

Math Circles - Pigeonhole Principle - Fall 2022

Exercises

Graph Theory

1. Show that in a group of 100 people, at least two people must know the same number of people (assuming that knowing each other is mutual, i.e., if I know you then you also know me).
2. Prove that, in a group of 6 children (all of whom know each other), there will always either be a group of 3 children who are all friends, or a group of 3 children who do not get along.

Colouring the Plane

1. Colour the plane with two colours. Prove that there must be an equilateral triangle whose vertices are all the same colour.
2. Colour the plane with two colours. Prove that on any line segment, there must be three points of the same colour such that one point is exactly halfway in between the other two points.
3. Colour the plane with n colours. Prove that one can find a rectangle whose four corners are all the same colour.