Grade 7/8 Math Circles

November 17, 2021

Different Base Counting Systems - Problem Set

1. Convert the following numbers into the decimal counting system.

   (a) 1001001_2 (b) 101001011_2 (c) 11110101_2 (d) $71FA_{16}$ (e) ACE_{16}

2. Convert the following numbers to into the binary counting system.

   (a) 47_{10} (b) 156_{10} (c) 222_{10} (d) $71C3_{16}$

3. Convert the following numbers to into the hexadecimal counting system.

   (a) 58_{10} (b) 7180_{10} (c) 101101010010_2 (d) 100000100_2

4. It is possible to perform addition in different base counting systems. In base 10, we perform addition by first adding the rightmost digit, and if the sum exceeds the number of available digits, we "carry over" 1 to the next highest place value. Addition works the same way in a different base, the only difference being the number of available digits there are to use.

   Suppose we want to calculate $101 + 11$ in binary. We will start by adding the right most digit. $1 + 1 = 10$ in binary, so we carry over a 1 to the next place value, while the current place value becomes 0.

$$\begin{array}{r} {\color{blue}1} \\ 101 \\ +11 \\ \hline 0 \end{array}$$

   We carried a 1 to the second place value, so the next place value becomes $1 + 0 + 1 = 10$, and we carry over a 1 to the next place value and put a 0 in the current place value.

$$\begin{array}{r} {\color{blue}1} \\ 101 \\ +11 \\ \hline 00 \end{array}$$

Finally, the next place value is $1 + 1 = 10$, and so the final answer is $101 + 11 = 1000$.

$$\begin{array}{r} 101 \\ +11 \\ \hline 1000 \end{array}$$

Perform the following calculations in binary. Think of what each place value represents and how to carry values over each place value (as with base 10 addition).

(a) $\begin{array}{r} 10 \\ +1 \\ \hline \end{array}$ (b) $\begin{array}{r} 1001 \\ +110 \\ \hline \end{array}$ (c) $\begin{array}{r} 10 \\ +10 \\ \hline \end{array}$ (d) $\begin{array}{r} 110 \\ +10 \\ \hline \end{array}$ (e) $\begin{array}{r} 111 \\ +11 \\ \hline \end{array}$

5. Similar to the question above, perform the following calculations in hexadecimal. Think of what each place value represents and how to carry values over each place value (as with base 10 addition).

(a) $\begin{array}{r} 17 \\ +6 \\ \hline \end{array}$ (b) $\begin{array}{r} \text{AB} \\ +\text{C} \\ \hline \end{array}$ (c) $\begin{array}{r} \text{71D} \\ +\text{F} \\ \hline \end{array}$ (d) $\begin{array}{r} \text{A} \\ +\text{B} \\ \hline \end{array}$ (e) $\begin{array}{r} \text{F} \\ +1 \\ \hline \end{array}$

You may refer to the hexadecimal to decimal conversion chart below.

Hex Digit	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal Value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

6. It is also to perform subtraction in different base counting systems. In a counting system with any base, we start by subtracting from the rightmost digit. If we obtain a negative difference, we can "borrow 1" from the next highest place value.

Perform the following subtractions in binary and hexadecimal. Think of what each place value represents and how to borrow values from the next highest place value (as with base 10 addition).

(a) $\begin{array}{r} 1111 \\ -10 \\ \hline \end{array}$ (b) $\begin{array}{r} 1001 \\ -11 \\ \hline \end{array}$ (c) $\begin{array}{r} \text{17A} \\ -5 \\ \hline \end{array}$ (d) $\begin{array}{r} \text{1D} \\ -\text{B} \\ \hline \end{array}$ (e) $\begin{array}{r} \text{A37D1} \\ \text{E9C} \\ \hline \end{array}$

7. Convert 152_{10} to a base-5 number. You may use the same method that we used for hexadecimal numbers in the lesson.

8. Is it possible for a counting system to be base 0? What about base 1?

9. Here are some problems regarding 5 digit numbers. State all your answers in the decimal counting system.

   (a) How many different 5-digit binary numbers are there? Note that the leading digit must be a 1 since we truncate all leading 0's. *(Hint: how would you do this in base 10?)*

   (b) How many different 5-digit binary numbers are there that have 1 as their last digit?

   (c) How many different 5-digit base B numbers are there for any given B? Again, the leading digit cannot be 0. You may assume that $2 \leq B \leq 10$.

10. Suppose there is a base-26 counting system where the digits are the English alphabet, in the order from a to z. The conversion chart is as follows.

Letter	a	b	c	d	e	f	g	h	i	j	k	l	m
Decimal Value	0	1	2	3	4	5	6	7	8	9	10	11	12
Letter	n	o	p	q	r	s	t	u	v	w	x	y	z
Decimal Value	13	14	15	16	17	18	19	20	21	22	23	24	25

Translate the following secret message from base-10 to base-26. The numbers are separated by spaces, and each number represents a "word".

7838998 381 9171, 126182 8 44932!

You can use the chart below to help you.

Exponent Value	26^4	26^3	26^2	26^1	26^0
Numerical Value	456976	17576	676	26	1

Hint: you can either use the chart above to find teh greatest power of 26 that is less than or equal to the decimal number and use the subtraction algorithm (pages 3 and 4 of the lesson pdf), or use the long division algorithm (page 7 and 8 of the lesson pdf) and convert the remainders to base-26, which is much faster to compute.

Note: these types of secret messages are known as **cryptography**, where mathematical concepts or rules are used to encrypt messages into forms that are hard to decipher for anyone who is unaware of the rules. In this case, the rules refer to the base-26 counting system, as introduced above.