



CEMC Math Circles - Grade 11/12

November 25 - December 2, 2020

Dwellings with many Doors



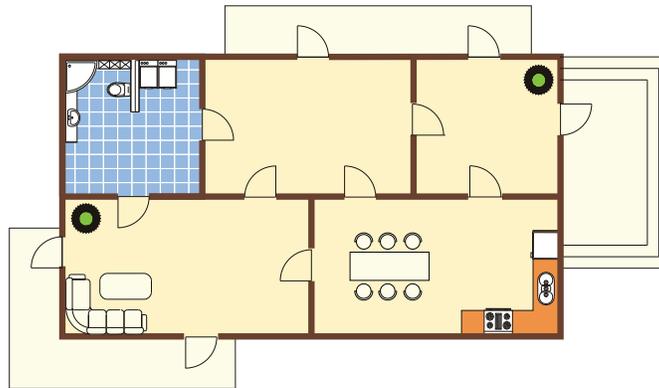
For the last week, we're going to have a look at a very different kind of problem. In fact, it's related to a very important historical problem, as well as the fields of topology and combinatorics. These are about houses with many doors, but you'll see that they're very general kind of problem.

Disclaimer: Don't try to google answers to these problems! Solutions will be posted later in the week— it's much more fun to work them out yourself.

Problem 1. You live in a large house with several rooms— the floor plan is illustrated below. One day, in the midst of your infinite boredom, you wonder about the following question:

Can you walk through your house in such a way that you go through each door *exactly once*? If so, how many different ways are there to do this?

Remarks: This isn't a trick question. The gaps in the walls are doors, and you can pass through each door exactly once. You can start and end wherever you would like.



Problem 2. Your butler becomes tired of you endlessly wandering around the house, and shuts the doors to the bathroom (the room in the top left corner) Does this help matters? In other words, can you now walk through the house in such a way that you go through each remaining door exactly once? What if you insist on starting and ending at the same place?

If you can do either of these walks, how many different walks of each kind are there?

What is the smallest number of doors that you can close so that you can walk exactly once through each remaining door, and start and end in the same place?