

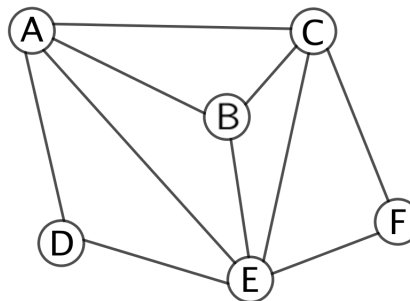
Grade 7/8 Math Circles

October 21, 2020

Graph Theory Proofs - Problem Set

1. For the graph to the right:

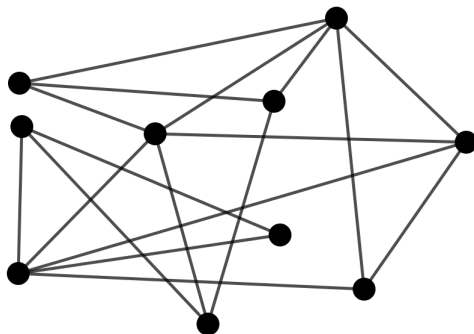
- Find a path from vertex D to vertex C.
- List the neighbours of vertex B.
- Find a 4-colouring.
- Explain why a 3-colouring can't be found.



2. For each part listed below, draw a possible graph with the given properties:

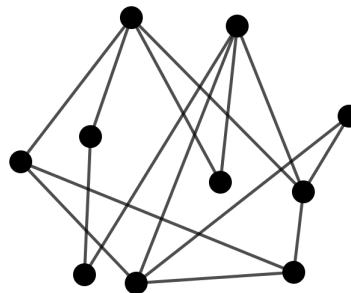
- Exactly 6 vertices and 15 edges.
- A 2-colouring, exactly 6 vertices and 9 edges.
- Non-planar.
- No 2-colouring.
- No 6-colouring.

3. Prove that the following graphs are planar. *Hint: Click on the graph names to access a movable graph.*



Graph 1:

<https://www.geogebra.org/m/es5vpja2>

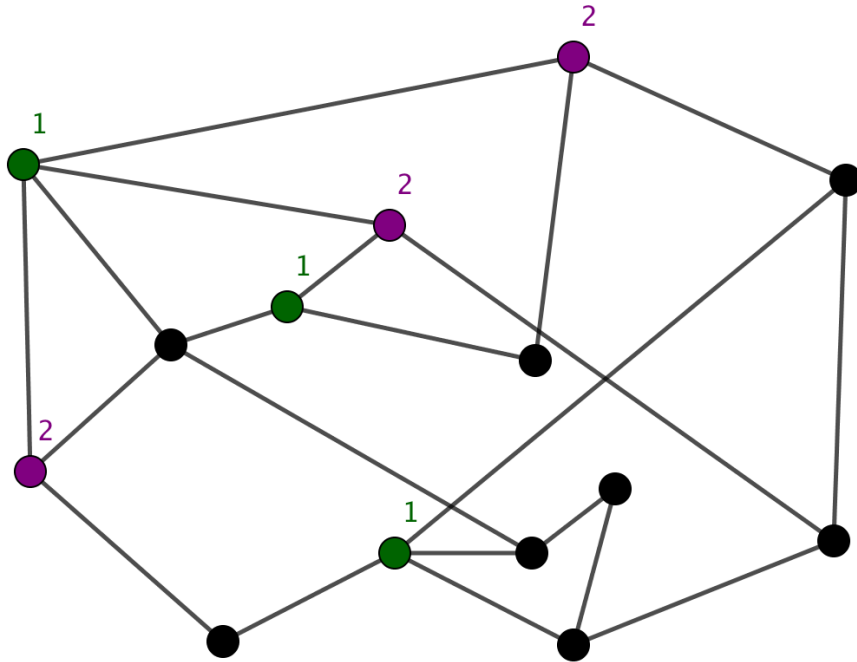


Graph 2:

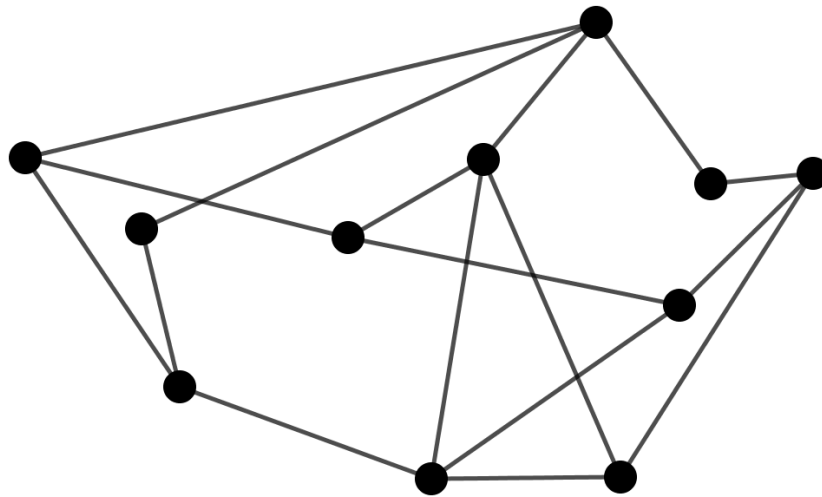
<https://www.geogebra.org/m/qkjuvfkq>

4. Find an edge subdivision of a $K_{3,3}$ in the following graphs to prove that they are non-planar. For the first graph, a recommended 6 vertices, split into 2 groups, has been given.

Graph 1

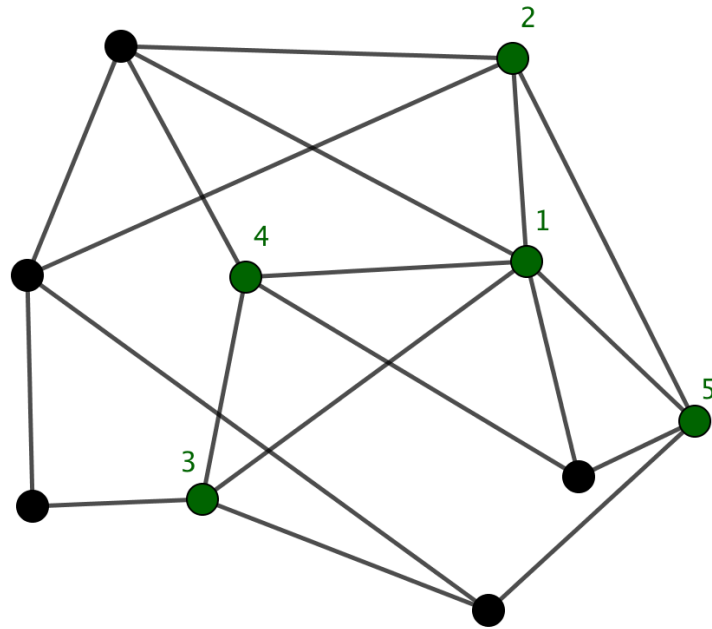


Graph 2

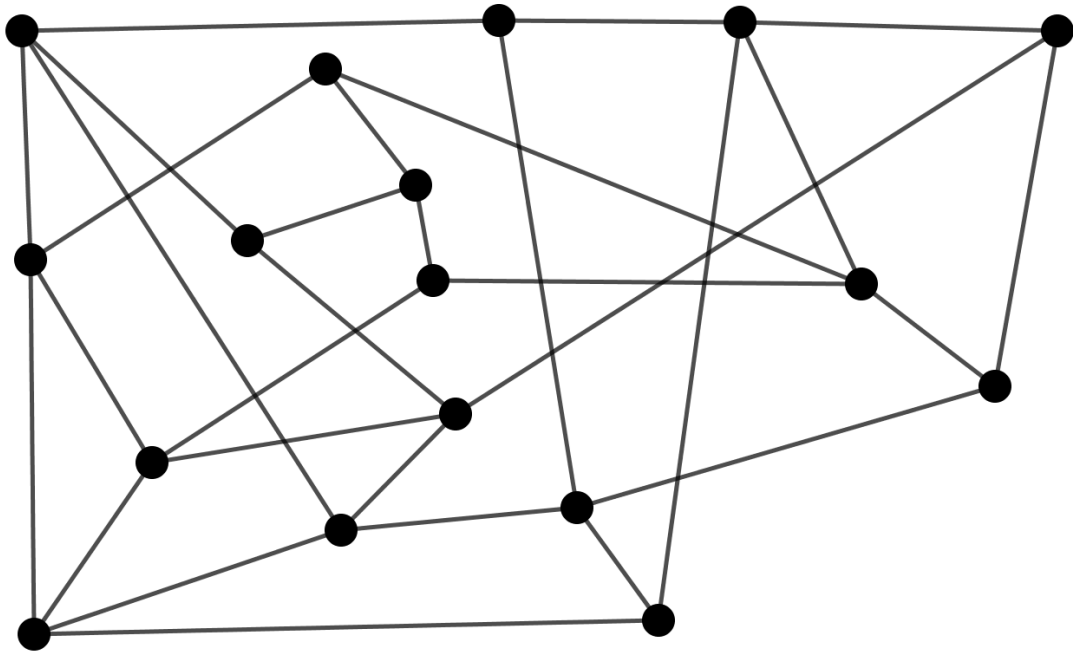


5. Find an edge subdivision of a K_5 in the following graphs to prove that they are non-planar. For the first graph, a recommended 5 vertices has been given.

Graph 1



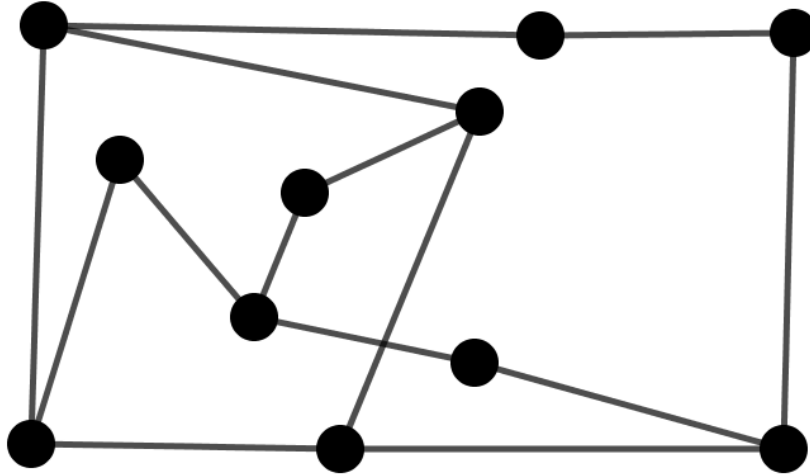
Graph 2



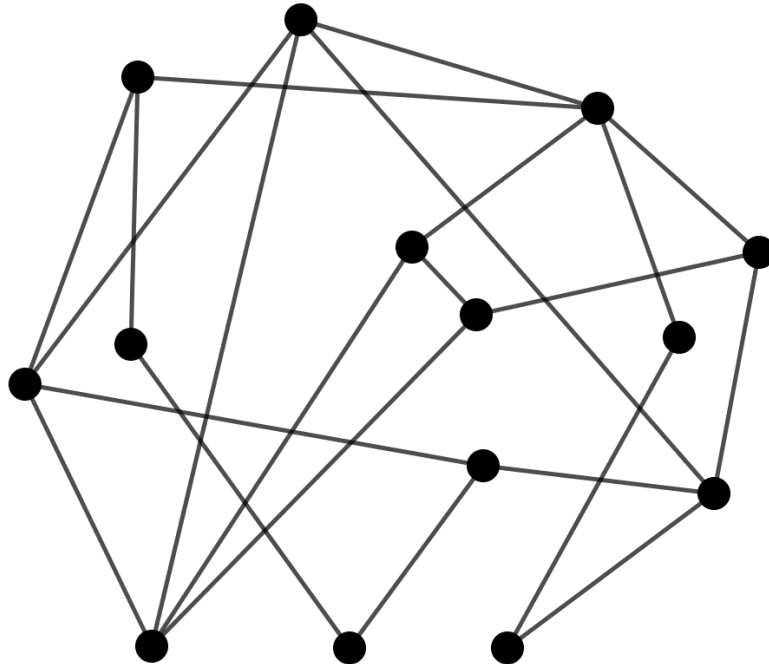
6. Prove that the following graphs are non-planar.

Graph 1

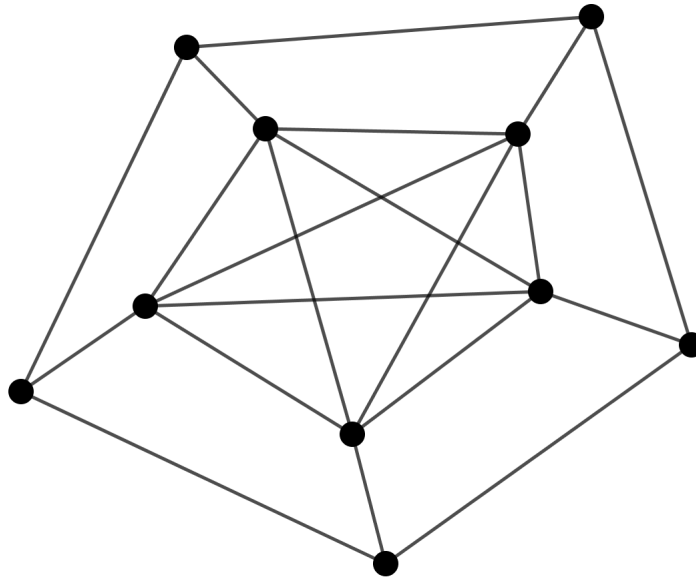
Our example graph from earlier



Graph 2



7. Find the chromatic number for the graphs in problem 6. Prove that the number you find is indeed the chromatic number. *Hint: To prove that a chromatic number, k , is correct, we need to show that the graph is k -colourable and that it is not $(k - 1)$ -colourable.*
8. Prove that the graph below is not 4-colourable.
Hint: Find a K_5 graph as a part of the graph (Not an edge subdivision). Explain why this means it is not 4-colourable.



9. *Timetabling (BCC 2018)*

This is one of the BCC problems that we tried last week. This week, let's try to use what we have learned about colouring to answer the problem.

Bebras Tech offers the following evening classes: Computing (**C**), Geography (**G**), Language (**L**), Math (**M**), and Science (**S**).

Three beavers would like to sign up for these courses: Xavier wants to take **C**, **L** and **M**; Yvette wants to take **C**, **G**, and **S**; Zoey wants to take **L**, **M** and **S**.

Bebras Tech wants to squeeze these courses into as few evenings as possible such that each course is offered on exactly one evening, and beavers can take at most one course per evening.

What is the least number of evenings needed for Bebras Tech to schedule these courses?

Hint: Let each course be represented by a vertex. Represent each possible conflict as an edge. Assigning a vertex a colour is then the same as assigning a course a night.

10. Word of your skills in timetabling has gotten around and the university has asked for your help in scheduling the following courses, with conflicts represented as X's in the below table:

Table 1: Course Conflicts

	Algebra	Computing	Calculus	English	French	Physics
Algebra		X	X	X		X
Computing	X				X	X
Calculus	X			X		X
English	X		X		X	
French		X		X		
Physics	X	X	X			

Each course is to be assigned one time slot. Represent this situation with a graph and use your knowledge of colouring to find and prove what the minimum number of timeslots would be.