



Grade 6 Math Circles

Oct 11/12/13, 2022

Counting - Problem Set

1. A palindrome is a whole number which is the same whether you read it normally (left to right) or backwards (right to left). For example 6, 22, 313, 4884, 54745, and 612216 are all palindromes. How many 5-digit palindromes are there?
2. At the Gauss School of Mathematics, there are 17 members of the math team. Each of these members is taking at least one foreign language class. The school only offers French and German as its foreign language classes. 13 members of the math team members are taking French and 4 are taking both French and German. How many math team members are taking German?
3. You have 2 pairs of shoes, 3 pairs of socks, 5 pairs of pants, and 9 shirts. An outfit consists of one pair of shoes, one pair of socks, one pair of pants, and one shirt. How many outfits can you make?
4. Evaluate the following:
 - (i) $\frac{19!-18!}{17!}$
 - (ii) ${}_{178}P_1 \times {}_{100}P_1$
 - (iii) ${}_9C_4 + {}_9C_5 - {}_{10}C_5$
 - (iv) ${}_{10}C_5 + {}_{10}C_6 - {}_{11}C_6$
5. The Friday Math Circles Grade 7/8 class has four grade 7 students and three grade 8 students. How many ways can they be seated in a row of 7 chairs such that at least two grade 7 students are sitting next to each other?
6. For each of the following words, how many ways can we rearrange the letters in the word? (eg. in Example 3, we saw that there are 24 ways of rearranging the letters of the word MATH)
 - (a) COUNT
 - (b) CIRCLES
 - (c) ONONONONONO
7. How many ways can 5 students be seated at a round table? Two seatings are considered the same if, for every student, the person sitting on their right is the same in both seatings (i.e. can be achieved by rotation).



8. (*Gauss 2019 Gr. 8 # 21*) In Jen's baseball league, each team plays exactly 6 games against each of the other teams in the league. If a total of 396 games are played, how many teams are in the league?
9. A regular deck has 52 cards. There are 13 cards of each suit: Hearts, Diamonds, Spades, and Clubs. In each suit the 13 cards are numbered 2, 3, ..., 10, J, Q, K, A. A flush consists of a group of 5 cards of the same suit. How many different flushes are there in a regular deck?
10. (*Fermat 2019 # 18*) How many 7-digit positive integers are made up of the digits 0 and 1 only, and are divisible by 6?
11. An ant starts at $(0, 0)$ on the coordinate plane. The ant can repeatedly move either up one unit or to the right one unit. That is, if the ant is at (x, y) then it can either move to $(x + 1, y)$ or $(x, y + 1)$. How many paths could the ant take to reach $(5, 5)$?
12. Find a formula for the number of diagonals in a convex polygon with n sides.
Recall: A convex polygon is a polygon such that every angle is less than 180° . A diagonal in a convex polygon is a line segment between two non-adjacent vertices (two vertices are called adjacent if the line segment between them is a side of the polygon).