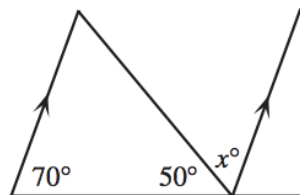


Intermediate Math Circles February 13: Contest Preparation Problems (Geometry)

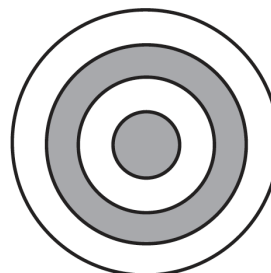
February 15, 2019

1. (#6, 2002 Pascal Contest)
Find the value of x .



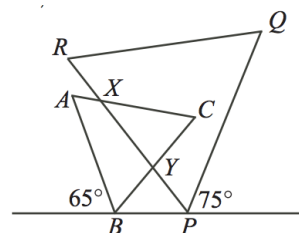
2. (#17, 2003 Cayley Contest)

In the diagram, the four circles have a common centre, and have radii of 1, 2, 3, and 4. What is the ratio of the total area of the shaded regions to the area of the largest circle?



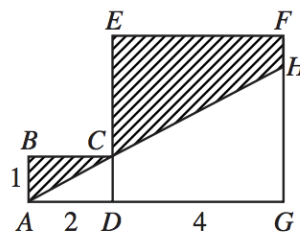
3. (#15, 2007 Cayley Contest)

In the diagram, $\triangle ABC$ and $\triangle PQR$ are equilateral. Find $\angle CXY$.



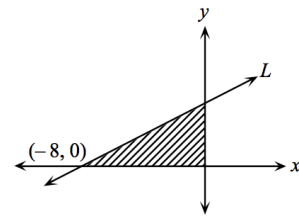
4. (#18, 1998 Pascal Contest)

In the diagram, $DEFG$ is a square and $ABCD$ is a rectangle. A straight line is drawn from A , passes through C , and meets FG at H . Find the area of the shaded region.



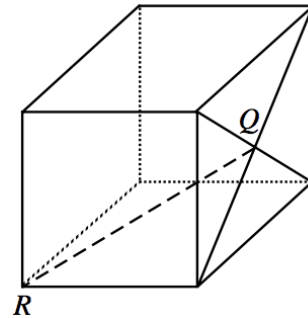
5. (#15, 2001 Pascal Contest)

The line L crosses the x -axis at $(-8, 0)$. The area of the shaded region is 16. What is the slope of the line L ?



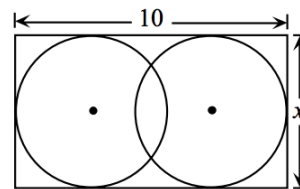
6. (#21, 1998 Pascal Contest)

Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. Find the length of QR .



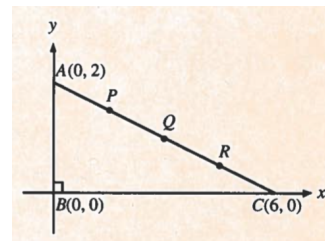
7. (#19, 2001 Pascal Contest)

Two circles with equal radii are enclosed by a rectangle, as shown. The distance between their centres is $\frac{2x}{3}$. Find x .



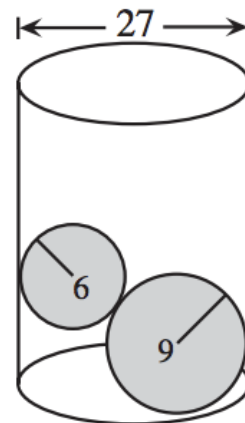
8. (#20, 1995 Cayley Contest)

Points P , Q , and R divide side AC into four equal parts. Calculate the slope of BR .



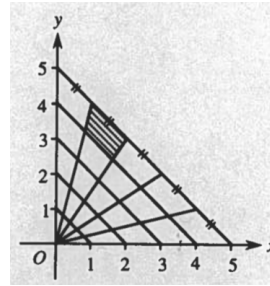
9. (#24, 2002 Cayley Contest)

A cylinder, which has a diameter of 27 and a height of 30, contains two lead spheres with radii 6 and 9, with the larger sphere sitting on the bottom of the cylinder, as shown. Water is poured into the cylinder so that it just covers both spheres. What is the volume of water required? [Give your answer in exact form.]



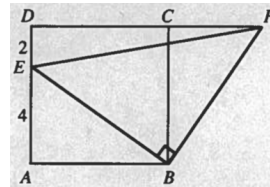
10. (#22, 1990 Cayley Contest)

The five marked segments are equal in length. Find the area of the shaded region.



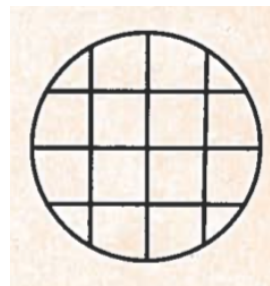
11. (#24, 1990 Cayley Contest)

$ABCD$ is a square. If $EB \perp BF$, $DE = 2$, and $AE = 4$, what is the area of $\triangle EBF$?



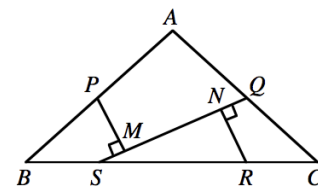
12. (#19, 1995 Cayley Contest)

Two perpendicular diameters are drawn in a circle of radius 2. All possible chords parallel to and at a distance of 1 unit from these diameters are drawn. What is the total length of the six chords in the diagram (including the diameters)? [Note: A *chord* is a line segment connecting two points on the circumference of a circle.]



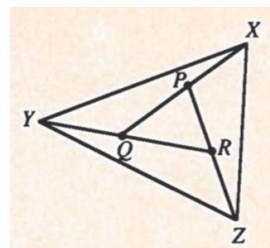
13. (#25, 2000 Pascal Contest)

$\triangle ABC$ is an isosceles triangle in which $AB = AC = 10$ and $BC = 12$. The points S and R are on BC such that $BS : SR : RC = 1 : 2 : 1$. The midpoints of AB and AC are P and Q respectively. Perpendiculars are drawn from P and R to SQ meeting at M and N respectively. What is the length of MN ?



14. (#21, 1995 Cayley Contest)

$\triangle PQR$ has side QP extended to X so that $QP = PX$, PR extended to Z so that $PR = RZ$, and RQ extended to Y so that $RQ = QY$. If the area of $\triangle XYZ$ is 420, what is the area of $\triangle PQR$?



15. (#23, 1995 Cayley Contest) Three circles are tangent to the line QR and to each other as shown. The two larger circles each have the same radius. What is the ratio of the radius of the smaller circle to the radius of one of the larger circles?

