Grade 6 Math Circles

November 3rd, 2021

Linear Relations - Solutions

Exercise Solutions

Activity 1

The link here will take you to the Desmos Graphing Calculator. Take some time to play around with it and understand how it works. Once you feel comfortable with it, use it to graph the following equations.

(a) $x = -7$

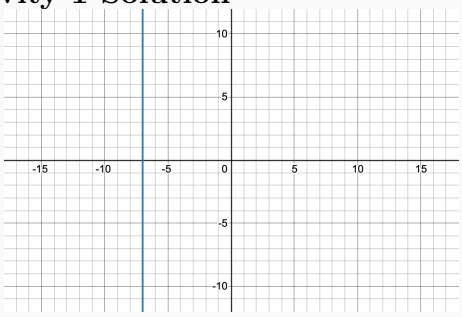
(b) $y = 4$

(c) $2x - 3y = 5$

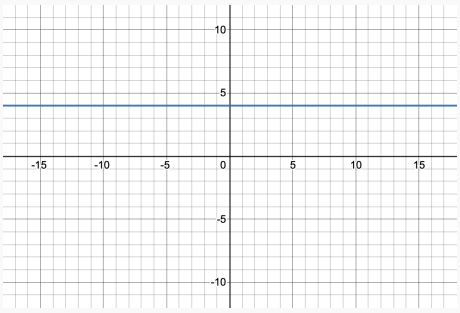
(d) $-x = y - 1$

Activity 1 Solution

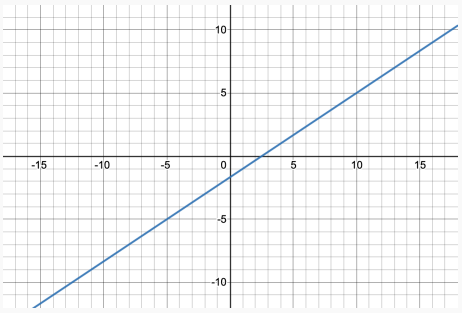
(a)



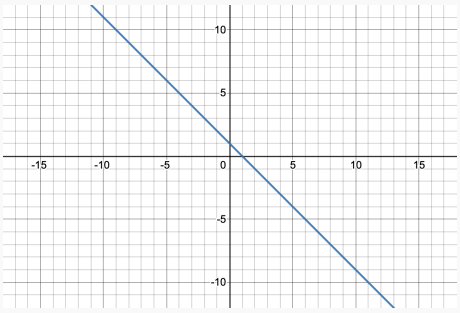
(b)



(c)



(d)



Activity 2

Determine if the following equations are linear relations or not.

(a) $y = x$

(b) $y = 0$

(c) $7x - 4y = 19$

(d) $y = yx + x$

Activity 2 Solution

(a) $y = x$ is a linear relation with $m = 1$ and $b = 0$.

(b) $y = 0$ is a linear relation with $m = 0$ and $b = 0$.

(c) $7x - 4y = 19$ is a linear relation. We can show this by rewriting it first, as shown below:

$$\begin{aligned} 7x - 4y &= 19 \\ -4y &= -7x + 19 \\ y &= \frac{7}{4}x - \frac{19}{4} \end{aligned}$$

Here, we can clearly see that the equation is in the form $y = mx + b$, with $m = \frac{7}{4}$ and $b = -\frac{19}{4}$.

(d) $y = yx + x$ is NOT a linear relation. Here, we have $m = y$ and $b = x$, which means that m and b are variables. As stated above, m and b must be constant numerical values, not variables.

Activity 3

For the following points on a line, calculate m and determine if the line is increasing, decreasing, horizontal, or vertical.

(a) $(9, -2)$ and $(9, 4)$

(b) $(1, -1)$ and $(4, -10)$

(c) $(-9, -11)$ and $(0, 7)$

(d) $(0, -1)$ and $(100, -1)$

Activity 3 Solution

(a) We substitute $(9, -2)$ and $(9, 4)$ into the equation for m to get:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{9 - 9} = \frac{6}{0} \text{ (undefined!)}$$

Thus, m is undefined, so the line is vertical.

(b) We substitute $(1, -1)$ and $(4, -10)$ into the equation for m to get:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-10)}{1 - 4} = \frac{9}{(-3)} = -3$$

Thus, $m < 0$, so the line is decreasing.

(c) We substitute $(-9, -11)$ and $(0, 7)$ into the equation for m to get:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-11)}{0 - (-9)} = \frac{18}{9} = 2$$

Thus, $m > 0$, so the line is increasing.

(d) We substitute $(0, -1)$ and $(100, -1)$ into the equation for m to get:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-1)}{0 - 100} = \frac{0}{(-1)00} = 0$$

Thus, $m = 0$, so the line is horizontal.

Activity 4

The cost of a certain electrician is as follows: an initial flat fee of \$200, and then an hourly fee of \$45.

(a) Represent this as a linear relation. Be sure to state what each component represents.

(b) What is the cost if the electrician works for 6 hours?

(c) How long would the electrican have to work for the cost to be \$605?

Activity 4 Solution

(a) First let us define variables to represent unknown values. We have that the cost of the electrician depends on the number of hours worked. So we can say that the number of hours is the independent variable x, and the cost of the electrian is the dependent variable y.

Next, we have that the cost increases by \$45 for each hour the electrician works, which means that if x increases by 1, then y increases by 45, so $m = 45$. Additionally, we have to include the initial flat fee of \$200, which is constant for any number of hours, so $b = 200$. Thus, we get the following equation to represent the cost of the electrician for any number of hours:

$$y = 45x + 200$$

(b) Next, to find the cost if the electrian works for 6 hours, we simply substitute $x = 6$ into the equation and solve for y, which gives:

$$\begin{aligned} y &= 45x + 200 \\ &= 45(6) + 200 \\ &= 270 + 200 \\ &= 470 \end{aligned}$$

Thus, the electrician working for 6 hours costs \$470.

(c) Finally, to find the number of hours worked by the electrian for the cost to be \$605, we

substitute $y = 605$ into the equation and solve for x, which gives:

$$\begin{aligned} 45x + 200 &= y \\ 45x + 200 &= 605 \\ 45x &= 605 - 200 \\ 45x &= 405 \\ x &= \frac{405}{45} \\ x &= 9 \end{aligned}$$

Thus, the cost is \$605 if the electrician works for 9 hours..

Problem Set Solutions

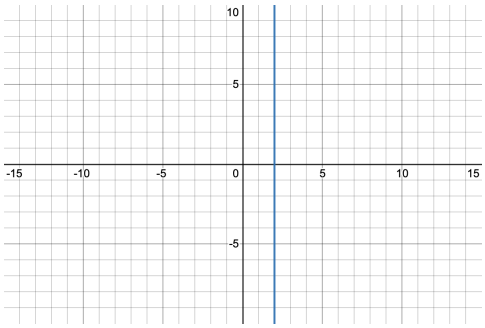
1. Complete the table below with the values of x and y from the equation $-2x - 5y = 1$.

x	-3	12		-43			0	
y			7		29	-51		0

Solution:

x	-3	12	-18	-43	-73	127	0	$-\frac{1}{2}$
y	1	-5	7	17	29	-51	$-\frac{1}{5}$	0

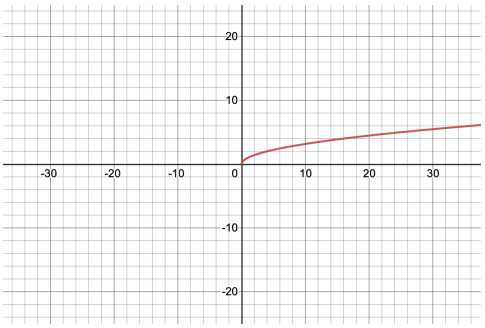
2. **TRUE** or **FALSE**. The slope of the following line is 0.



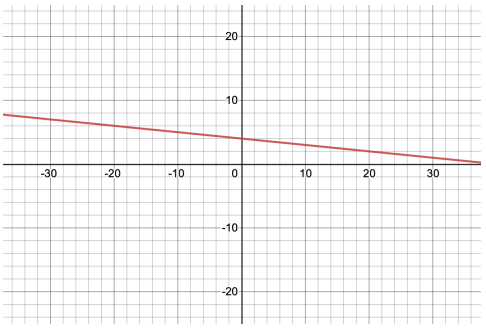
Solution: FALSE. Lines that have a slope of 0 are horizontal. Lines that are vertical, as this one is, have a slope that is undefined.

3. Which of the following graphs are linear relations?

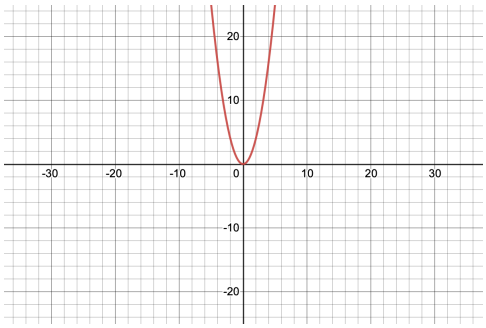
(a)



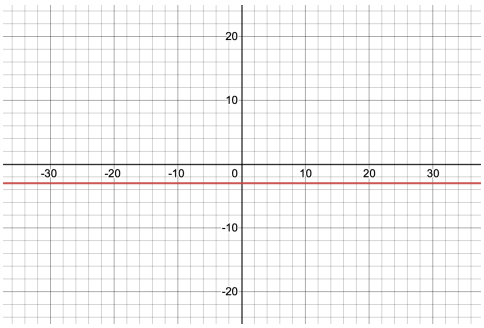
(b)



(c)



(d)



Solution:

(a) Not a linear relation, since the graph is not a straight line.

(b) Linear relation, since the graph is a straight line.

(c) Not a linear relation, since the graph is not a straight line.

(d) Linear relation, since the graph is a straight line.

4. What is the equation of the line that passes through the points $(-12, 3)$ and $(81, -59)$? (Hint: calculate the slope first, then the y-intercept).

Solution: Our first step is to determine the slope using the formula for m, which gives:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-59 - 3}{81 - (-12)} \\ &= \frac{-62}{93} \\ &= -\frac{2}{3} \end{aligned}$$

Then, we substitute $m = -\frac{2}{3}$, $x = -12$ and $y = 3$ into $y = mx + b$ (Note: we could also

use $x = 81$ and $y = -59$) to get:

$$\begin{aligned} mx + b &= y \\ \left(-\frac{2}{3}\right)(-12) + b &= 3 \\ 8 + b &= 3 \\ b &= 3 - 8 \\ b &= -5 \end{aligned}$$

Thus, we have that the equation of the line is $y = -\frac{2}{3}x - 5$.

5. Does the line that passes through the points $(3, 3)$ and $(-32, -11)$ also pass through the point $(249, 101)$?

Solution: Our first step is to determine the equation of the line, so we use the formula for m to get:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-11 - 3}{-32 - 3} \\ &= \frac{-14}{-35} \\ &= \frac{2}{5} \end{aligned}$$

Then, we substitute $m = \frac{2}{5}$, $x = 3$ and $y = 3$ into $y = mx + b$, which gives:

$$\begin{aligned} mx + b &= y \\ \left(\frac{2}{5}\right)(3) + b &= 3 \\ \frac{6}{5} + b &= 3 \\ b &= 3 - \frac{6}{5} \\ b &= \frac{15}{5} - \frac{6}{5} \\ b &= \frac{9}{5} \end{aligned}$$

So, we get the equation of the line to be $y = \frac{2}{5}x + \frac{9}{5}$. Now, we can substitute $x = 249$ into the equation and see if the y-value is 101.

$$\begin{aligned} y &= \frac{2}{5}x + \frac{9}{5} \\ &= \frac{2}{5}(249) + \frac{9}{5} \\ &= \frac{498}{5} + \frac{9}{5} \\ &= \frac{507}{5} \\ &= 101.4 \end{aligned}$$

Thus, we have that the line does not pass through $(249, 101)$.

6. A linear relation has a slope of $-\frac{3}{2}$ and contains the point $(8, 0)$. What is the value of y when $x = 0$?

Solution: There are two ways we can solve this problem.

(1) The first way is to realize that the value of y when $x = 0$ is the y-intercept b. So, we substitute the values $m = -\frac{3}{2}$, $x = 8$ and $y = 0$ into $y = mx + b$ to get:

$$\begin{aligned} mx + b &= y \\ \left(-\frac{3}{2}\right)(8) + b &= 0 \\ -12 + b &= 0 \\ b &= 12 \end{aligned}$$

Thus, the value of y when $x = 0$ is 12.

(2) The second way is to use the formula for calculating m, where we substitute $m = -\frac{3}{2}$ and the points $(8, 0)$ and $(0, y_2)$, to get:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= m \\ \frac{y_2 - 0}{0 - 8} &= -\frac{3}{2} \\ \frac{y_2}{-8} &= -\frac{3}{2} \\ y_2 &= -\frac{3}{2}(-8) \\ y_2 &= 12 \end{aligned}$$

Thus, the value of y when $x = 0$ is 12.

7. If a line has a slope of 2 and passes through the points $(2, 7)$ and $(a, 3a)$, what is the value of a?

Solution: There are two ways to solve this problem.

(1) For the first solution, we want to determine the equation of the line, which means we need to determine the value of b. So, we substitute $m = 2$, $x = 2$ and $y = 7$ into

$y = mx + b$ to get:

$$\begin{aligned} mx + b &= y \\ (2)(2) + b &= 7 \\ 4 + b &= 7 \\ b &= 7 - 4 \\ b &= 3 \end{aligned}$$

So, the equation of the line is $y = 2x + 3$. Now, we substitute $x = a$ and $y = 3a$ into this equation to get:

$$\begin{aligned} y &= 2x + 3 \\ 3a &= 2a + 3 \\ 3a - 2a &= 3 \\ a &= 3 \end{aligned}$$

(2) For the second solution, we use the formula for calculating m, where we substitute $m = 2$ and the points $(2, 7)$ and $(a, 3a)$, to get:

$$\begin{aligned} \frac{y_2 - y_1}{x_2 - x_1} &= m \\ \frac{3a - 7}{a - 2} &= 2 \\ 3a - 7 &= 2(a - 2) \\ 3a - 7 &= 2a - 2(2) \\ 3a - 7 &= 2a - 4 \\ 3a - 2a &= -4 + 7 \\ a &= 3 \end{aligned}$$

8. An unspecified amount of money is deposited into a bank account. Nine days later, the balance in the bank account is \$16075. Twelve days after that, the balance in the bank account is \$20575. Assume this follows a linear relationship.

(a) What is the daily rate of money being deposited in the bank account?

(b) What was the initial balance of the bank account?

(c) What will the balance of the bank account be six weeks after the initial deposit?

(d) How long after the initial deposit will the balance in the bank account be exactly \$24700?

Solution:

(a) First, we want to define variables for this problem. In this case, the number of days after the initial deposit is the independent variable x, and the balance of the bank account is the dependent variable y. Then, we see that the balance is \$16075 after nine days, which gives us the ordered pair $(9, 16075)$. Twelve days after that the balance is \$20575, which gives another ordered pair $(9 + 12, 20575) = (21, 20575)$.

We then realize that the daily rate of money being deposited in the bank account is simply m, which we calculate using the formula from the lesson:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{20575 - 16075}{21 - 9} \\ &= \frac{4500}{12} \\ &= 375 \end{aligned}$$

Thus, the daily rate of money being deposited in the bank account is \$375.

(b) The initial balance of the bank account is just b in this case, which we can calculate by substituting $m = 375$, $x = 9$ and $y = 16075$ into $y = mx + b$, which gives:

$$\begin{aligned} mx + b &= y \\ (375)(9) + b &= 16075 \\ 3375 + b &= 16075 \\ b &= 16075 - 3375 \\ b &= 12700 \end{aligned}$$

Thus, the initial balance of the bank account is \$12700.

(c) Using the information from parts (a) and (b), we have the equation of this linear relation to be $y = 375x + 12700$. Then, we have that six weeks is equal to 42 days, so we substitute $x = 42$ into the equation to get:

$$\begin{aligned} y &= 375x + 12700 \\ &= 375(42) + 12700 \\ &= 15750 + 12700 \\ &= 28450 \end{aligned}$$

Thus, the balance after six weeks is \$28450.

(d) To determine how long after the initial deposit the balance will be exactly \$24700, we substitute $y = 24700$ into the linear equation to get:

$$\begin{aligned} 375x + 12700 &= y \\ 375x + 12700 &= 24700 \\ 375x &= 24700 - 12700 \\ 375x &= 12000 \\ x &= \frac{12000}{375} \\ x &= 32 \end{aligned}$$

Thus, the balance will be exactly \$24700 after 32 days.