



Grade 6 Math Circles

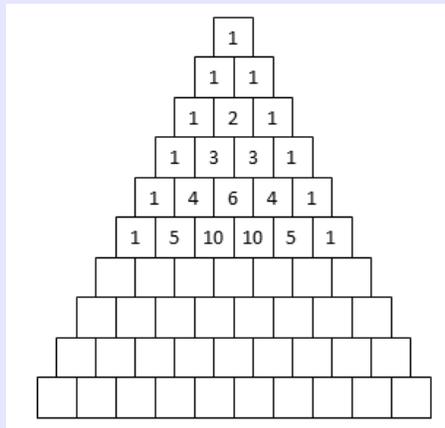
November 22/23/24, 2022

Pascal's Triangle Solutions

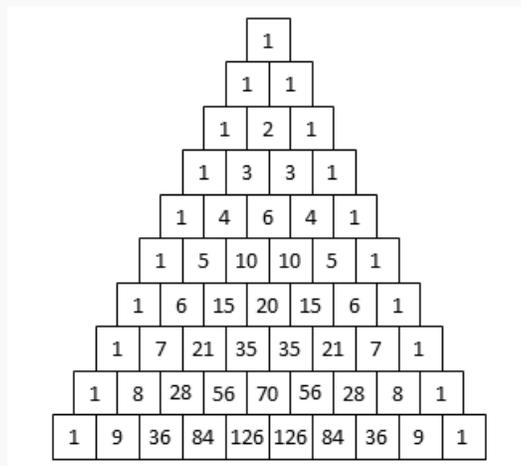
Exercise Solutions

Exercise 1

Fill in the empty rows using the addition rule.



Exercise 1 Solution





Exercise 2

What is the sum of the entries in the 7th row of Pascal's triangle?

Exercise 2 Solution

The sum of row 7 is equal to $2^7 = 128$.

Exercise 3

Use Pascal's triangle to find 11^7 .

Exercise 3 Solution

Row 7 is 1, 7, 21, 35, 35, 21, 7, 1.

So, $11^7 = 1(10^0) + 7(10^1) + 21(10^2) + 35(10^3) + 35(10^4) + 21(10^5) + 7(10^6) + 1(10^7) = 19487171$.

Exercise 4

What is the 12th Fibonacci number?

Exercise 4 Solution

To find the 12th Fibonacci number, we find the sum of the entries in the 12th diagonal of our triangle which is shifted to the left. In our above image, we found up to the 11th Fibonacci number, so we look at the next diagonal for the 12th. Remember that we need the entire diagonal, so we need one more row in the above image.



$$\begin{aligned}
6^6 &= \frac{(6 \times 5 \times 4 \times 3 \times 2 \times 1) \times (6 \times 15 \times 20 \times 15 \times 6 \times 1)}{5 \times 10 \times 10 \times 5 \times 1} \\
&= \frac{((3 \times 2) \times 5 \times (2 \times 2) \times 3 \times 2) \times ((3 \times 2) \times (5 \times 3) \times (5 \times 2 \times 2) \times (5 \times 3) \times (3 \times 2))}{5 \times (5 \times 2) \times (5 \times 2) \times 5} \\
&= \frac{(5 \times 5 \times 5 \times 5) \times (3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)}{(5 \times 5 \times 5 \times 5) \times (2 \times 2)} \\
&= (3 \times 3 \times 3 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2) \\
&= 729 \times 64 \\
&= 46656
\end{aligned}$$

Exercise 6

Below is a checkerboard with one checker within a square on the bottom row. The checker can only move diagonally to a square above it. How many different ways can the checker move to the opposite side of the board?

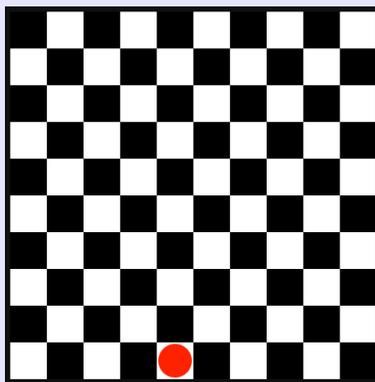
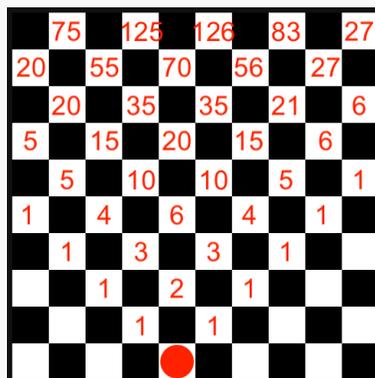


Figure: Retrieved from [Wikipedia](#).

Exercise 6 Solution

We can count the possible paths to each square and continue on until we reach the end.

Figure: Retrieved from [Wikipedia](#).

So, looking at the entries along the edge, there are $75 + 125 + 126 + 83 + 27 = 436$ different ways to reach the opposite side of the board.

Notice that the numbers match the entries in Pascal's triangle, except along the edges where the board ends before the triangle would. This is because we are using the same addition principle to find the number of paths.

Exercise 7

Find the entry of Pascal's triangle given each row number and term number.

- | | | |
|------------------|------------------|-------------------|
| a) row 0, term 0 | c) row 4, term 4 | e) row 28, term 0 |
| b) row 6, term 3 | d) row 9, term 1 | f) row 8, row 7 |

Exercise 7 Solution

- | | |
|---|--|
| a) $\binom{0}{0} = \frac{0!}{0!(0-0)!} = \frac{1}{1} = 1$ | d) $\binom{9}{1} = \frac{9!}{1!(9-1)!} = \frac{9!}{8!} = \frac{9 \times 8!}{8!} = 9$ |
| b) $\binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6!}{3! \times 3!} = \frac{6 \times 5 \times 4 \times 3!}{6 \times 3!} = 5 \times 4 = 20$ | e) $\binom{28}{0} = \frac{28!}{0!(28-0)!} = \frac{28!}{28!} = 1$ |
| c) $\binom{4}{4} = \frac{4!}{4!(4-4)!} = \frac{4!}{4! \cdot 0!} = \frac{4!}{4!} = 1$ | f) $\binom{8}{7} = \frac{8!}{7!(8-7)!} = \frac{8 \times 7!}{7! \times 1!} = 8$ |

**Exercise 8**

Audrey is painting her room and she has 6 different paint colours: red, orange, yellow, green, blue, and purple. Use Pascal's triangle to determine how many different colour combinations can she use to paint her room if she uses...

- a) 1 colour b) 4 colours c) 6 colours

Exercise 8 Solution

- a) We want to choose 1 colour from 6 colours. The number of ways of doing this is equal to $\binom{6}{1}$. Looking at Pascal's triangle, we see that the entry with row number 6 and term number 1 is 6. So, there are 6 different ways of choosing 1 colour.
- b) We want to choose 4 colour from 6 colours. The number of ways of doing this is equal to $\binom{6}{4}$. Looking at Pascal's triangle, we see that the entry with row number 6 and term number 4 is 15. So, there are 15 different ways of choosing 4 colours.
- c) We want to choose 6 colour from 6 colours. The number of ways of doing this is equal to $\binom{6}{6}$. Looking at Pascal's triangle, we see that the entry with row number 6 and term number 6 is 1. So, there is only one way of choosing all 6 colours.



Problem Set Solutions

1. Create Pascal's triangle with rows 0 to 14. Refer to this triangle for the rest of the problems.

Problem 1 Solution

													1													
											1	1														
									1	2	1															
							1	3	3	1																
					1	4	6	4	1																	
			1	5	10	10	5	1																		
	1	6	15	20	15	6	1																			
	1	7	21	35	35	21	7	1																		
	1	8	28	56	70	56	28	8	1																	
	1	9	36	84	126	126	84	36	9	1																
	1	10	45	120	210	252	210	120	45	10	1															
	1	11	55	165	330	462	462	330	165	55	11	1														
	1	12	66	220	495	792	924	792	495	220	66	12	1													
	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1												
	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1											

2. What is the sum of the 12th row of Pascal's triangle?

Problem 2 Solution

The sum of row 12 is equal to $2^{12} = 4096$.



3. Use Pascal's triangle to find 11^9 .

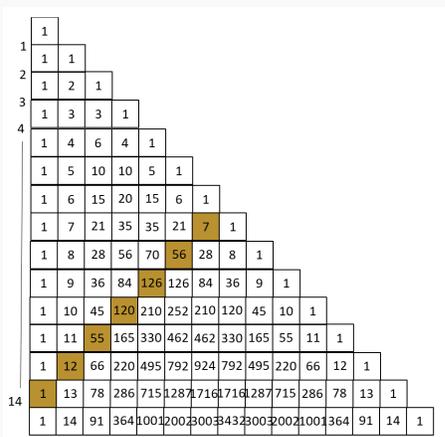
Problem 3 Solution

Row 9 is 1, 9, 36, 84, 126, 126, 84, 36, 9, 1.

$$\begin{aligned} \text{So, } 11^9 &= 1(10^0) + 9(10^1) + 36(10^2) + 84(10^3) + 126(10^4) + 126(10^5) + 84(10^6) + 36(10^7) + 9(10^8) + 1(10^9) \\ &= 2357947691 \end{aligned}$$

4. Find the 14th Fibonacci number using Pascal's Triangle.

Problem 4 Solution

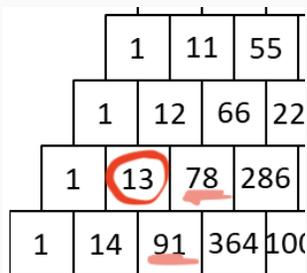


Since $1 + 12 + 55 + 120 + 126 + 56 + 7 = 377$, the 14th Fibonacci number is 377.

5. Use Pascal's triangle to find 13^2 .



Problem 5 Solution



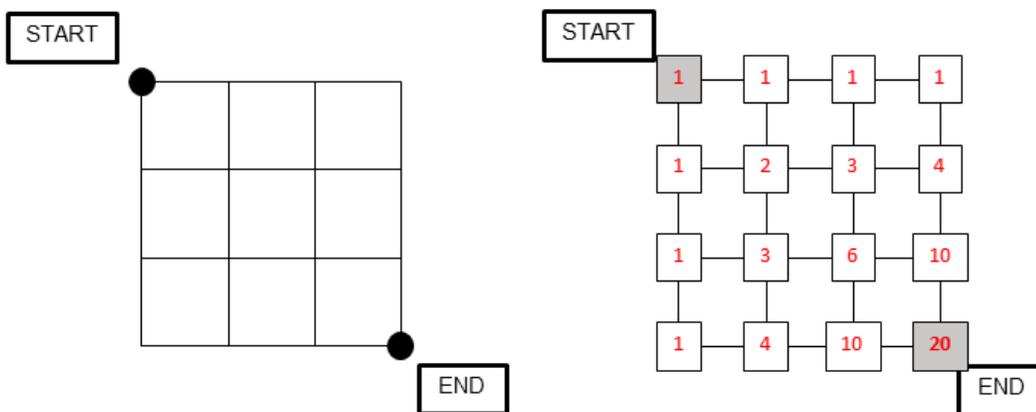
Referring back to the triangle from Problem 1, $13^2 = 78 + 91 = 169$.

6. Try building a triangle using our addition rule but starting with a number other than 1. What do you notice about your triangle? Does it relate at all to Pascal's triangle?

Problem 6 Solution

Each entry is just the number it would have been for Pascal's triangle multiplied by the number you started with.

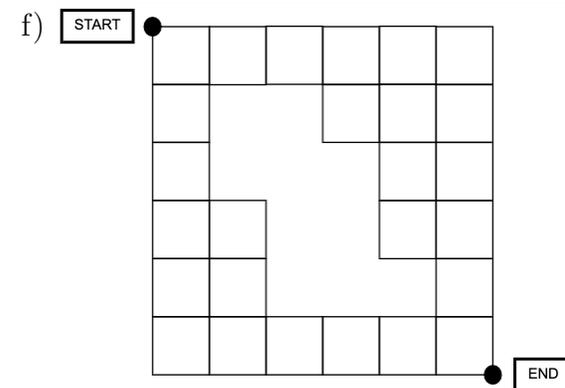
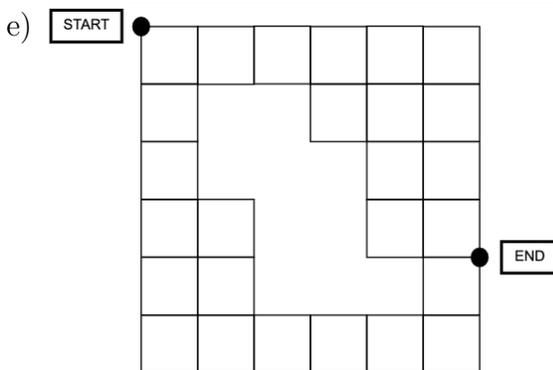
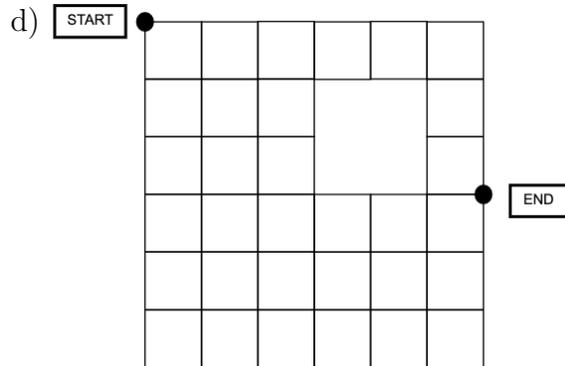
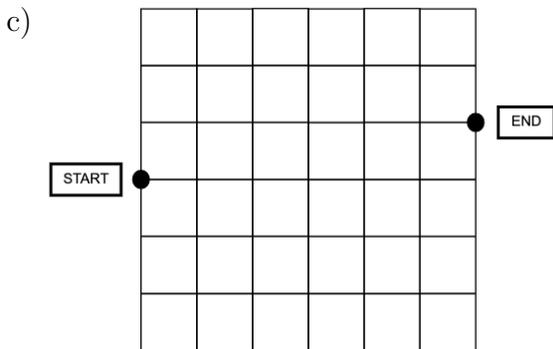
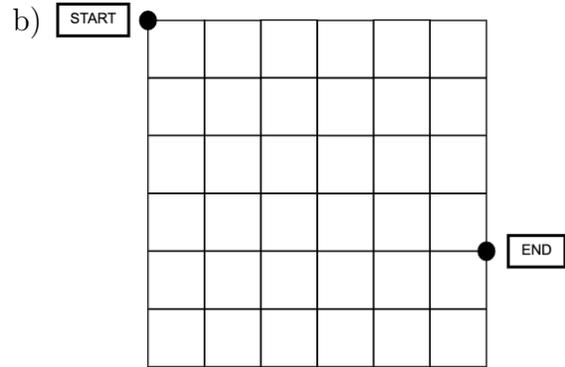
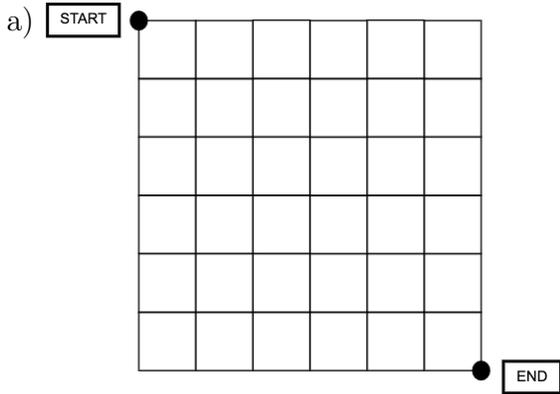
7. For each of the grids below, count how many paths can be taken from the dot marked START to the dot marked END by moving along the lines and only moving down or to the right. Below is an example.



We can write the number of paths which can be taken to get to each point. So, the number of

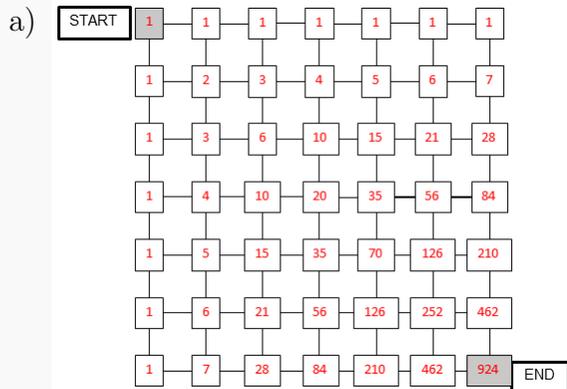


paths which can be taken to get to one specific point, a , is just the sum of the number of paths for each of the two points which can be taken to get to a . In the above example, there are 20 different paths from START to END.

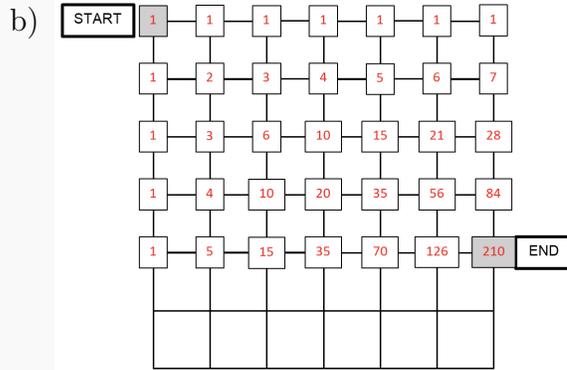




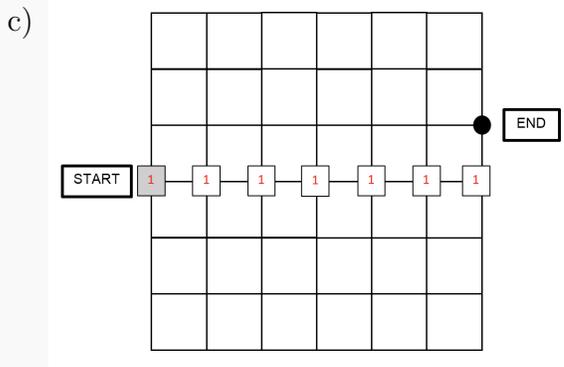
Problem 7 Solution



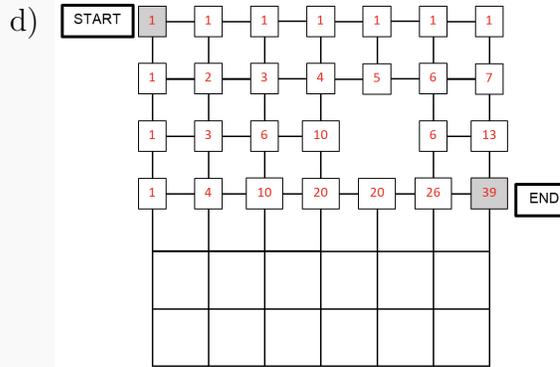
There are 924 different paths.



There are 210 different paths. Notice that we don't need to check any points below the End point because movement is only allowed down and to the right, so points below the End point will have no effect on it.



There are no possible paths because the End point is above the Start point, but movement is only allowed down and to the right.



There are 39 different paths.



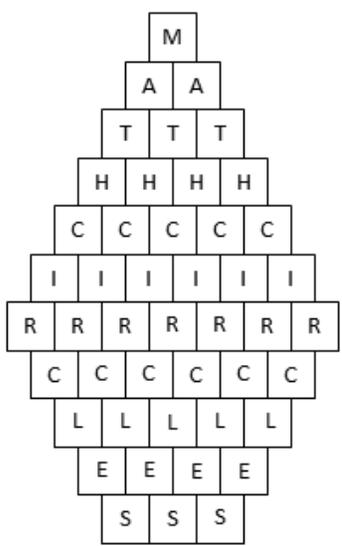
e)

There are 79 different paths.

f)

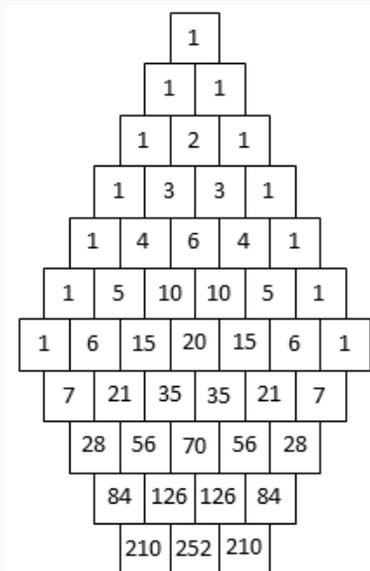
There are 227 different paths.

8. Using the following diagram, how many ways can you make a path that spells MATH CIRCLES by starting at M and moving downwards?



Problem 8 Solution

What we're counting is how many paths (as defined in the lesson) we can take to reach each S. Since the number of paths is equal to the entry in Pascal's triangle we can replace the letters with the entries from Pascal's triangle.



The number of ways you can write MATH CIRCLES by ending with the first S is 210, the number of ways you can write it by ending with the second S is 252, and the number of ways you can write it by ending with the third S is 210. In total, the number of ways we can make a path that spells MATH CIRCLES is $210 + 252 + 210 = 672$.

9. A student council is made up of 6 students in grade 5 and 8 students in grade 6. They want to choose 4 students to speak about various topics in the next assembly.
- How many ways can they choose the 4 students without restrictions?
 - How many ways can they choose the 4 students if they can only choose grade 6 students?
 - How many ways can they choose the 4 students if there must be an equal number of grade 5 and grade 6 students?

Problem 9 Solution

- We want to choose 4 students from 14 students and the number of ways of doing this is $\binom{14}{4}$. Looking at Pascal's triangle, the entry with row number 14 and term number 4 is 1001. So, there are 1001 ways of choosing the 4 students.
- We want to choose 4 students from 8 students and the number of ways of doing this is $\binom{8}{4}$. Looking at Pascal's triangle, the entry with row number 8 and term number



4 is 70. So, there are 70 ways of choosing the 4 students.

- c) We want to choose 2 students from 6 students and 2 students from 8 students. The number of ways of doing this is $\binom{8}{2} + \binom{7}{2}$. Using Pascal's triangle, we get $28 + 21 = 49$. So, there are 49 ways of choosing the 4 students.