



Grade 6 Math Circles
February 12th/13th
Angles

Warm-Up

Draw and label an example of the following:

- An Acute Angle
- A Right Angle
- An Obtuse Angle
- A Straight Angle
- A Reflex Angle
- A Full Rotation

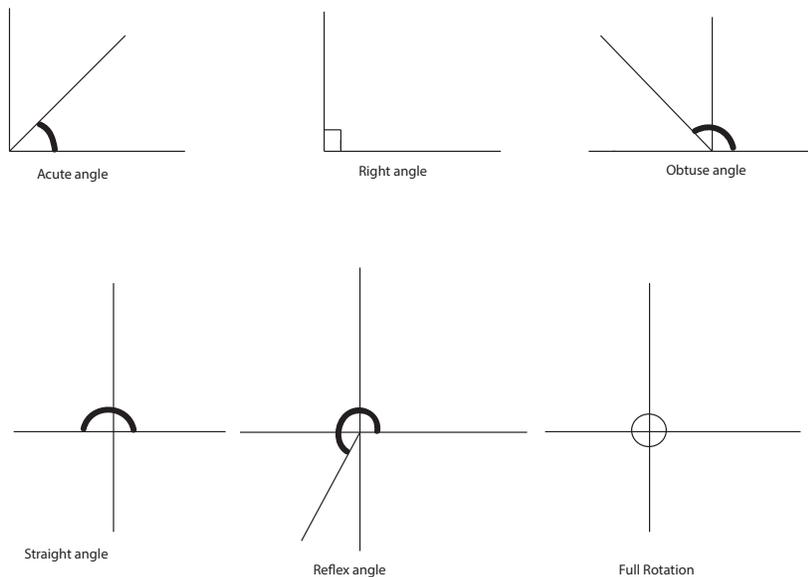
See the lesson below

Types of Angles

As can be seen in the warm-up, the space between two lines measured from a point of intersection is called an **angle**. We can classify angles based on their size:

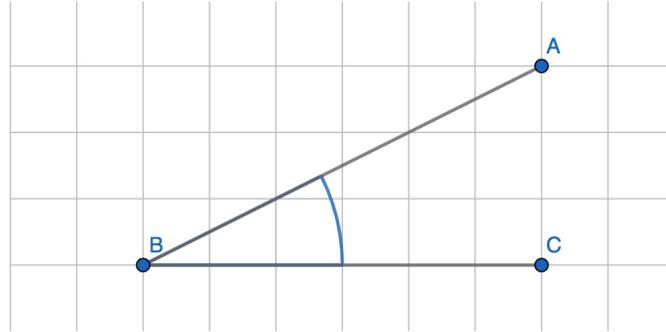
Angle	Classification
less than 90°	Acute angle
$= 90^\circ$	Right angle
greater than 90° and less than 180°	Obtuse angle
$= 180^\circ$	Straight angle
greater than 180° and less than 360°	Reflex angle
$= 360^\circ$	Full rotation

Here are some examples of what these angles look like:



Labelling Angles

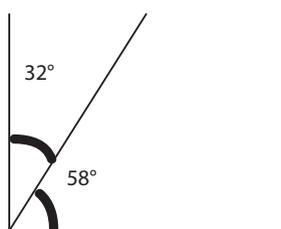
Angles are labeled according to the three points involved, in order in-middle-out. For example, the angle below could be called either $\angle ABC$ or $\angle CBA$.



Classifying Angles

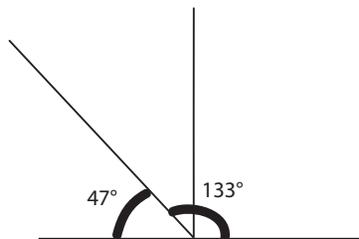
Additionally, there are some other ways that we classify angles:

- **Complementary angles** are two angles that add up to 90° . These angles can be beside each other or apart. When complementary angles are together, they make a right angle.
- **Supplementary angles** are two angles that add up to 180° . They can also be beside each other or apart. When supplementary angles are together, they make a straight angle.
- In addition, the angles around any intersection point always add up to 360° . The **full rotation** around a point makes a circle, so naturally it is 360° .



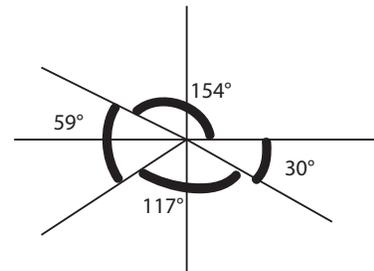
$$58^\circ + 32^\circ = 90^\circ$$

Complementary angles



$$133^\circ + 47^\circ = 180^\circ$$

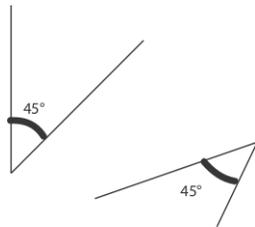
Supplementary angles



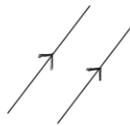
$$154^\circ + 59^\circ + 117^\circ + 30^\circ = 360^\circ$$

Full rotation

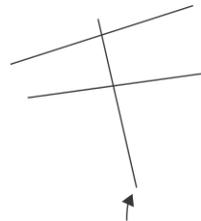
- **Congruent** angles mean that angles are of equal measure, even if they are not in the same place.
- Two lines are **parallel** if they never meet and are always the same distance apart. When looking at a diagram, if there are arrows along two or more lines, those lines are parallel to one another.
- A **transversal** is a line that crosses at least two other lines.



Congruent angles

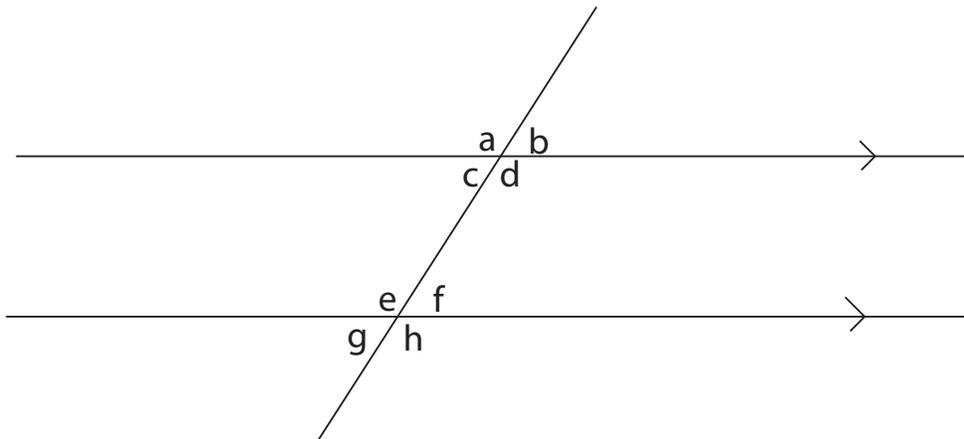


Parallel lines



Transversal

Angle Properties



Alternate angles are angles that are on opposite sides of the transversal, but in between the lines. When the lines are parallel, alternate angles are congruent.

In the diagram above, d and e are alternate angles, and c and f are alternate angles.

Which angles from the diagram above satisfy this property?

$$d = e \text{ and } c = f$$

The easiest way to think of alternate angles is the “**Z pattern**”. If you draw the letter “z” from either direction of the top parallel line to the bottom, the angles which represent the insides of the “z” are congruent.

Corresponding angles are angles that are on the same side of the transversal, and on matching sides of their own line. When the lines are parallel, corresponding angles are congruent.

In the diagram above, a and e are corresponding angles, b and f are corresponding angles, c and g are corresponding angles, and d and h are corresponding angles.

Which angles from the diagram above satisfy this property?

$a = e$, $b = f$, $c = g$ and $d = h$

The easiest way to think of corresponding angles is the “**F pattern**”. If you draw the letter “F”, backwards or forwards, on the transversal, then the angles on top or below the horizontal lines will be congruent.

Opposite angles are angles that are opposite each other when two lines cross. Opposite angles are equal. In the diagram above, a and d , b and c , e and h , and f and g are all opposite angles.

Which angles from the diagram above satisfy this property?

$a = d$, $b = c$, $e = h$, and $f = g$

The easiest way to think of opposite angles is the “**X pattern**”. Picture an X between the transversal and the line. The angles on top and on the bottom of the “X” are congruent, as are the angles on either side.

Interior angles are angles that are on the same side of the transversal, in between the lines. When the lines are parallel, interior angles are supplementary (remember, this means that they add up to 180°).

Which angles from the diagram above satisfy this property?

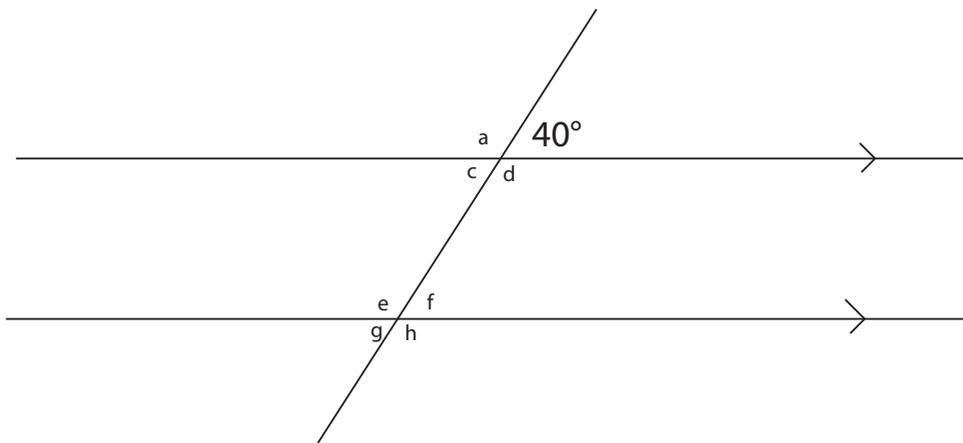
In the diagram above, d and f are interior angles, and c and e are interior angles.

$c + e = 180^\circ$, and $d + f = 180^\circ$

The easiest way to think of interior angles is the “**C pattern**”. If the space between the two parallel lines and the transversal is a very robotic letter “C”, then the two angles in the “C” are supplementary.

Notice that since $a = d$ and $d = h$ (because they are opposite and corresponding angles), $a = h$. Similarly, $b = c$ and $c = g$, so $b = g$.

Exercise. Find angles a , c , d , e , f , g and h .



$a = 180^\circ - 40^\circ = 140^\circ$, supplementary angle with the given angle

$c = 40^\circ$, because of opposite angles property with the given angle

$d = 140^\circ$, because of opposite angles property with a

$e = 140^\circ$, because of alternate angles property with d

$f = 40^\circ$, because of alternate angles property with c

$g = 40^\circ$, because of opposite angles property

$h = 140^\circ$, because of opposite angles property

This is only one way of thinking of the solution. There are alternative ways to find the correct answers.

Triangles

The sum of angles in a triangle is always 180° .

This leads to different types of triangles, categorized by angles:

Equilateral triangles: Triangles with 3 equal sides and therefore 3 equal angles

When looking at a diagram, if there is a dash mark through two or more lines, those lines are of equal length.

Exercise.

Since there are 180° in a triangle and all angles are equal in an equilateral triangle, what is this angle?

60°

Isosceles triangles: Triangles with 2 equal sides and therefore 2 equal angles

Scalene triangles: Triangles with no equal sides and therefore no equal angles

Right triangles: Triangles with 1 right angle

Exercise.

Isosceles right triangles have 2 equal sides, and a right angle. Can you find their other angle?

45°

Acute triangles: Triangles in which all angles are acute, less than 90°

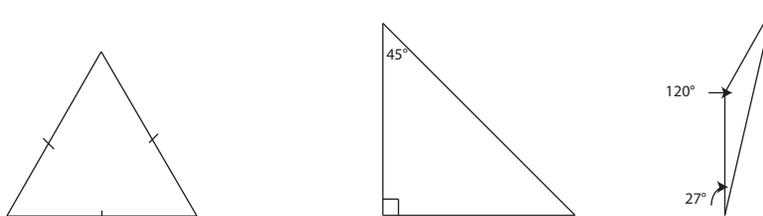
Obtuse triangles: Triangles in which 1 angle is obtuse, greater than 90°

Exercise.

Why is it impossible to have a triangle with a reflex angle?

The sum of the angles in the triangle is 180° , therefore there cannot be an angle greater than 180° .

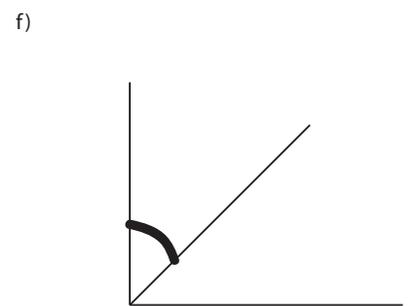
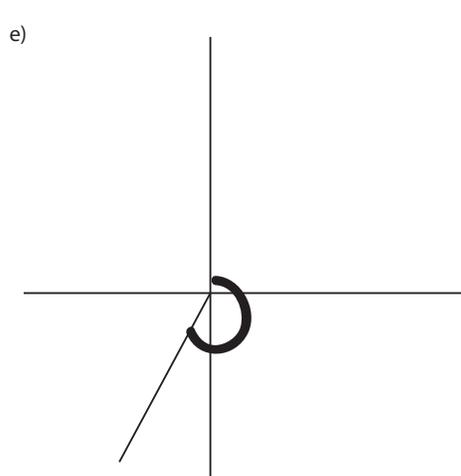
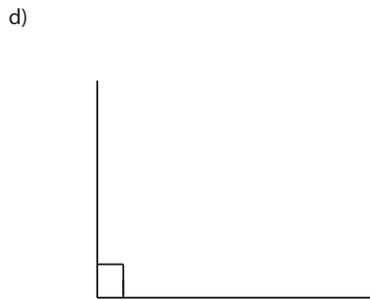
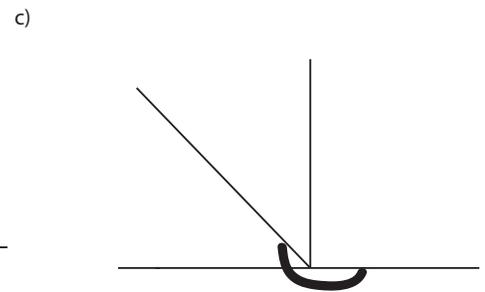
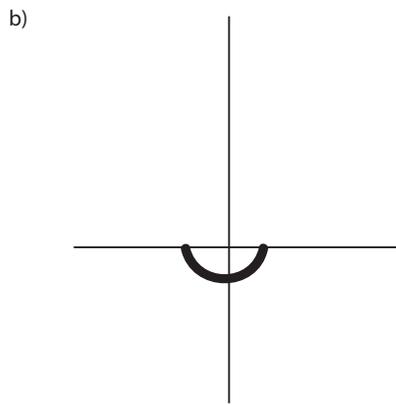
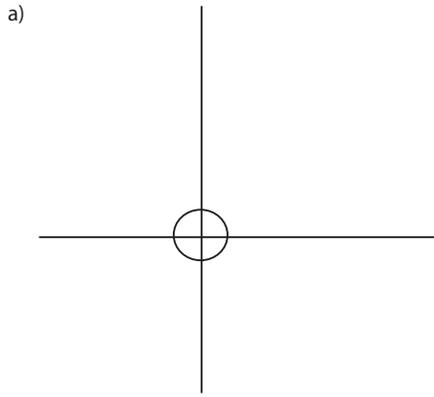
Classify the triangle by number of similar angles (equilateral, isosceles, scalene) and size of angle (right, acute, obtuse). Then determine the missing angle(s):



Equilateral Acute, Isosceles Right, Scalene Obtuse

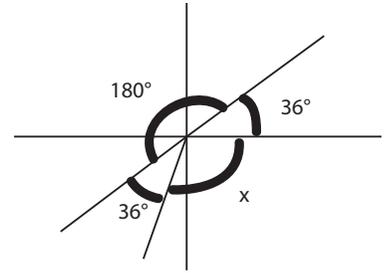
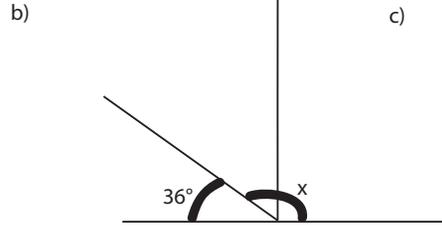
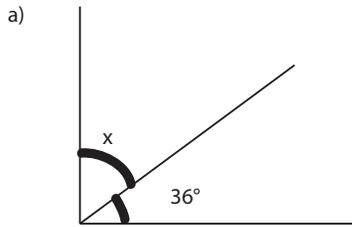
Problem Set

1. Classify the angle.



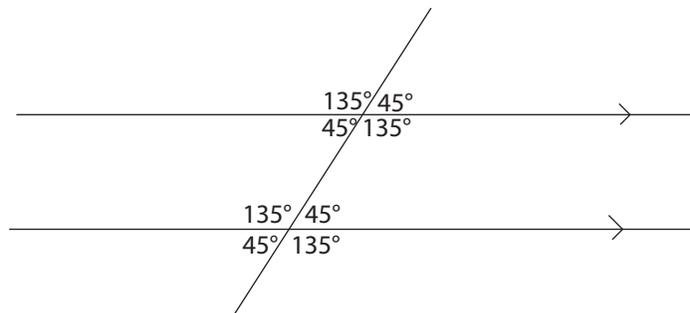
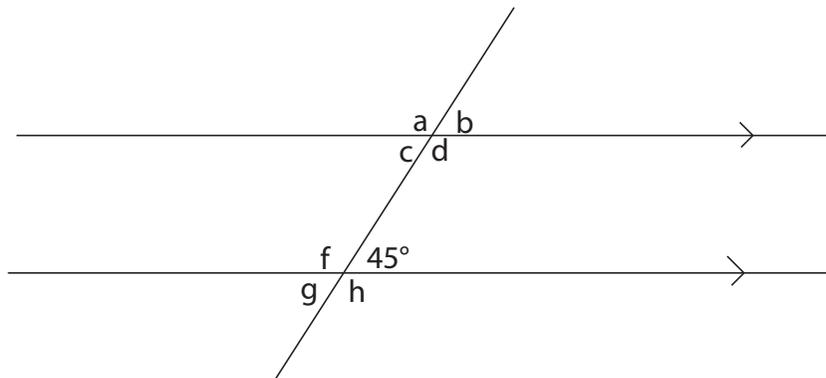
- a) Full Rotation
- b) Straight Angle
- c) Reflex Angle
- d) Right Angle
- e) Obtuse Angle
- f) Acute Angle

2. Find the missing angle, x , in each diagram.
 (Note: Diagrams may not be to scale)



- a) $x = 90^\circ - 36^\circ = 54^\circ$
 b) $x = 180^\circ - 36^\circ = 144^\circ$
 c) $x = 360^\circ - 180^\circ - 36^\circ - 36^\circ = 108^\circ$

3. Find angles a , b , c , d , f , g and h .



- $b = 45^\circ$, because of corresponding angles property with given angle
 $c = 45^\circ$, because of alternate angles property with given angle
 $d = 180^\circ - 45^\circ = 135^\circ$, because of interior angles property with given angles

$a = 135^\circ$, because of opposite angles property with d

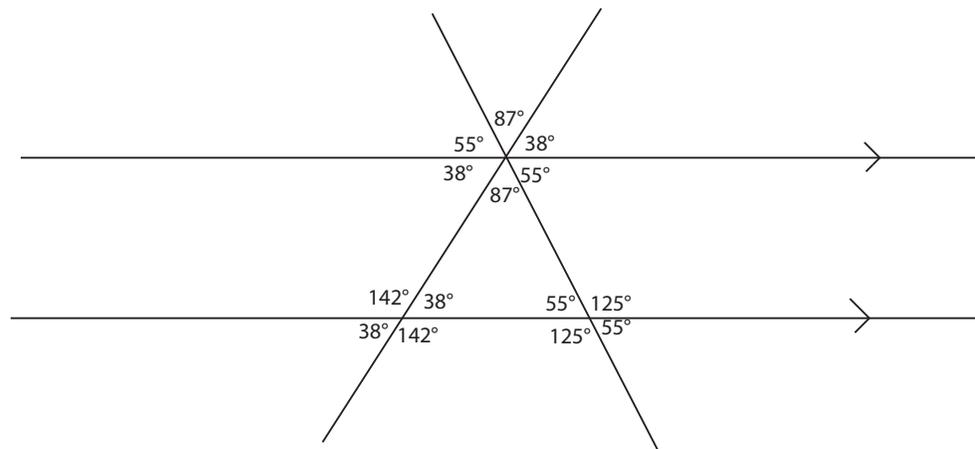
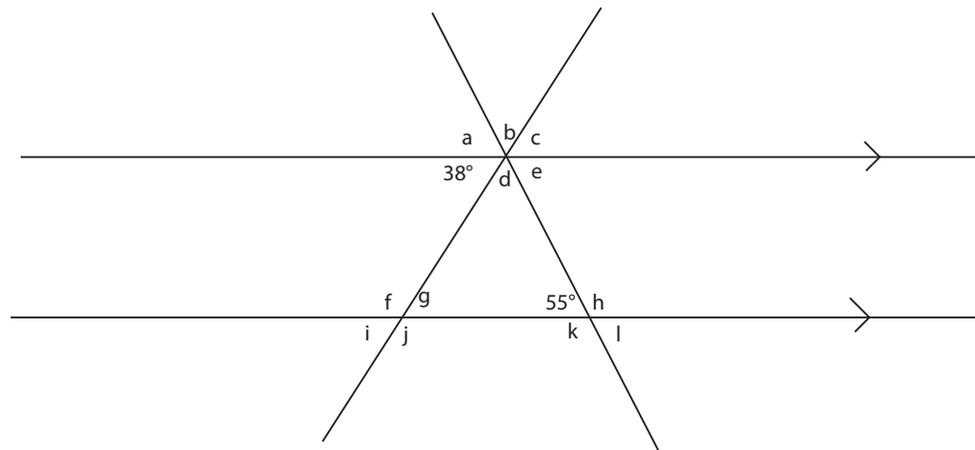
$f = 180^\circ - 45^\circ = 135^\circ$, because of supplementary angles property with given angle

$g = 45^\circ$, because of opposite angle property

$h = 180^\circ - 45^\circ = 135^\circ$, because of supplementary angles property with given angle

This is one way of thinking of the solution. There are alternative ways to find the correct answers.

4. Find angles $a, b, c, d, e, f, g, h, i, j, k$ and l .



This is one way of thinking of the solution. There are alternative ways to find the correct answers.

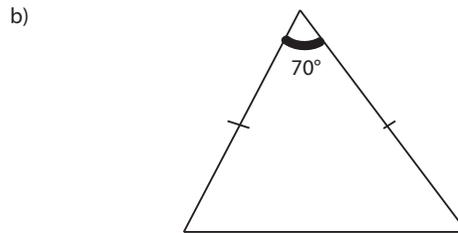
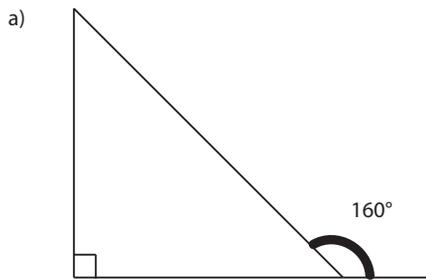
$a = 55^\circ$, because of corresponding angles property with given angle (55°)

$c = 38^\circ$, because of opposite angles property with given angle (38°)

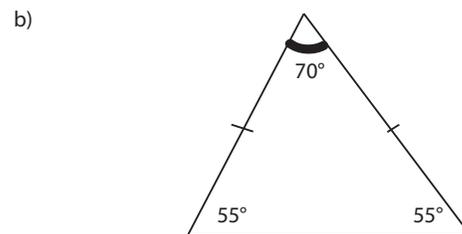
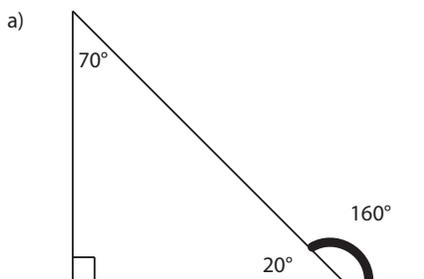
$e = 55^\circ$, because of alternate angles property with given angle (55°)
 $f = 180^\circ - 38^\circ = 142^\circ$, because of interior angles property with given angle (38°)
 $g = 38^\circ$, because of alternate angles property with given angle (38°)
 $h = 180^\circ - 55^\circ = 125^\circ$, because of supplementary angles property with given angle (55°)
 $i = 38^\circ$, because of corresponding angles property with given angle (38°)
 $k = 180^\circ - 55^\circ = 125^\circ$, because of supplementary angle property with given angle (55°)
 $l = 55^\circ$, because of opposite angles property with given angle (55°)

$b = 180^\circ - 38^\circ - 55^\circ = 87^\circ$, because of the formation of a straight angle with c and e
 $d = 87^\circ$, because of opposite angles property with b
 $j = 142^\circ$, because of opposite angles property with f

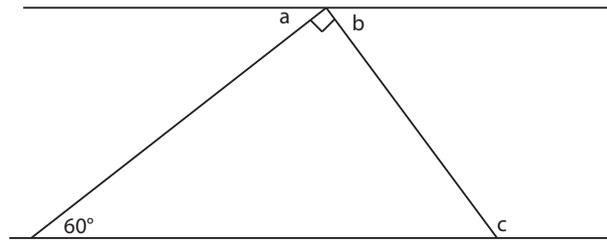
5. Classify the triangle and find the missing angle(s):
 (Note: Diagrams may not be to scale)



- a) Scalene right
 b) Isosceles acute



6. Find angles a , b , and c if the top and bottom lines are parallel.



$$a = 60^\circ, b = 30^\circ, c = 150^\circ$$

Find the unknown angle in the triangle using the fact that the interior angles of a triangle sum to 180° . This angle will be 30° .

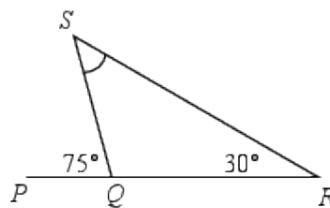
$c = 180^\circ - 30^\circ = 150^\circ$, because of supplementary angles

$b = 180^\circ - 150^\circ = 30^\circ$, because of interior angles property with c

$a = 180^\circ - 90^\circ - 30^\circ = 60^\circ$, because a , b , and the right angle from the triangle form a straight angle.

This is one way of thinking of the solution. There are alternative ways to find the correct answers.

7. In the diagram shown, PQR is a straight line segment. What is the measure of $\angle QSR$?

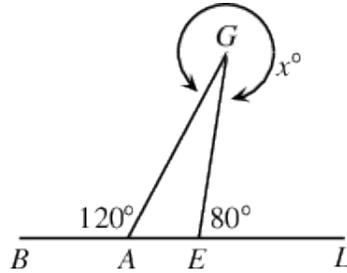


$$\angle QSR = 45^\circ$$

2014 Gauss Contest (Grade 7) #17.

https://www.cemc.uwaterloo.ca/contests/past_contests.html for full solutions.

8. Four points B , A , E , and L are on a straight line as shown. G is a point off the line so that $\angle BAG = 120^\circ$ and $\angle GEL = 80^\circ$. If the reflex angle at G is x , then what does x equal?



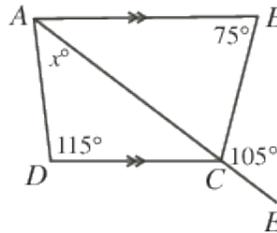
$$x = 340^\circ$$

2005 Gauss Contest (Grade 8) #13.

https://www.cemc.uwaterloo.ca/contests/past_contests.html for full solutions.

CHALLENGE

9. In the diagram, AB is parallel to DC and ACE is a straight line. What is the value of x ?

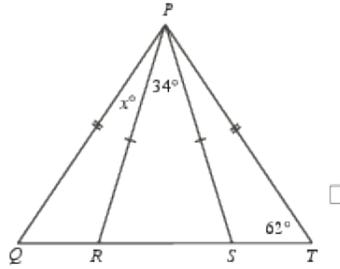


$$x = 35^\circ$$

2010 Gauss Contest (Grade 8) #21.

https://www.cemc.uwaterloo.ca/contests/past_contests.html for full solutions.

10. In the diagram, points R and S lie on QT . Also, $\angle PTQ = 62^\circ$, $\angle RPS = 34^\circ$, and $\angle QPR = x^\circ$. What is the value of x ?

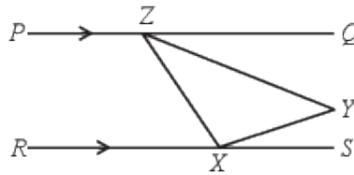


$$\angle QZY = 30^\circ$$

2012 Pascal Contest #17.

https://www.cemc.uwaterloo.ca/contests/past_contests.html for full solutions.

11. In the diagram, PQ is parallel to RS . Also, Z is on PQ and X is on RS . If Y is located between PQ and RS so that $\angle YXS = 20^\circ$ and $\angle ZYX = 50^\circ$, what is the measure of $\angle QZY$?

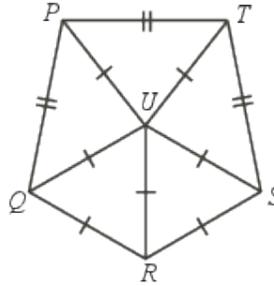


$$\angle QZY = 30^\circ$$

2013 Cayley Contest #19.

https://www.cemc.uwaterloo.ca/contests/past_contests.html for full solutions.

12. In the diagram, $\triangle QUR$ and $\triangle SUR$ are equilateral triangles. Also, $\triangle QUP$, $\triangle PUT$ and $\triangle TUS$ are isosceles triangles with $PQ = QU = SU = TU$ and $QP = PT = TS$. What is the measure of $\angle UST$ in degrees?



$$\angle QZY = 30^\circ$$

2012 Fermat Contest #12.

https://www.cemc.uwaterloo.ca/contests/past_contests.html for full solutions.