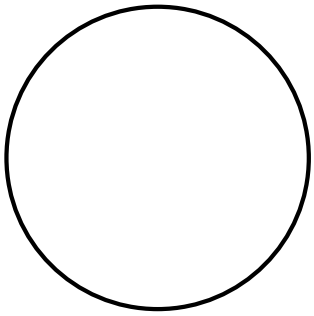
We've done a lot of learning and exploring new areas of math this term in Math Circles. For this final lesson, we're going to take a slightly different approach. Today, let's look at something many of you should already be familiar with, learn some more about it, but then focus on ***applications*** of the idea. Let's talk about symmetry!

You may have learned about symmetry already in school. You should be most familiar with symmetries in shapes, around something called an axis of symmetry. Let's review this idea, and then learn more!

Symmetry

Symmetry is not usually seen as a particular subject in math, but instead something that comes up over and over again in all areas of math, and in the real world. You should have already learned about symmetries in shapes. Let's look at some examples:





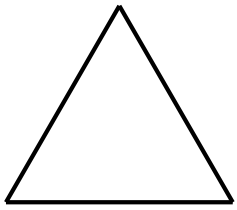
The ***axis of symmetry*** is:

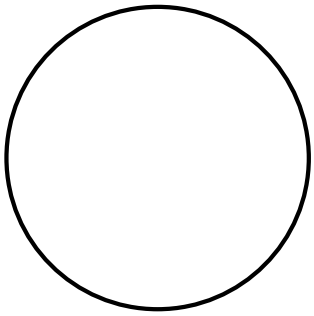
At least, that's the symmetry that you're already used to. This type of symmetry that involves drawing an axis through a shape is only one *type* of symmetry:

The formal way to say that the triangle has this symmetry is:

That said, we can of course have other types of symmetry. To understand this, we need to first talk about what a symmetry *really* is. ***Symmetry*** is an "_____ property". This means:

So what other types of symmetries might we have? Let's look back at the shapes.





Symmetry and the World

The focus of this lesson, after understanding symmetry, is to look into its applications in the real world.

It's important to note that the rest of this lesson does not cover universally accepted math concepts like we have in Math Circles so far. Instead we're taking the math concept of symmetry and using it to create our own understanding of music and art. This is much closer to being philosophy than math. Doing this is still important, however. Math doesn't happen in an empty world, and thinking about how different subjects interact with each other is an enriching part of any learning.

Symmetry and Beauty

In nature we see symmetry everywhere. From the patterns of butterflies and flowers, to the tiny atoms and molecules that make up everything, we find symmetry in every part of science.

Humans look symmetric on the outside, as much as biology allows us to be. Our symmetric bodies let us walk and interact the way we do with the world around us. Humans like symmetry, and like to find symmetry in world. As beings that are especially good at pattern recognition, we find symmetry pleasing, and easy to take in. In terms of evolution, it's believed that symmetric features were a sign of good health and strength, and so we evolved to recognize it and find it good. That's why in our art, that humans make for themselves, we can still find symmetry, and enjoy it.

To understand how deep our relationship to symmetry is, it might be cool to look for symmetries that are <u>not</u> visual, and see how we can apply the same ideas to them that we did to all the other symmetries so far.

Symmetry and Music

Time Signatures

Let's start with something simple, and work our way up. Listen to sound of a metronome. Notice that the sound repeats consistently every bar. Changing the time signature may change how many beats are in each bar, and how long they are, but the beats would repeat all the same. What connection can we make to symmetry? What kind of symmetry does sound of the metronome have?

Rhythm

What is a ***rhythm***? Rhythm is a regular repeated pattern. Usually in music it refers more to "length". In other words, a repeated pattern of notes that have a particular length is usually called a rhythm, with less attention to what the notes actually are. Try making your own rhythm, and listen carefully to some of your favourite songs, trying to find the rhythm in them. What symmetry do we find here? How can we relate this to our experience of music?

Motif

Talking about rhythm leads us to the concept of motif. In art in general, motif often refers to some dominant idea or feature. In music, it usually refers to a repeating part of the piece that is the "smallest unit of the musical idea". In other words, it while it does repeat in a piece, it **also** conveys the idea of the piece. If you think of music as having a message, a motif is the smallest snippet that still tells you that message. The most famous example of this is in Beethoven's 5th Symphony, which starts with the motif - give it a listen! What can we say about the symmetry of a motif, and its relation to how we experience music?

Songs

What are some symmetric features of a song? How does this impact our experience of the song? Listen to some songs you enjoy and think about it!

Try listening to Bohemian Rhapsody by Queen. What do you think of it? Does it sound weird to you?

Another very different genre of music where we can find symmetries is rap music. Rap is historically very lyrically driven. Many if not most rap songs have a consistently repeating beat in the background, with lyrics over top. Lyricism in rap is centered around rhyming. This is its own form of symmetry. Analyzing rhyme patterns in rap can reveal complex symmetries with respect to reflections and translations, over instrumentals that are very symmetric in time.

Asymmetries and Music

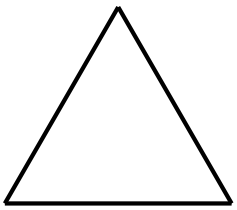
Symmetries are pleasing and we like them, but they're not always the best thing. If music or art is too symmetric, then it also becomes uninteresting. Humans are good at taking symmetry in all at once. The music doesn't tell a story anymore, it doesn't give us a reason to listen closely, if everything is so symmetric that it always sounds the same. Making good music or art means you have to balance symmetry and repitition, which we like and are important, with ***asymmetries***, or features of the art that are not symmetric. Asymmetry keeps the song interesting, while symmetry keeps it familiar and comfortable. Keeping this balance is something that Bohemain Rhapsody, for example, does really well, in a very interesting way.

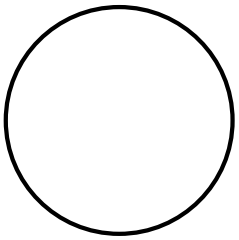
Next time you're listening to music try thinking about this, and think about **why** you enjoy the song.

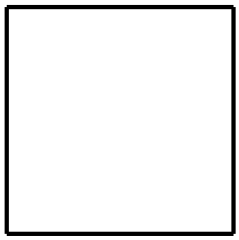
Problems

1. What is symmetry? What does it mean for something to be symmetric?

2. Find the symmetries of the following shapes. Make sure to include what type of symmetry it is.

(a) 

(b) 

(c) 

3. If you recall back to the coordinate systems lesson, remember that we talked about certain shapes that we "care about" enough to have coordinate systems based around them. Let's explore the symmetries of these shapes. Think about how this might connect to what we talked about in that lesson.

   (a) What are the symmetries of a perfect sphere?

   (b) What are the symmetries of a cylinder?

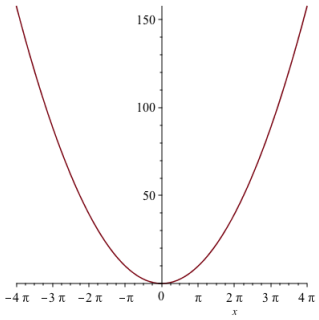
   (c) What are the symmetries of an infiitely long cylinder? (ie. a pole that goes on forever)

   (d) What are the symmetries of a 2D plane?

4. Recall that you had a lesson on matrices earlier this term. The ***transpose*** of a matrix is the matrix you get when you take each row and turn it into a column (more details in the matrix lesson). What do you think a matrix would look like if it's *symmetric* with respect to taking the transpose?

5. As we've learned, we can analyze graphs qualitatively to understand quickly some of the information they convey. One of the the analyses we can do on graphs is to look at if they are ***even*** or ***odd***.
   Even means that the graph is symmetric with respect to reflections about the y-axis.
   Odd means that the graph is symmetric with respect to rotations of 180° about the origin (0,0).
   State whether the following graphs are even or odd.

(a)



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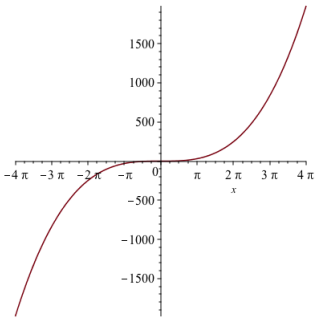

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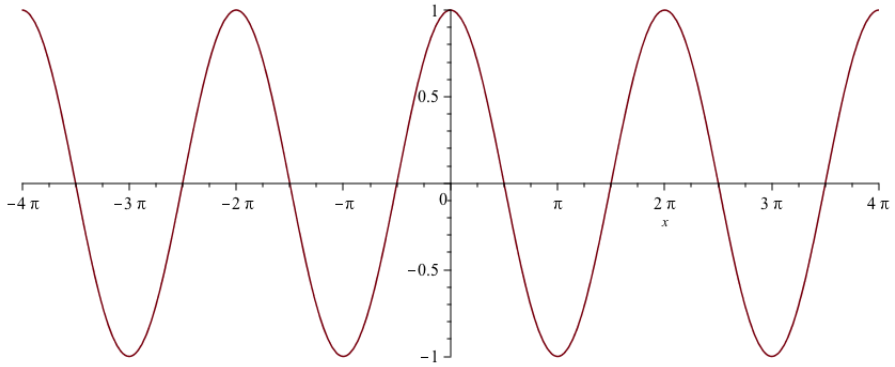

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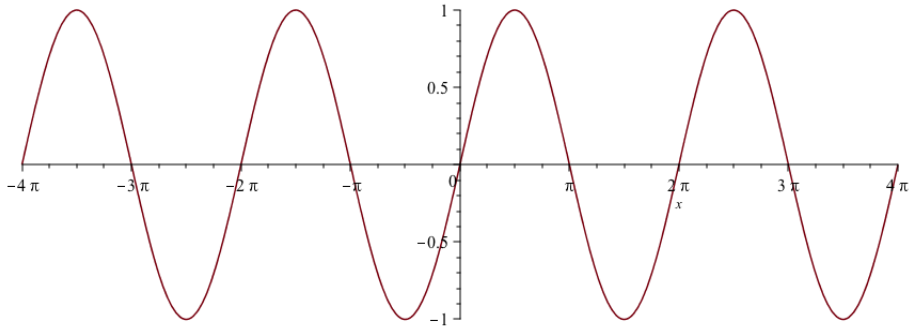

6. Look for symmetries and asymmetries in your music!