Grade 6 Math Circles

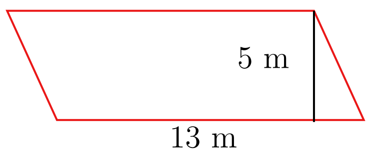
October 3/4/5, 2023

Area, Volume, and Optimization - Problem Set

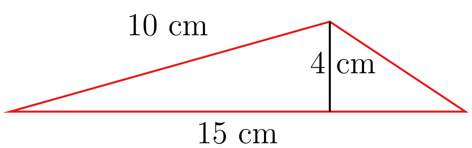
1. Calculate the area of the following shapes.

   (a) A rectangle with sides of length 8 cm and 6 cm.

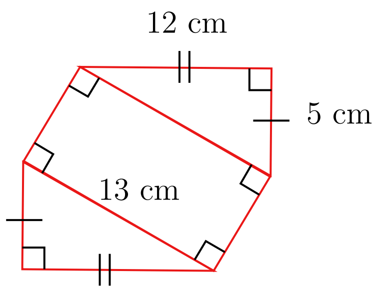
   (b)



   (c)



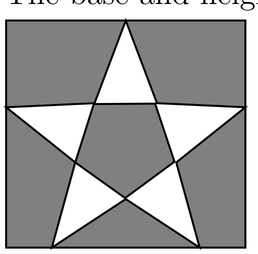
   (d) The perimeter of the following shape is 40 cm.



   (e) Calculate the area of the grey region given the following information:

   The area of the square is 100 cm^2.

   The base and height of each triangle is 3 cm and 4 cm, respectively.

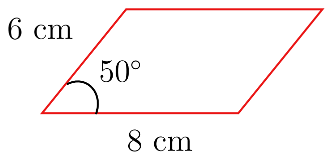


2. Determine both the maximum area and the dimensions or angle that produce the maximum area of the following shapes.

   (a) A rectangle with a perimeter of 28 cm.

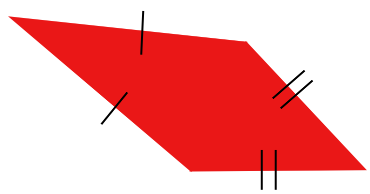
   (b) A triangle with a base of 9 cm, a height of 5 cm, and another side of length 6 cm.

   (c) The following parallelogram has its side lengths fixed. Change the angle to optimize the area.



3. You formed parallelograms and rectangles from pairs of congruent triangles in today's lesson. You might have noticed you can form rectangles from right triangles, but not from acute or obtuse triangles. Why is this?

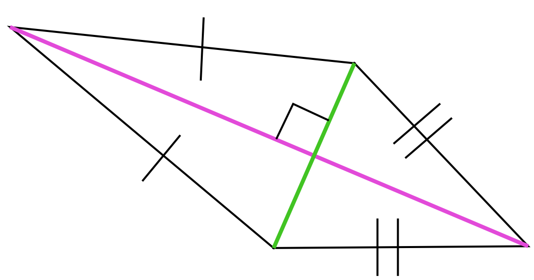
4. (a) Draw two congruent scalene triangles. Record the base and height of your triangles using a ruler. Recall that a scalene triangle is one with no equal sides. Form a **kite** using these two triangles. A kite is a quadrilateral with two pairs of equal adjacent sides.



Example kite

   (b) Calculate the area of your kite using the base and height of the triangles you measured.

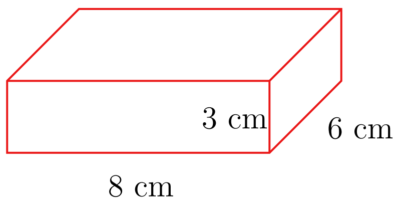
   (c) Calculate the area of this kite by dividing it into triangles.



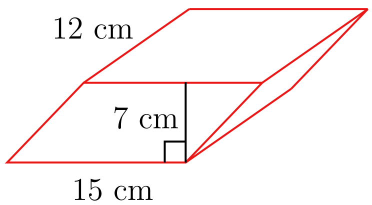
The green line has a length of 8 cm and the purple line has a length of 14 cm.

5. Calculate the volume of the following solids.

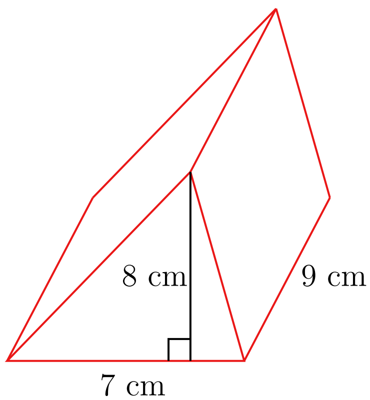
(a)



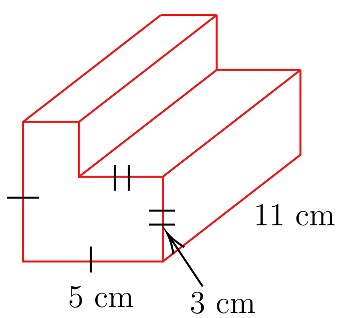
(b)



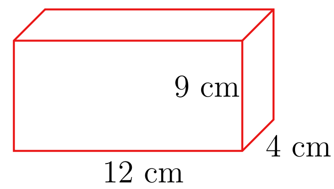
(c)



(d)



6. (a) Calculate the surface area of this rectangular prism.



(b) Determine both the maximum volume and the dimensions that produce the maximum volume of the above rectangular prism while keeping surface area constant.

(c) Recalculate the surface area of the optimized rectangular prism. It should be the same as the answer to part (a).

7. Farmer Peter wants to build a fence around two rectangular fields on his property for his crops. He has two types of crop; corn and beans. There must be a 4 meter wide alley between his corn field and bean field. Both fields must have the same area. He has 200 meters of fencing available and must use all of it. The fencing only surrounds the outside of the fields and not the alley between the fields.

   What are the dimensions of each field such that the area of the fields is maximized? What is the area of each field? What is the area of the total fenced area?

   Draw a diagram to help with your answer.

8. CHALLENGE PROBLEM

(a) Brian wants to install an in-ground pool in his backyard. He wants it to be 2 m deep, 10 m long, and as wide as possible given that he has only 110 m^2 of material to make the walls and bottom of the pool. How many meters wide can he make the pool? What is the volume of the pool?

(b) Brian remembers that he also needs to add a tiled border around the edge of the pool. He wants it to be 1 m wide around the entire pool. He decides to buy 26 m^2 of extra material for the border. Given the pool must be the same depth and same length as part (a), show that the maximum width of the pool with the border is 4.5 m. What is the volume of the pool?