UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

Fall 2018 - October 23/24/25

Qualitative Graph Analysis

What do we mean when we say Qualitative Graph Analysis? Let's look at the words separately. Qualitative means to describe something based on size, appearance, value, etc. A graph is a method of organizing and presenting information. Analysis refers to a detailed examination of the parts or structure of something. When we analyze things, we break them down into smaller parts and try to understand their meaning. We also try to connect the smaller parts together to further understand the whole.

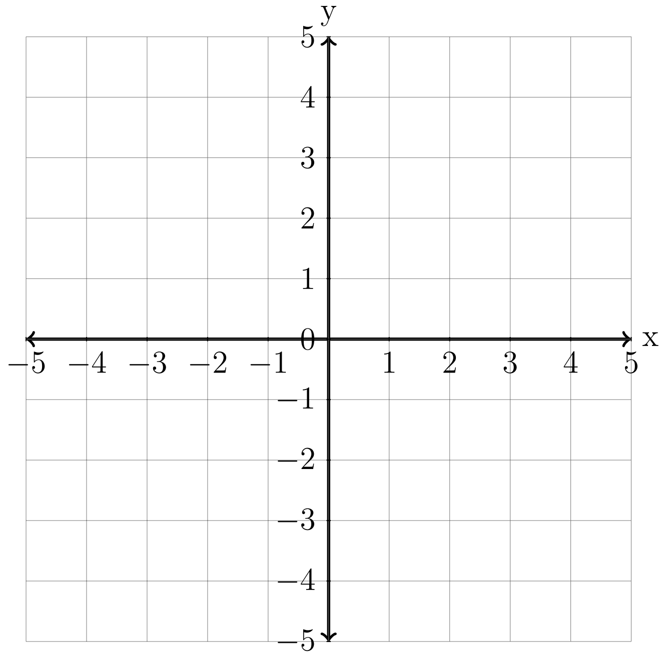
So, if we reflect on the words we just defined, what is qualitative graph analysis? Qualitative graph analysis involves looking at the appearance and size of graphs (and their parts) and trying to understand what is being represented. You might think that the analysis part sounds rather difficult but, we just did analyze something. We analyzed the title, Qualitative Graph Analysis, and discovered what we would be learning about today.

Plotting Points on the Cartesian Plane

Before we begin looking at graphs we need to understand how to plot points in the Cartesian plane. The Cartesian plane is formed by 4 quadrants. These four quadrants are separated by two axes, the x axis (horizontal) and the y axis (vertical).

In the Cartesian plane we can plot points that have negative values. Say we have a point $P = (x, y)$:

- If $x > 0$ and $y > 0$, then P is in the __**first**__ quadrant
- If $x > 0$ and $y < 0$, then P is in the __**fourth**__ quadrant
- If $x < 0$ and $y > 0$, then P is in the __**second**__ quadrant
- If $x < 0$ and $y < 0$, then P is in the __**third**__ quadrant



Plotting points with negative values is simple. You still travel horizontally along the x axis before traveling vertically on the y axis to find your point, but if the point $P = (x, y)$ has a negative value for x then you move left instead of right. If y has a negative value then you move down instead of up.

Example 1: Plot and label the following points on a Cartesian plane using the grid below.

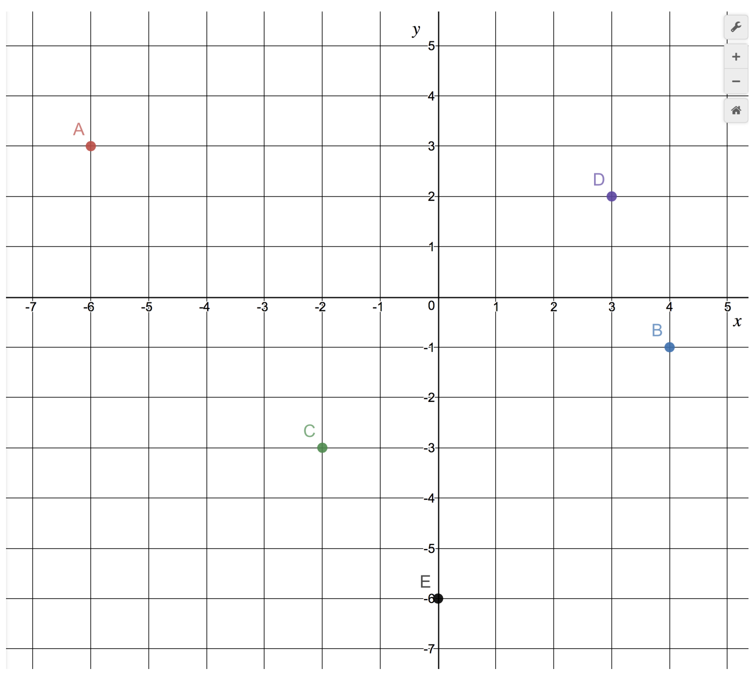
(a) A = (-6,3)

(b) B = (4,-1)

(c) C = (-2,-3)

(d) D = (3,2)

(e) E = (0,-6)

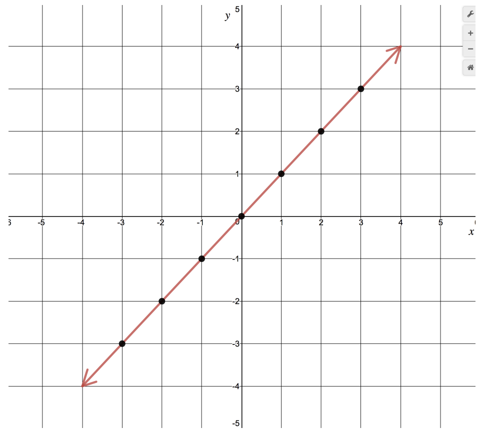


Common Patterns in Graphs

Linear

A graph that is linear has data points that when connected, form a line. The most basic line that can be drawn has points which are represented by the equation $y = x$.

x	$y = x$
-3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3



State whether this linear graph increases or decreases. On what interval? Increases on the interval $(-\infty, +\infty)$.

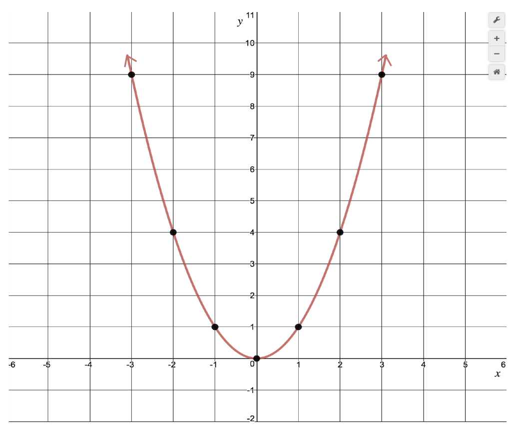
Is there a maximum value. If so, what is it? There is no maximum.

Is there a minimum value. If so, what is it? There is no minimum.

Quadratic

A graph that is quadratic has data points that when connected, form a parabola. The most basic parabola that can be drawn has points which are represented by the equation $y = x^2$.

x	$y = x^2$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



Describe the shape of a parabola. A parabola is U-shaped.

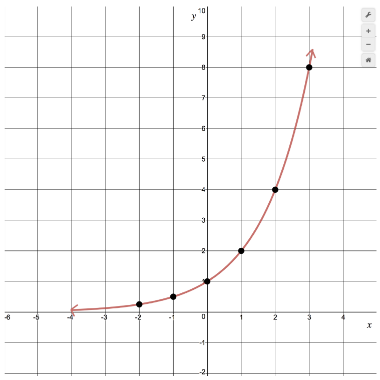
State whether this graph increases or decreases. On what interval? Increases on the interval $[0, +\infty)$. Decreases on the interval $(-\infty, 0]$.

Is there a maximum value. If so, what is it? There is no maximum

Is there a minimum value. If so, what is it? The minimum value is 0.

Exponential

Given the following points, plot an exponential graph on the grid below: (-2,0.25), (-1,0.5), (0,1), (1,2), (2,4), (3,8)



Special features of exponential graphs:

- approaches the x axis but never touches it
- doesn't increase constantly (increases rapidly)

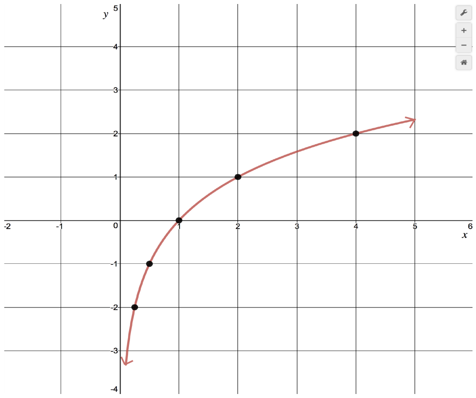
State whether this graph increases or decreases. On what interval? Increases on the interval $(-\infty, +\infty)$.

Is there a maximum value. If so, what is it? There is no maximum.

Is there a minimum value. If so, what is it? There is no minimum.

Logarithmic

Given the following points, plot a logarithmic graph on the grid below: (0.25,-2), (0.5,-1), (1,0), (2,1), (4,2)



Special features of logarithmic graphs:

- approaches the y axis but never touches it
- doesn't increase constantly (increases slowly)

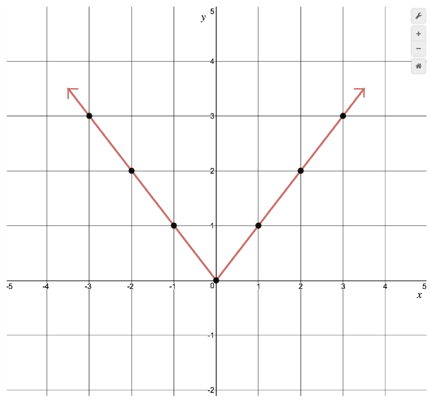
State whether this graph increases or decreases. On what interval? Increases on the interval $(0, +\infty)$.

Is there a maximum value. If so, what is it? There is no maximum.

Is there a minimum value. If so, what is it? There is no minimum.

Absolute Value

Given the following points, plot an absolute value graph on the grid below: (-3,3), (-2,2), (-1,1), (0,0), (1,1), (2,2), (3,3)



Special features of absolute value graphs:

- looks like a "V"
- increases or decreases constantly

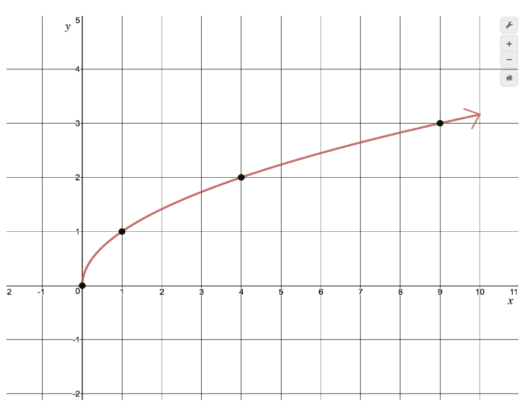
State whether this graph increases or decreases. On what interval? Increases on the interval $[0, +\infty)$. Decreases on the interval $(-\infty, 0]$.

Is there a maximum value. If so, what is it? There is no maximum.

Is there a minimum value. If so, what is it? The minimum value is 0.

Square Root

Given the following points, plot a square root graph on the grid below: (0,0), (1,1), (4,2), (9,3)



Special features of square root graphs:

- has a definite end (has no x values less than zero)
- doesn't increase constantly (increases slowly)

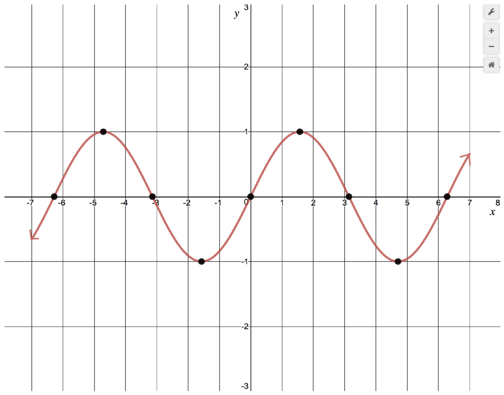
State whether this graph increases or decreases. On what interval? Increases on the interval $[0, +\infty)$.

Is there a maximum value. If so, what is it? There is no maximum.

Is there a minimum value. If so, what is it? The minimum value is 0.

Trigonometric Sine

Given the following points, plot a trigonometric sine graph on the grid below:
(-6.28, 0), (-4.71,1), (-3.14,0), (-1.57,-1), (0,0), (1.57,1), (3.14,0), (4.71,-1), (6.28,0)



Special features of trigonometric sine graphs:

- has a wave pattern that repeats itself
- y values range from -1 to 1

State whether this graph increases or decreases. The graph both increases and decreases.

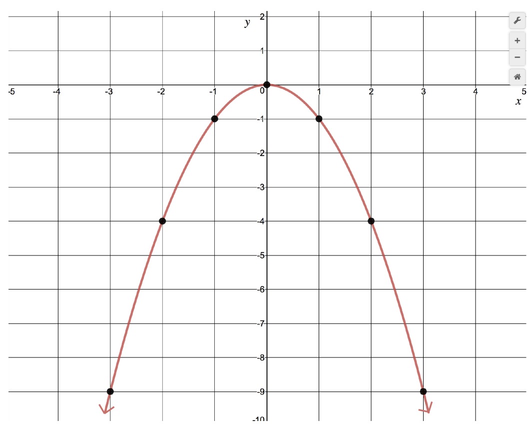
Is there a maximum value. If so, what is it? The maximum value is 1.

Is there a minimum value. If so, what is it? The minimum value is -1.

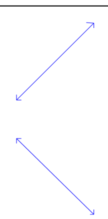
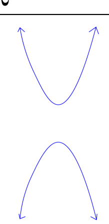
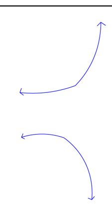
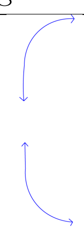
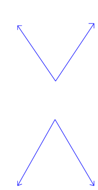
Example 2: Use the table of values to determine points to plot on the graph below. Connect the points and state which pattern the graph has (linear, etc).

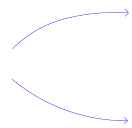
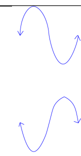
x	$y = -x^2$
-3	-9
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9

Quadratic



Example 3: In the table below draw the shapes the graphs take.

Graph	Linear	Quadratic	Exponential	Logarithmic	Absolute Value
Shapes					

Graph	Square Root	Trig (Sine)
Shapes		

Analyzing Graphs from Real-Life

Graph 1

When is the graph increasing? The graph is increasing on the interval $[0, 0.625]$.

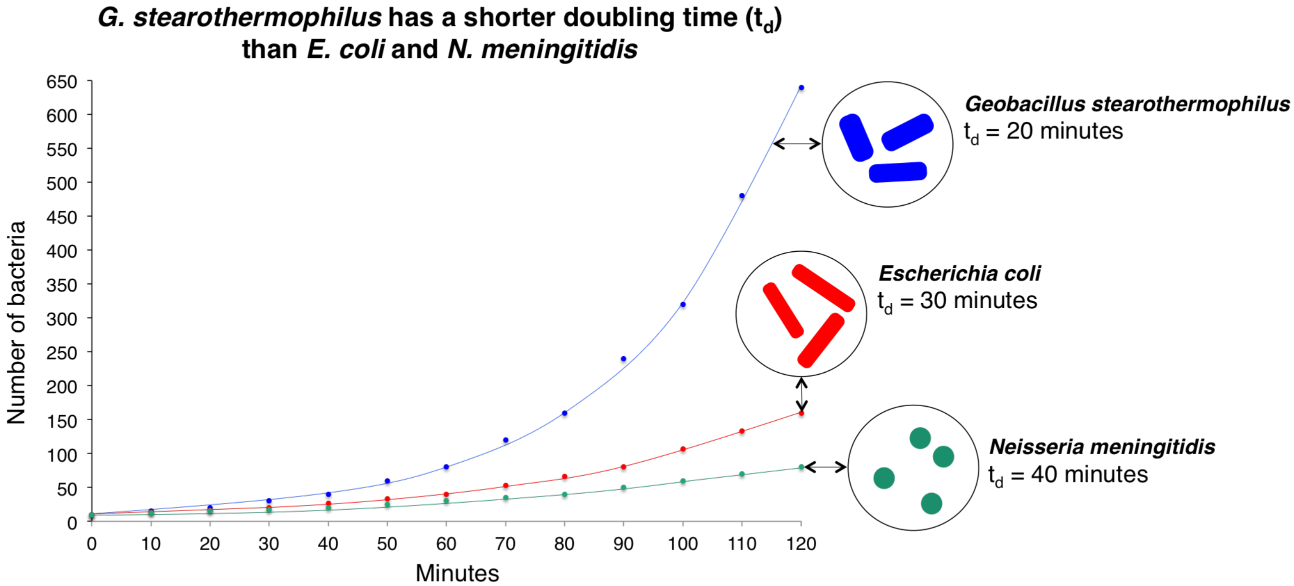
When is the graph decreasing? The graph is decreasing on the interval $[0.625, 2.5]$.

Is there a maximum value? What does it represent? The maximum value is 56.25 and it is the maximum height of 56.25 ft.

What graph pattern does this you remind you of? A quadratic graph because it has the shape of a parabola but flipped upside down.

What could this graph be modeling? Why might the graph start with a height of 50 ft? This graph could be modeling the height of a ball when it is thrown. The graph might start at 50 ft because the ball is thrown off a building or hill that is 50 ft tall.

Graph 2



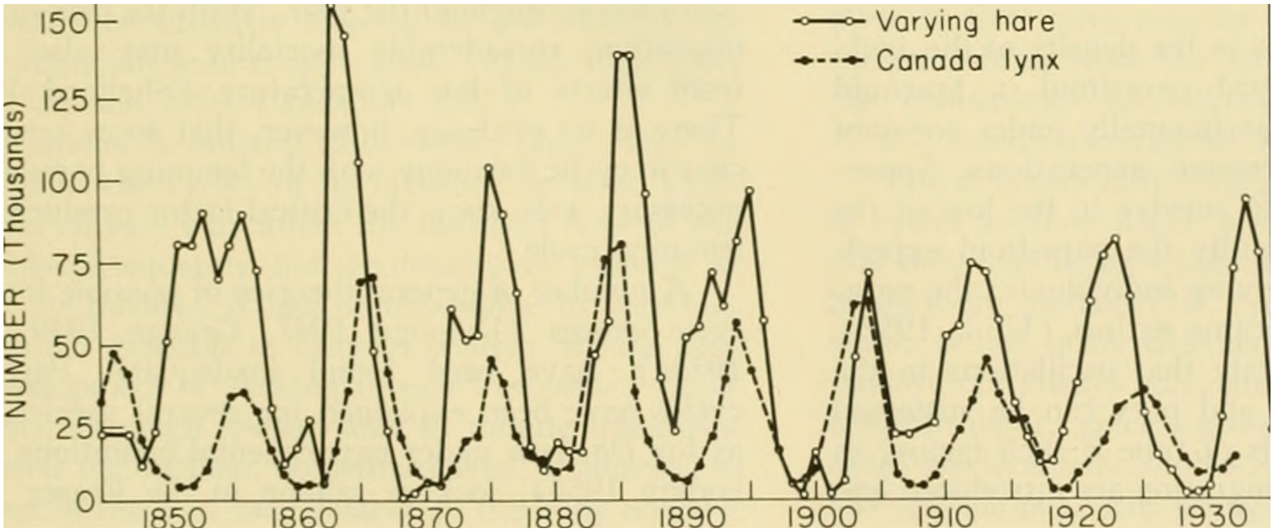
When is the graph increasing? Is it increasing at a constant rate? The graph is increasing on the interval $[0, +\infty)$. It is not increasing at a constant rate.

When is the graph decreasing? The graph is never decreasing.

Is there a minimum value? What does it represent? There is a minimum value. It represents the initial or starting population of the bacteria.

What graph pattern does this you remind you of? An exponential graph.

Graph 3



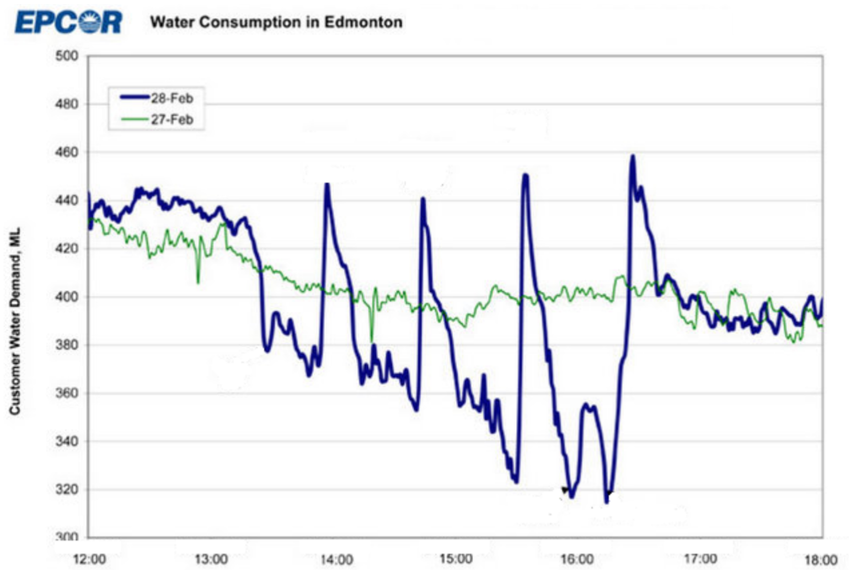
Sometimes graphs have harder patterns to analyze. Analyze this graph. Can you find any patterns? **Hint:** Look at how and when the graph increases and decreases.

Both the hare population and lynx population show a consistent trend of increase and decrease. Looking closer we see that when the hare population increases, the lynx population increases shortly after and when the lynx population increases, the hare population begins to decrease. When the hare population is smaller, the lynx population decreases. Soon after the lynx population starts to decrease, the hare population starts to increase. After this the cycle starts again.

What year had the most hares? What year had the most lynxes? 1866 was the year with the most hares. 1885 was the year with the most lynxes.

What words describe the type of relationship the Canada Lynx and Varying Hare have? The Canada Lynx and Varying Hare have a predator-prey relationship.

Graph 4



This graph shows the use of water in Edmonton on two days in February 2010.

What is different about the way water was used on February 27 and February 28? The water use was much mored varied on Feb. 28 than on Feb. 27.

At 14:45, what is the water demand approximately? At 14:45 the water demand is approximately 352 ML/min.

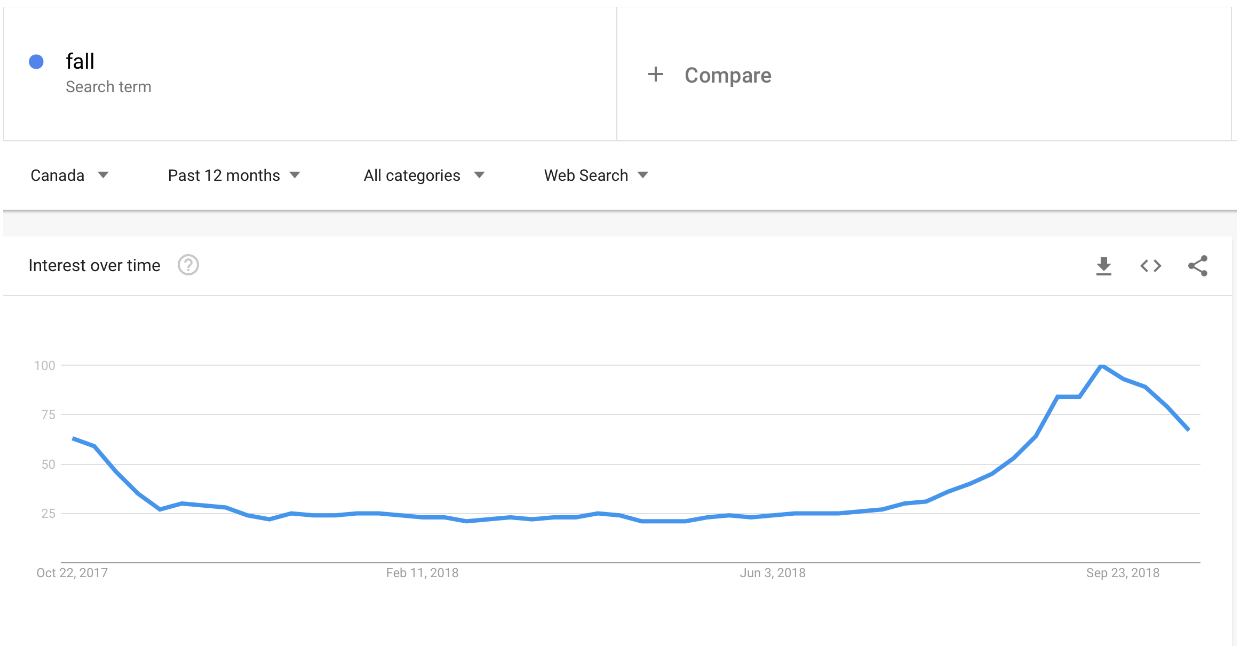
In the graph, the vertical axis begins at 300 ML/min, rather than 0 ML/min. How do you think the look of the graph would change if its scale went from 0 to 500 ML/min instead of starting from 300 ML/min? Why do you think it was presented this way?
If the vertical axis ranged from 0 to 500 ML/min, the ups and downs of the graph would not look as extreme, since they would only span one quarter of the vertical space of the graph. By showing a vertical range from 300 to 500 ML, the graph exaggerates the variations in water use during the hockey game. This technique of "selective scale" is often used with graphs in the media, in order to focus on trends and changes. By studying the scale of a graph carefully, you can interpret it more accurately.

What might be the reason that the use of water was so unusual on February 28, 2010? On Sunday, Feb 28, 2010, Canada won a gold medal in Olympic men's hockey. Epcor, Edmonton's water provider, noticed a significant change in water use patterns during the game than on a typical weekend. The graph correlates the water use patterns to the key points of the game (like the end of the first period, etc).

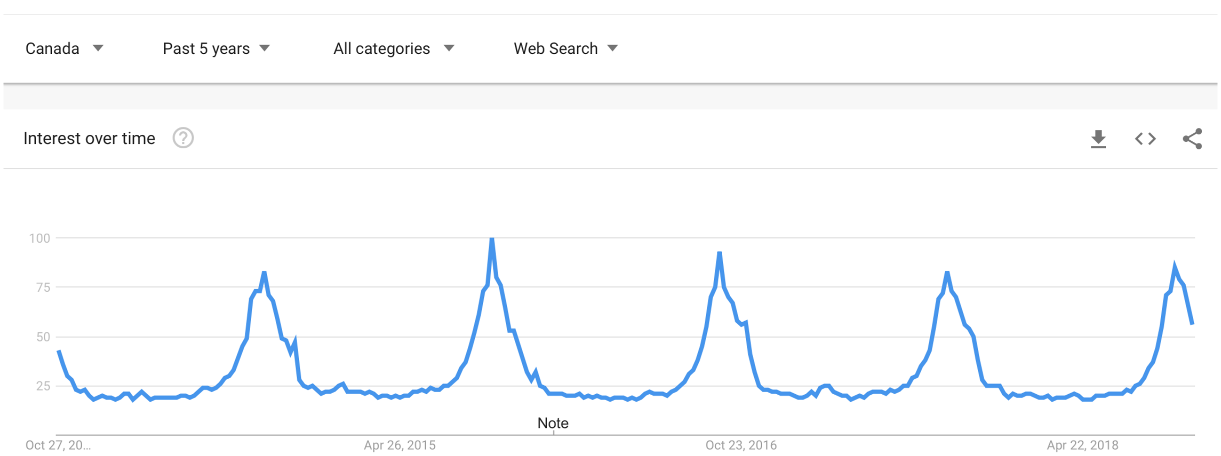
Google Trends

Google trends is a website that contains information about terms being searched on the Google search engine. All you have to do is type in a word or two and it will give you the statistics about how much that term has been searched. This information is presented in a graph. These statistics can be restricted by country and by the time period. Here are a couple examples.

Example A: If we look at the popularity of the term "fall" in the past 12 months, we get a graph that looks like this

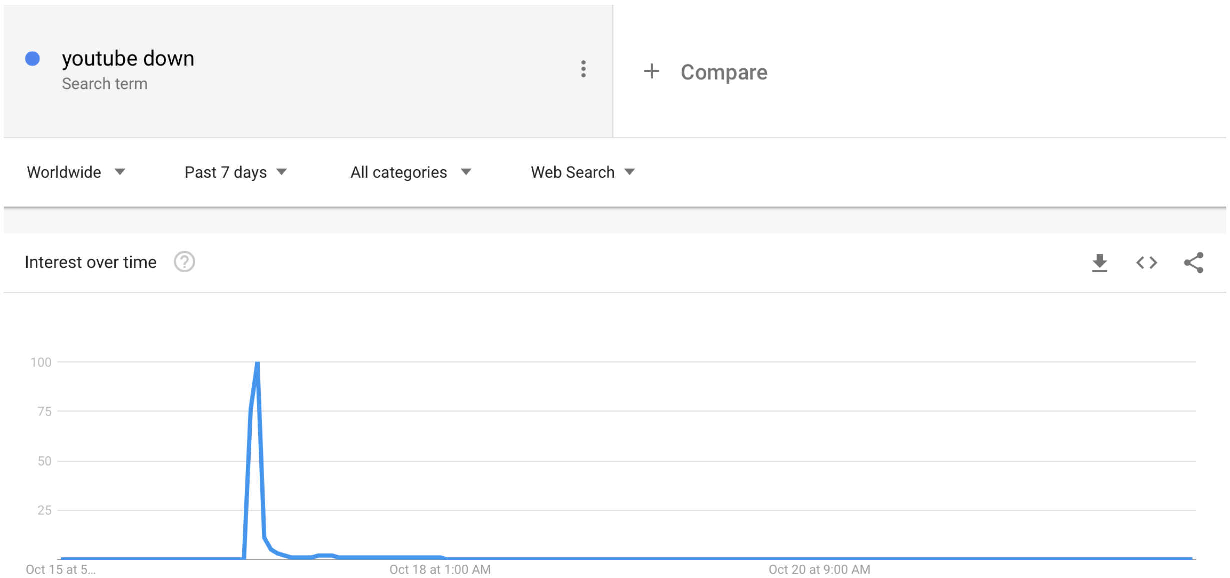


But if we change the timeline to the past five years, we get



This shows us that there is a pattern of people searching “fall” every year near/during Fall.

Example B: Just last week YouTube’s website crashed and was unable to play videos. Let’s look at the worldwide results for “youtube down” in the past seven days.



Between October 15, 2018 and October 18, 2018 we can see the spike where YouTube’s website was probably not working.

Problem Set

1. State which quadrant the point is in.

   (a) A = (6,-7) Fourth

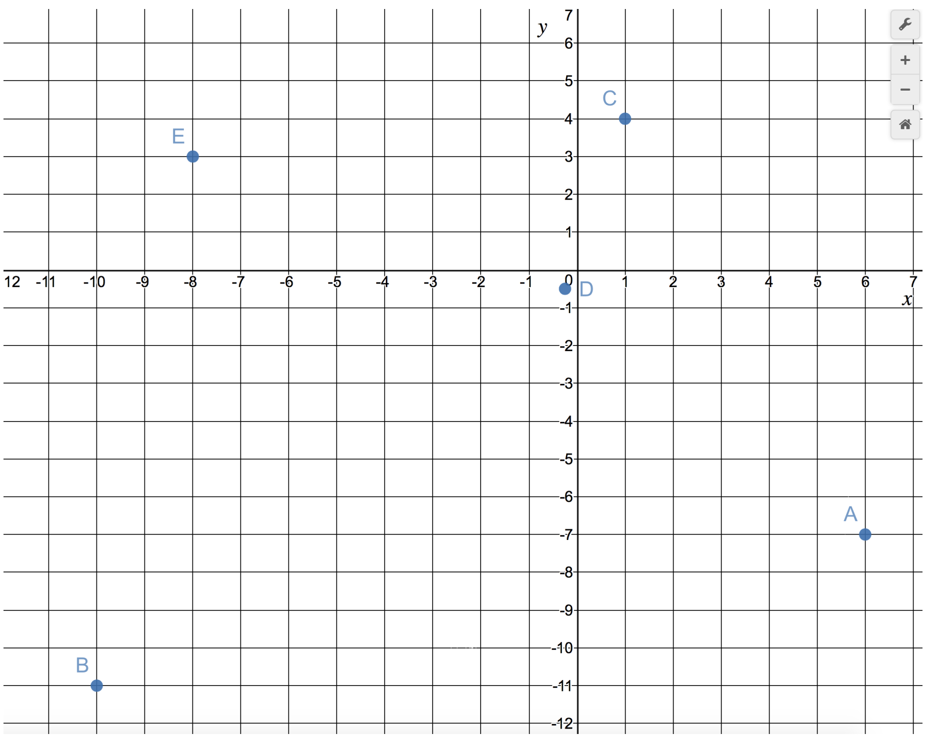
   (b) B = (-10,-11) Third

   (c) C = (1,4) First

   (d) D = (-0.25,-0.5) Third

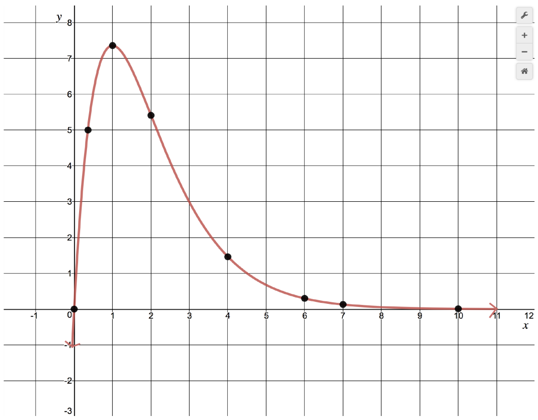
   (e) E = (-8,3) Second

2. Graph the points from question one on a Cartesian plane. Remember to label the x and y axes.



3. *Surge Graph*

Given the following points, plot a surge graph on the grid below: (0,0), (0.36,5), (1,7.36), (2,5.41), (4,1.46), (6,0.3), (7,0.13), (10,0.009). Make note of any special features that a surge graph has.



Special features of surge graphs:

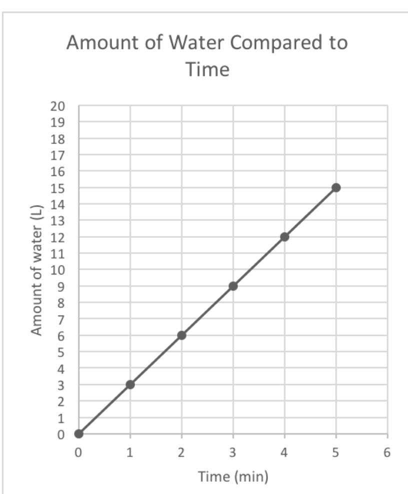
- as x goes to $+\infty$, the y values get closer and closer to zero
- has a bump at the beginning

(a) State whether this graph increases or decreases. On what interval does it? Increases on the interval $(-\infty, 1]$ and decreases on the interval $[1, +\infty)$.

(b) Is there a maximum value? If so, what is it? There is a maximum value. It is 7.36

(c) What are some real-life situations or phenomena that a surge graph could be used to represent? Surge graphs are often used to represent the concentration of a drug in the body versus hours since consumption. They can also be used to represent the body's reaction to an infection.

4. **Extrapolation** is estimating or concluding something by assuming that existing trends or patterns will continue. In mathematics, we do this by extending a graph (with a dashed line) past the data we already know. How much water is there at six minutes?



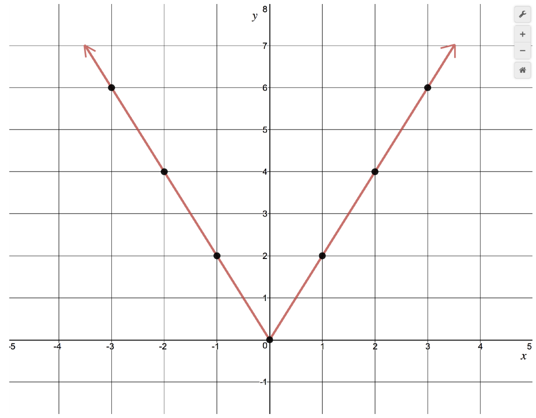
Extending the line on the graph, we get that at six minutes there will 18 L of water.

5. Using the table of values, plot the points the data represents and state which pattern the graph has (linear, etc).

   (a)

x	y
-3	6
-2	4
-1	2
0	0
1	2
2	4
3	6

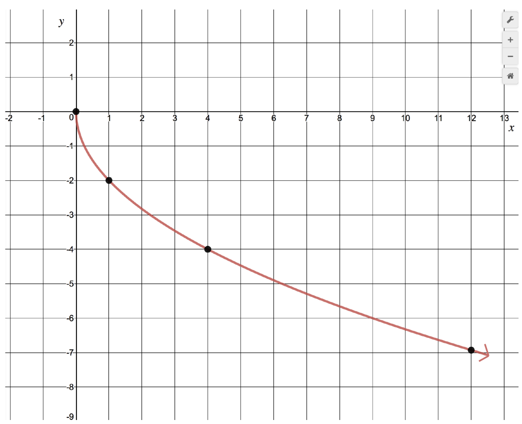
Absolute Value



   (b)

x	y
0	0
1	-2
4	-4
12	-6.93

Square Root



6. Choose three terms that you think will show an interesting graph on Google Trends for Canada in the past 12 months.
   Answers will vary for a and b. As such, the following sample solution only answers the question for one term instead of three.

   (a) What do you think each graph will look like? Why?
   I think the graph for fortnite will show that it is a extremely trending term. Thus, the graph will be increasing and have high values. I think this because fortnite has had increasing popularity in the news and other media. It is also one of the more popular online games out today.

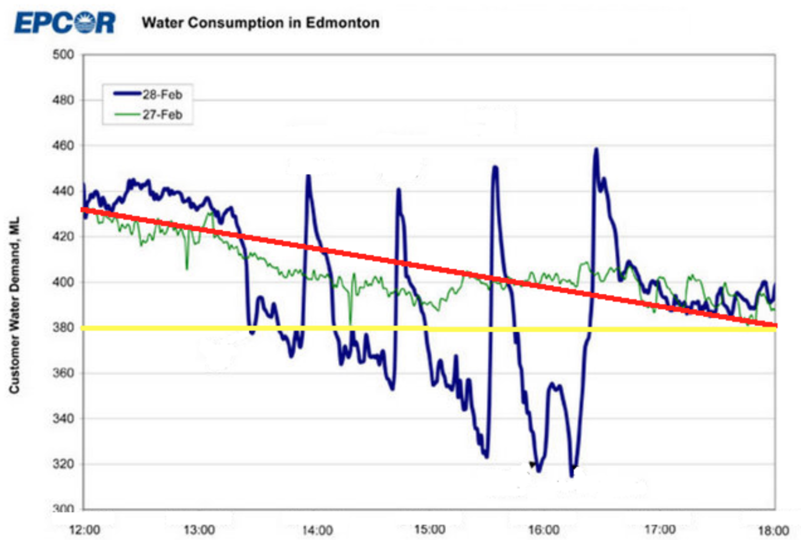
   (b) Use a computer and go to **https://trends.google.com/trends/?geo=CA** to see the actual graphs. Are they like you expected them to be? Why or why not?

For the most part the graph looks like I expected it to. However, there is more variability than I expected. This is probably due to fortnite releasing new features every once in a while that makes people who play or search fortnite online more interested.

(c) If you ran these terms on the United States instead of Canada do you expect the graph to look different? Why or why not?
I do not expect the graph to look that different because Canada and the United States have very similar video game cultures.

7. *** Using Graph 4, approximate the total water used on February 27 between 12:00 and 16:00. **Hint:** Pay attention to the scale of the graph
The total water use between 12:00 and 16:00 is represented by the area under the green “curve”. It is not trivial to find the area under such a strangely-shaped curve. Calculus gives techniques for determining the area under some familiar curves if we can recognize their shape or equation, but in this case we have neither a familiar shape nor an equation.

One way to approximate the water use on Feb. 27 between 12:00 and 16:00 is to draw a straight line that is close to the shape of the graph. That line could start at 430 ML at 12:00 and end at 380 ML at 16:00. It appears as if the curve is both above and below the line in equal proportions. The total water use is approximated by the area under the red line. That area consists of two regions: a right-angled triangle between the red (upper, “diagonal”) line and the yellow (horizontal) line, and a rectangle below the yellow line.



We must also consider the units of the graph. The vertical axis of the graph as published by Epcor is labelled as ML. Interpreting the graph this way would mean that at any particular instant we can measure the volume of water used. In fact, we must measure the water used at regular units of time. Epcor clarified that the points on this graph represent the volume of water used in each one-minute interval. The unit for the vertical axis should actually read ML/min.

The base of the triangle has a length of 4 hours = 240 minutes, while the height of the triangle is 50 ML/min. The area of the triangle is thus

$$\frac{1}{2}(240 \text{ min}) \times 50 \text{ ML/min} = 6000 \text{ ML}$$

The rectangle has a width of 240 min and a height of 380 ML/min, for an area of

$$240 \text{ min} \times 380 \text{ ML/min} = 91200 \text{ ML}$$

The total water use on Feb. 27 from 12:00 to 16:00 is thus 6000 + 91 200 = 97 200 ML.

Note that if we had left the vertical axis unit as ML, our calculation would give as a unit the ML hour, when what we actually want is a volume of water, measured in ML. This is a good lesson in careful interpretation of data presented in the media.