Grade 7/8 Math Circles
Week of 13th November
Types of Numbers

Exercise Solutions

1. From the sets we've already looked through, we have that
   (a) $11 \in \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
   (b) $\sqrt{4} = 2 \in \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
   (c) $\frac{-2}{1} = -2 \in \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
   (d) $\sqrt{11} \in \bar{\mathbb{Q}}, \mathbb{R}$
   (e) $4.333\bar{3} \in \mathbb{Q}, \mathbb{R}$
   (f) $\pi \in \bar{\mathbb{Q}}, \mathbb{R}$
2. We can solve for x, and the solutions are imaginary,
   (a) $x = \pm 3i$
   (b) $x = \pm 2i$
   (c) $x = \pm\sqrt{5}i$
3. Graphing these out, we have

4. Computing these expressions using our formula for complex addition, we have that
   (a) $(1 + 6i) + (3 + 4i) = 4 + 10i$
   (b) $(4 + 2i) - (8 - 3i) = -4 + 5i$
   (c) $(3 + 2i) + ((2 - i) + (3 - 2i)) = 8 - i$

5. Using our formula for multiplication and division for complex numbers, we can evaluate the expressions
    (a) $\frac{1+i}{1-i} = i$
    (b) $(2-3i)\cdot\left(\frac{1}{2}-\frac{1}{3}i\right) = -\frac{13}{6}i$
    (c) $((4+i)-(2+2i))\cdot(1-i) = 1-3i$
    (d) $(2-3i)\cdot((3-i)+(2+2i)) = 13-13i$
    (e) $\frac{1-3i}{5-2i} = \frac{11}{29}-\frac{13}{29}i$
    (f) $\left(\frac{3}{4}i\right)\cdot\left(\frac{4}{3}i+\frac{1}{3}\right) = -1+\frac{1}{4}i$
6. Using the formula for the modulus, we have that
    (a) $|z| = |1-i| = \sqrt{2}$
    (b) $|z| = |\sqrt{2}+\sqrt{2}| = 2$
    (c) $|z| = |1+i| = \sqrt{2}$
    (d) $|z| = \left|\frac{-1-i}{1-i}\right| = |-i| = 1$
7. We can compute the following set theoretic expressions
    (a) $\mathbb{N}\cap\mathbb{I} = \varnothing$
    (b) $\mathbb{N}\cup\mathbb{Z} = \mathbb{Z}$
    (c) $\mathbb{Q}\cup\bar{\mathbb{Q}} = \mathbb{R}$
    (d) $\mathbb{R}\cap\mathbb{I} = \varnothing$
    (e) $\mathbb{N}\cap\mathbb{Z} = \mathbb{N}$
    (f) $\mathbb{R}\cup\mathbb{I} = \mathbb{C}$

Problem Set Solutions

1. Using our sets we derived in the lesson, we have that
    (a) $3+3i \in \mathbb{C}$
    (b) $\pi \in \bar{\mathbb{Q}}, \mathbb{R}$
    (c) $0 \in \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{I}, \mathbb{C}$
    (d) $-\frac{17}{\sqrt{2}} \in \bar{\mathbb{Q}}, \mathbb{R}$
    (e) $\sqrt{5}i \in \mathbb{I}, \mathbb{C}$
2. We know that elements of $\bar{\mathbb{Q}}$ are numbers that cannot be written as a fraction, thus we have that only $\sqrt{5}$ is irrational.

3. Evaluating the following expressions, we have that
   (a) $(2+3i)+(3-\frac{1}{2}i)=5+\frac{5}{2}i$
   (b) $(2-4i)-(3+4i)=-1-8i$
   (c) $(1-2i)\cdot(2+2i)=6-2i$
   (d) $(3-4i)+((1-3i)\cdot(1+2i))=10-5i$
4. Evaluating the following expression gives,
   (a) $\frac{1+2i}{2-i}=i$
   (b) $|4+7i|=\sqrt{65}$
   (c) $\frac{5-4i}{3+4i}=-\frac{1}{25}(1+32i)$
   (d) $\frac{3-4i}{5+12i}=-\frac{1}{169}(33+56i)$
5. The following statements are either true or false,
   (a) The product of two irrational numbers is always irrational. $\mathbb{F}:\sqrt{2}\cdot\sqrt{2}=2$
   (b) The product of two integers always an integer. **T**
   (c) The product of two complex numbers is always complex. **T**
   (d) The product of two natural numbers is always a real number. **T**
6. Solving for x, we have that
   (a) $x^2+1=0 \implies x=\pm i$
   (b) $x^2=-36 \implies x=\pm 6i$
   (c) $x^2+2=0 \implies x=\pm\sqrt{2}i$
   (d) $x^2+1=\frac{1}{2} \implies x=\pm\frac{1}{\sqrt{2}}i$
7. Given $x=5$ and $y=4$, we have that
   (a) $\sqrt{x+y}=\sqrt{5+4}=\sqrt{9}=3\in\mathbb{Q}$
   (b) $\sqrt{x-y}=\sqrt{5-4}=\sqrt{1}=1\in\mathbb{Q}$
   (c) $\sqrt{x\cdot y}=\sqrt{5\cdot 4}=\sqrt{20}\in\bar{\mathbb{Q}}$
   (d) $\sqrt{x/y}=\sqrt{5/4}\in\bar{\mathbb{Q}}$
8. Evaluating the following set expressions, we have that
   (a) $A\cap\bar{A}=\varnothing$
   (b) $\mathbb{N}\cup\mathbb{I}=\varnothing$
   (c) $\mathbb{R}\cap\mathbb{C}=\mathbb{R}$
   (d) $\{a,b,c,d,e,...\}\cap\{a,e,i,o,y,u\}=\{a,e,i,o,y,u\}$