



Intermediate Math Circles

February 12, 2020

Equations and Algebra: Problem Set

1. The symbol $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$ equals $ad - bc$.

If $\begin{array}{|c|c|} \hline x-1 & 2 \\ \hline 3 & -5 \\ \hline \end{array} = 9$, determine the value of x (2004 Pascal #18).

2. In the diagram, each of the five boxes is to contain a number. For every box other than the two on the ends, the number in that box must be the average of the number in the box to the left of it and the number in the box to the right of it. Determine the values of x , y and z (2010 Pascal #21).

8	x	y	26	z
---	-----	-----	----	-----

3. In each row of the table, the sum of the first two numbers equals the third number. Also, in each column of the table, the sum of the first two numbers equals the third number. What is the sum of the nine numbers in the table? (2006 Pascal #21)

m	4	$m+4$
8	n	$8+n$
$m+8$	$4+n$	6

4. A bank teller has some stacks of bills. The total value of the bills in each stack is \$1000. Every stack contains at least one \$20 bill, at least one \$50 bill, and no other types of bills. If no two stacks have the same number of \$20 bills, what is the maximum possible number of stacks that the teller could have? (2015 Cayley #19)
5. There are four unequal, positive integers a, b, c and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$? (1998 Cayley #23)
6. If x and y are positive integers with $xy = 6$, what is the sum of all the possible values of $\frac{2^{x+y}}{2^{x-y}}$? (2019 Cayley #19)
7. Suppose that x and y are positive numbers with

$$\begin{aligned} xy &= \frac{1}{9} \\ x(y+1) &= \frac{7}{9} \\ y(x+1) &= \frac{5}{18} \end{aligned}$$

What is the value of $(x+1)(y+1)$? (2011 Cayley #21)



8. Five positive integers are listed in increasing order. The difference between any two consecutive numbers in the list is three. The fifth number is a multiple of the first number. How many different such lists of five integers are there? (2007 Cayley #22)
9. Suppose that a , b and c are three numbers with

$$a + b = 3$$

$$ac + b = 18$$

$$bc + a = 6$$

Determine the value of c (2009 Cayley #22).

10. Solve the following system of equations (2000 Cayley #25 - modified)

$$x + xy + xy^2 = 26$$

$$x^2y + x^2y^2 + x^2y^3 = 156$$