UNIVERSITY OF WATERLOO

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Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

Fall 2018 - October 9/10/11

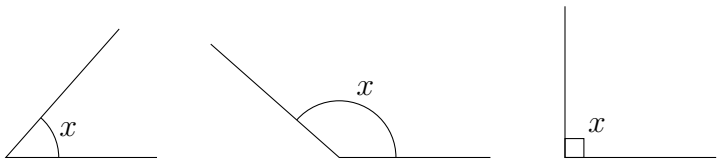
Angles and Light

Today we will be learning about angles. We will learn some angle properties that will allow us to determine the measure of angles without using a protractor. First, we need to recall some basic knowledge about angles.

Angles

The Basics

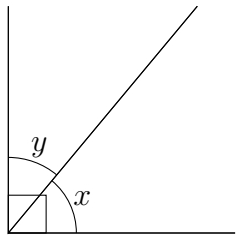
Acute Angle | Obtuse Angle | Right Angle



$x < 90°$ | $x > 90°$ | $x = 90°$

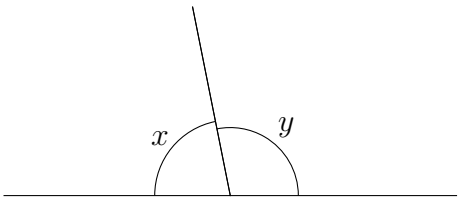
Complementary Angles

Angles are complementary if they add up to 90°



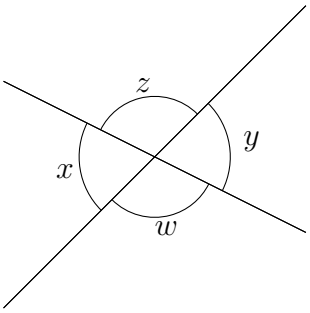
Supplementary Angles

Angles are supplementary if they add up to 180°



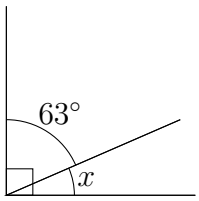
Opposite Angles

Opposite angles are the angles that are opposite each other when two lines intersect. Using what we know about supplementary angles, what property do opposite angles have?

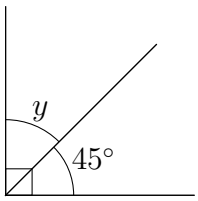


Example 1: Find the unknown complementary angle.

(a)

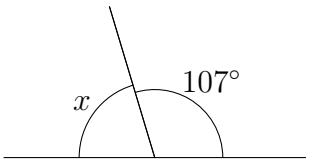


(b)



Example 2: Find the unknown angles.

(a)



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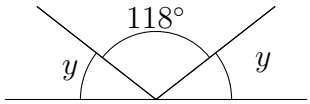

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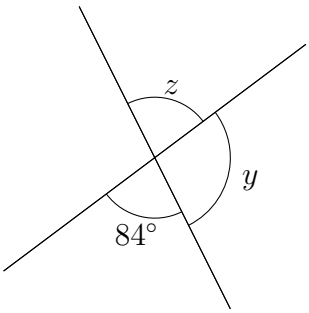
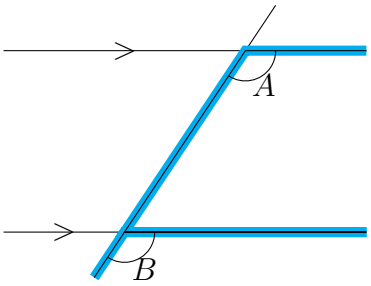


Parallel Lines and Angles

A traversal is a line that intersects two or more lines. If these lines are parallel, the angles around the lines have very nice properties.

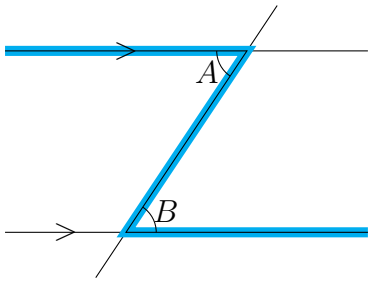
Corresponding Angles

Corresponding angles are angles that are on the same side of the traversal and on the same side of each parallel line. These angles are equal to each other. These types of angles are often referred to as the "F" pattern.



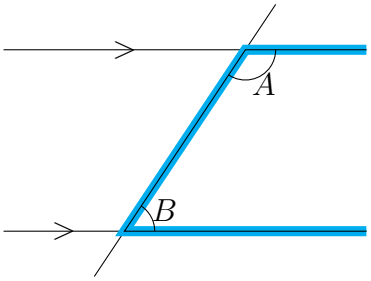
Alternate Angles

Alternate angles are angles that are on opposite sides of the traversal and on opposite sides of the parallel lines. These angles are equal to each other. These types of angles are often referred to as the "Z" pattern.



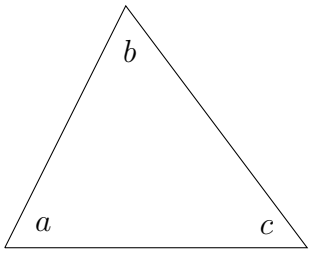
Co-interior Angles

Co-interior angles are angles on the same side of a traversal and on opposite sides of the parallel lines. These angles add up to 180°. This is sometimes referred to as the "C" pattern.



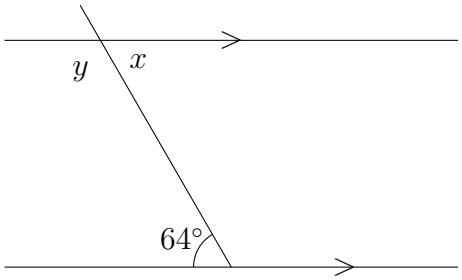
Interior Angles of a Triangle

Triangles have three interior angles. These three angles add up to 180°.



Example 3: Find all the missing angles in the diagrams and state the angle properties you used.

(a)



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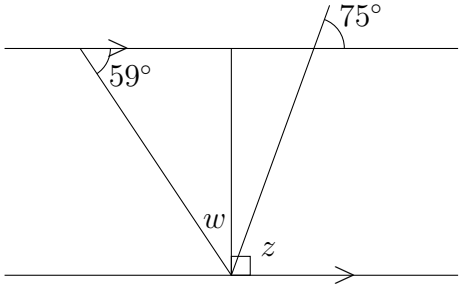
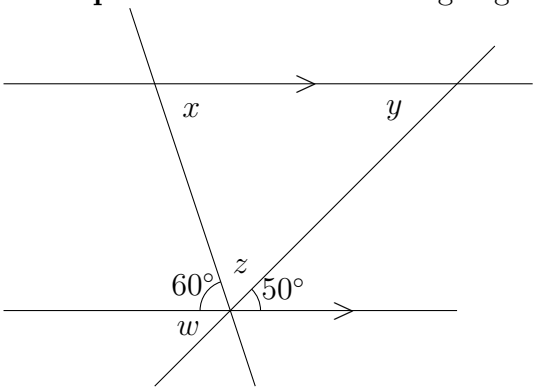
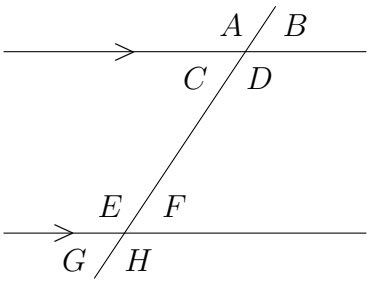


Example 4: Find all the missing angles w, x, y, and z in the diagram using angle properties.



Thinking Question

How many angles need to be known in order to determine all the angles in the diagram?



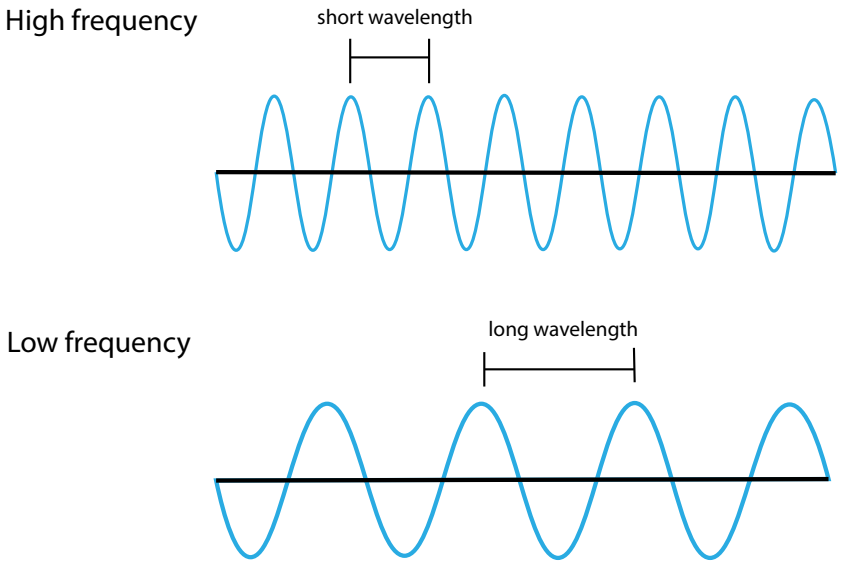
Light

Like sound, light travels in a wave. The light that we can see is only a small part of the electromagnetic spectrum. The electromagnetic spectrum is organized by frequency and wavelength. **Frequency** is the number of waves in a unit of time. **Wavelength** is the distance between corresponding points of two consecutive waves. This is, it is the distance it takes for the wave's shape to repeat.

Wavelength and frequency are inversely related. The higher the frequency, the shorter the wavelength. Also, the longer a wavelength, the lower the frequency. The relationship between wavelength and frequency also involves the speed of light and can be represented by the equation

$$c = \lambda f$$

where c is the speed of light, λ is the wavelength and f is the frequency.



To represent light in diagrams, we use a line with an arrow in the direction the light is heading. You can picture this as the beam of light that comes from a flashlight or laser. On diagrams, we also draw the normal. **The normal** is the direction perpendicular (at right angles) to a surface. It is drawn as a dashed perpendicular line at the point where the beam of light hits the surface.

What happens when a ray of light hits a flat mirror?

incident ray (I): _____

reflected ray (R): _____

angle of incidence (θ_i): _____

angle of reflection (θ_r): _____

point of incidence: _____

The Law of Reflection states that: _____

_____

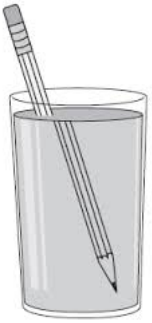
Example 5: A ray of light hits a flat mirror. It strikes the mirror with an angle of incidence of 40°.

(a) What is the angle of reflection?

(b) Draw a diagram to represent the situation. Label the normal (N), the incident ray (I), and the reflected ray (R).

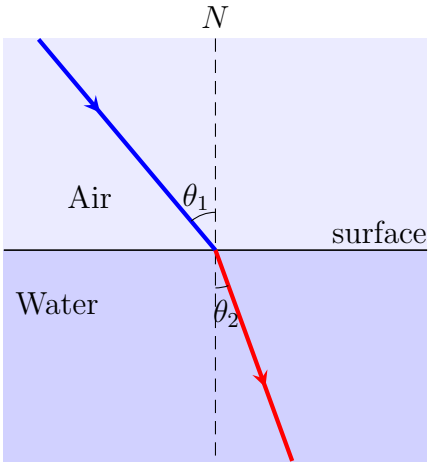
What happens when light travels from air to water?

When a pencil is placed in a cup of water on an angle, the pencil looks as if it has been broken into two pieces. This "Broken Pencil" phenomenon can be explained by refraction.

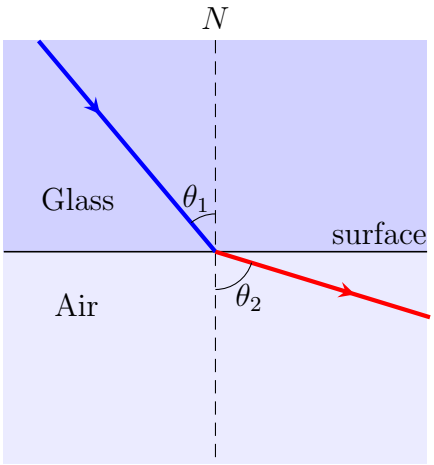


Retrieved from http://www.solpass.org/science4-5/light/print/5-3-light-released-questions-key.pdf?section=study-9

Refraction is the bending of light as it crosses the boundary separating two media. The light bends because it changes speed, either speeding up or slowing down depending on the material.



Light refracts towards the normal if the light wave passes from a medium where it travels fast to a medium where it travels slow (a medium that is more optically dense).



Light refracts away from the normal if the light wave passes from a medium in which it travels slow to a medium where it travels fast (a medium that is less optically dense).

How do we know how optically dense a medium is?

To determine the optically density of a medium, we use the refractive index (or index of refraction). This is the speed of light in a vacuum divided by the speed of the light in the medium. The greater the refractive index, the slower light travels in that medium. For example, the refractive index of water is 1.333 which means light travels 1.333 times slower in water than it does in air. Refractive indices of common media are stored in a table below.

Medium	Refractive Index (n)
Vacuum	1
Ice	1.31
Water	1.333
Olive Oil	1.47
Halite (rock salt)	1.516
Diamond	2.42

Example 6: Given two different media, state which one light travels faster in.

(a) Water and Diamond

(b) Olive Oil and Ice

(c) Water and Ice

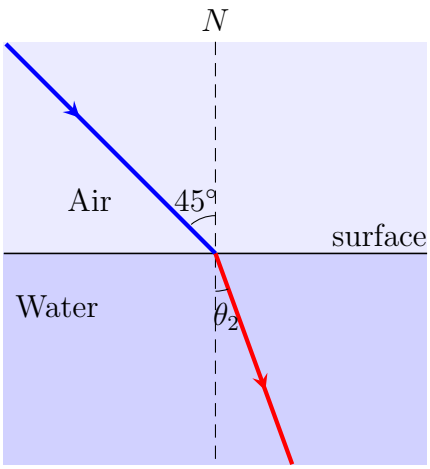
(d) Air and Halite

(e) Diamond and Olive Oil

Can we use the refractive index to determine the angle of refraction (θ_2)?
We can determine the angle of refraction (how much the light bends as it enters a new medium) by using Snell's Law. **Snell's Law** is stated as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where n_1 is the refractive index of medium 1 (the medium that the light ray starts in), θ_1 is the angle of incidence, n_2 is the refractive index of medium 2 (the medium that the light ray is refracted into), and θ_2 is the angle of refraction.



Let's use Snell's law on a specific case. We will use the example of the "broken pencil" and say that light is traveling from air to water with an angle of incidence of 45°.

Using the information we've been given, we get

$$1 \sin(45°) = 1.333 \sin \theta_2$$

(remember we assume air has the same refractive index as a vacuum). This equation can be solved. However, it can't be solved with the knowledge we have now. You will learn how to solve equations like this in high school. The solution is $\theta_2 = 32°$. This means that the angle of refraction is 32°.

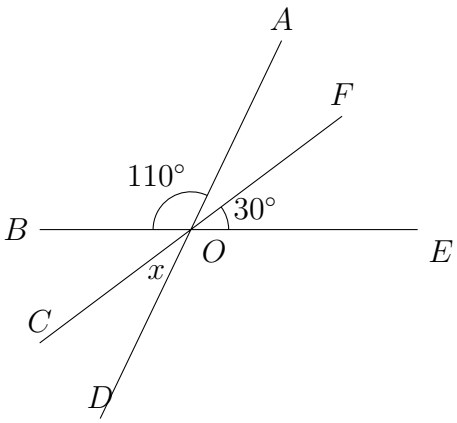
Problem Set

1. (a) What is the complementary angle of 57°?

   (b) What is the supplementary angle of 95°?

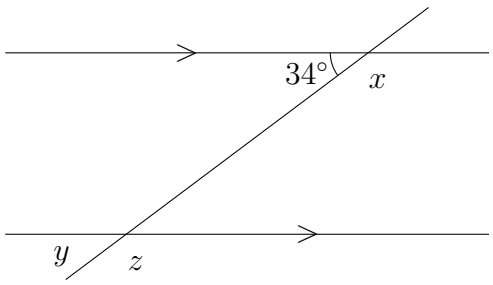
   (c) What is the opposite angle to 112°?

2. In the diagram, what is the value of x? *(Problems, Problems, Problems, Volume 6: page 40, question 4)*

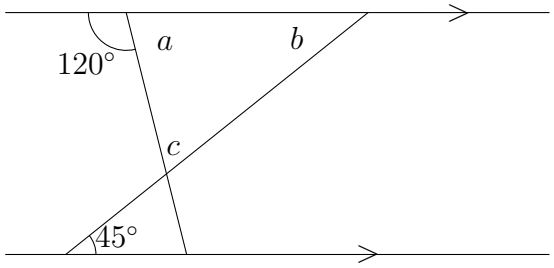


3. Find all the missing angles of the following diagrams and explain your reasoning.

   (a)



(b)



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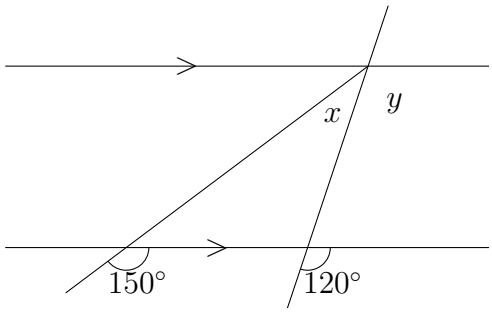

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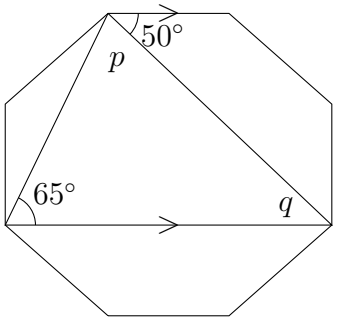
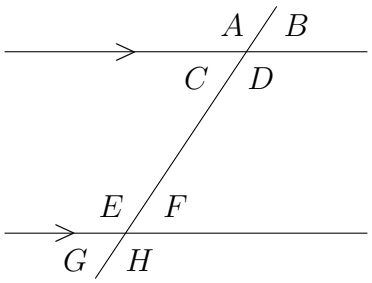


4. List all pairs of angles that are supplementary to each other in the diagram.

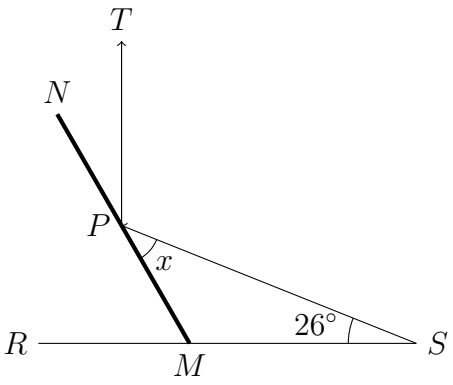


5. A ray of light is reflected off a flat mirror. The light is reflected off the mirror at an angle of 15° from the normal.

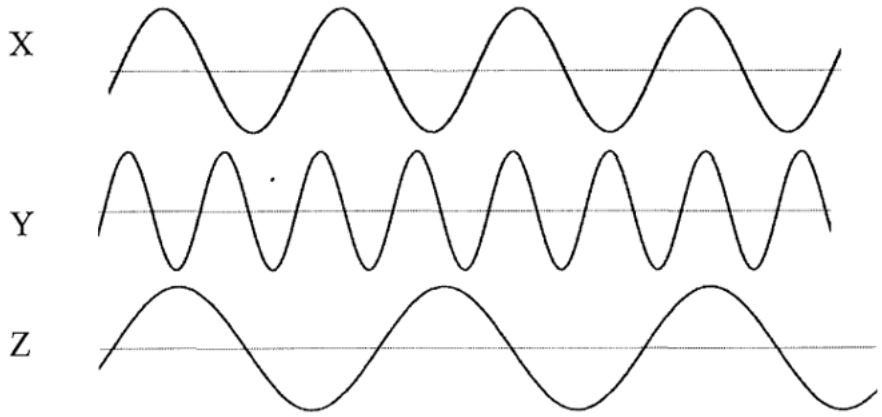
   (a) What is the angle of incidence?

   (b) Draw a diagram to represent the situation. Label the normal (N), the incident ray (I), and the reflected ray (R).

6. A beam of light shines from point S, reflects off a mirror MN at point P, and reaches point T so that PT is perpendicular to RS. What is the measure of $\angle SPM$? *(Problems, Problems, Problems, Volume 7: page 37, question 10)* **Hint:** Think about the Law of Reflection



7. List the waves (X, Y, and Z) in order from highest frequency to lowest frequency.

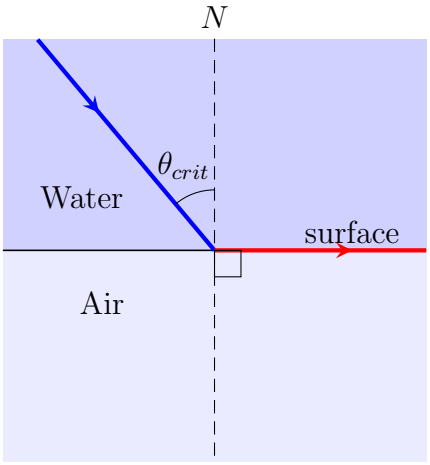


8. Sometimes the incident ray does not refract when it hits the boundary between two media. Instead, it is reflected back away from the boundary. This is called **total internal reflection** and occurs only if the following two statements are both true:

   (i) the light is coming from a more optically dense medium towards a less optically dense medium

(ii) the angle of incidence is greater than the critical angle

The **critical angle** (θ_{crit}) is the greatest angle at which light can strike the boundary between the two media without undergoing total internal reflection (being totally reflected into the first medium). For example, the critical angle from water to air is 48.6°. Given the critical angle and two media, state whether total internal reflection or refraction will occur and explain your reasoning.



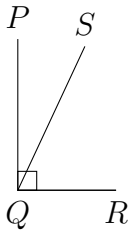
Note: To determine the relative optical densities of the media, use the table of refractive indices given in the lesson.

(a) Light travels from a diamond towards air at an angle of 16° from the normal. The critical angle is 24.4°.

(b) Light travels from water towards air at an angle of 54° from the normal. The critical angle is 48.6°.

(c) Light travels from ice to halite (the salt used to melt ice) at an angle of 42° from the normal. The critical angle is 59.8°.

9. * In the diagram, $\angle PQR$ is 90°, and $\angle RQS$ is 50° greater than $\angle PQS$. What is the measure of $\angle PQS$? *(Problems, Problems, Problems, Volume 1: page 5, question 2)*



10. ** When total internal reflection occurs and light is reflected back when it hits the boundary, do you think the angle of incidence will be equal to the angle of reflection? Why or why not? Use the knowledge gained from the lesson and the description of total internal reflection in question 8.