## Grade 7/8 Math Circles <br> March 9th, 2022 <br> Graph Theory and Search Algorithms Problem Set

1. What are the vertices and edges in the graph below? Include colour labels for the vertices.

2. List the neighbourhoods of all the vertices in the graph in question 1.
3. Do BFS on the graph below until all the vertices are coloured black. Start at vertex $H$ and pick neighbours from right-to-left (in other words, visually from the right side of the graph to the left). What order did you look at the vertices?

4. Do DFS on the graph in question 3 until all the vertices are coloured black. Start at vertex $E$ and pick neighbours alphabetically. What order did you look at the vertices?
5. Search for vertex $F$ starting from vertex $A$ using BFS for the following graph, picking neighbours alphabetically. What order did you look at the vertices?

6. For the graph in question 5 , would it be faster to find vertex $G$ from vertex $B$ using BFS or DFS if you pick neighbours from left-to-right? What about from right-to-left?
7. (a) Search through the whole graph below three times using BFS. Pick neighbours alphabetically and use the starting points $G, A$, and $E$. List the order that you look at the vertices for those three searches. What do you notice about the length of the lists of vertices?
(b) Search through the whole graph below three times using DFS. Pick neighbours alphabetically and use the starting points $G, A$, and $E$. List the order that you look at the vertices for those three searches. What do you notice about the length of the lists of vertices?

8. (a) Compare your lists from part (a) and part (b) in question 7 for each separate starting point. Which vertices can be found faster using BFS from the starting point $G$ ? What about for the staring points $A$ and $E$ ?
(b) Compare your lists from part (a) and part (b) in question 7 for each separate starting point. Which vertices can be found faster using DFS from the starting point $G$ ? What about for the staring points $A$ and $E$ ?
(c) Compare your lists from part (a) and part (b) in question 7 for each separate starting point. Are there any vertices found in the same order or at the same time between using BFS and DFS from the starting point $G$ ? What about for the staring points $A$ and $E$ ?
