Intermediate Math Circles
Fall 2019
Fun With Inequalities

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- We looked at some properties of "less than or equal to"
- We proved one of them

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- We solved word problems using linear inequalities with one variable

Plan for week 2

- Solve *Absolute Value Inequalities (Single Variable)* and *Rational Inequalities (Single Variable)*.
- Prove some properties of the Absolute Value Function.

Solving Absolute Value Inequalities (Single Variable)

What is *absolute value*?

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Another cool way of expressing *absolute value* is as follows

$$|b| = \sqrt{b^2}$$

Solving Absolute Value Inequalities (Single Variable)

Another way to think about *absolute value* is the distance from a *special point*. Sometimes that *special point* is zero, sometimes it is non-zero, and sometimes there are multiple *special point*. To find these *special point* we need to set what's contain in each absolute value to zero and solve.

Solving Absolute Value Inequalities (Single Variable)

Examples

Solve each of the following equations and inequalities.

1. $|x| = 12$
2. $|x| \geq 5$
3. $|x + 6| = 5$
4. $|x - 4| \geq 1$
5. $|x - 3| + |x + 6| < 13$

Proving some properties of the Absolute Value Function

Challenge

Prove the following:

1. $|-x| = x$
2. $|x| - |y| \leq |x - y|$
3. $||x| - |y|| \leq |x - y|$

Solving Rational Inequalities (Single Variable)

Before we deal with inequalities let's work with equalities first.

Practice

Solve each of the following equations.

1. $$\frac{4}{x} = \frac{5}{6}$$
2. $$\frac{-2}{x+4} = \frac{3}{x-1}$$

The most important thing to remember when solving rational equalities is

$$\boxed{\text{DON'T DIVIDE BY ZERO!}}$$

Solving Rational Inequalities (Single Variable)

Solving rational inequalities is what you would expect. It is similar to solving rational equalities, expect for that pesky property 7.

Just like when solving rational equalities you need to be aware when the denominator can be zero and exclude those values from your answer.

Example

Solve the inequality $\frac{4}{x} < -\frac{5}{6}$ algebraically.

Where is the error?

Example

For our example, can you spot the error in the following solution?

$$\frac{4}{x} < -\frac{5}{6}; \qquad x \neq 0$$

$$4(6) < -5(x)$$

$$24 < -5x$$

$$\frac{24}{-5} > \frac{-5x}{-5}$$

$$-\frac{24}{5} > x$$

$$x < -\frac{24}{5}$$

$$x < -4.8$$

Challenge

Solve the inequality $\dfrac{-2}{x+4} \geq \dfrac{3}{x-1}$ algebraically.

Thank you!