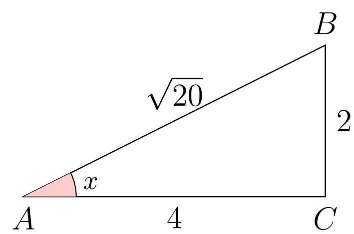
Grade 7/8 Math Circles

February 12-15, 2024

Trigonometric Ratios - Problem Set

1. Use the Pythagorean Theorem to find the missing side length of a right-angled triangle with a hypotenuse of 23.3 and another side length of 10.5.

2. Consider the triangle $\triangle ABC$ below:



   (a) Verify that $\triangle ABC$ is a right-angled triangle by showing that the Pythagorean Theorem holds, then determine the sine, cosine and tangent ratios of angle x.

   (b) What are three different (but similar) ways we can solve for the value of x?

3. Create a triangle that has two equal side lengths, an angle of 90°, and some other angle A.

   (a) What is $\tan(A)$? Do such triangles always have the same tangent ratio?

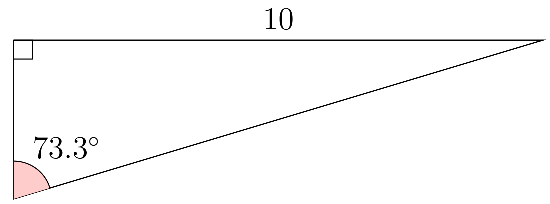
   (b) Solve for A. How could you have done this in your head? (*Hint*: the sum of angles in a triangle is 180°).

4. Maya walks 8m North then 15m East. How much less distance does she walk if she travels along a straight path from her starting to final position?

5. The area of a right-angled triangle is found by multiplying its base length, b, by its height, h, then dividing this product by 2. The formula is then

$$A = \frac{b \cdot h}{2}$$

   Use this to find the area of the following right-angled triangle:



6. Two pedestrians see a plane in the sky nearby. Pedestrian A sees the plane approaching them from the right at a distance of 40m away at an angle of 30° above the ground. Pedestrian B sees the plane flying away from them towards the left at an angle of 45° above the ground. How far is pedestrian B from the plane at this instant?

7. The sun emits a ray of light that strikes the top of a 120ft tall building, creating a 350ft long shadow past the building along the ground. What angle does the light make with the ground?

8. Recall the Cosine Law:

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$

Its formula is quite similar to the Pythagorean Theorem:

$$c^2 = a^2 + b^2$$

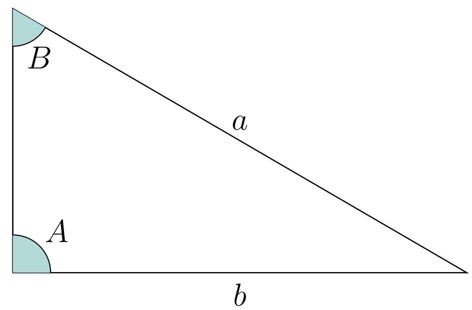
Describe the relationship between the two equations and explain how the Pythagorean Theorem is just a special case of the Cosine Law for positive values of a, b & c. What must angle C be?

9. A triangle is placed on a grid. Its three vertices on the (x, y) plane are point A at the coordinates $(-1, 1)$, B at $(2, -1)$, and C at $(-3, -3)$. Find the total perimeter of the triangle without finding any angles. (*Hint:* the distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

simply from using the Pythagorean Theorem).

10. Consider the triangle below.

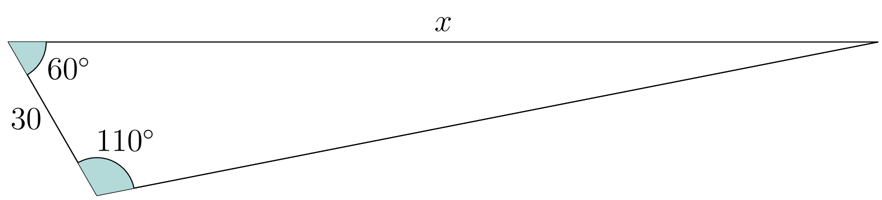


(a) What does the Sine Law tell us about this triangle?

(b) Given that $\sin(90°) = 1$, use (a) to prove the SOH in SOH CAH TOA. That is, for a right-angled triangle, the sine of an angle within the triangle is equal to the ratio of the angle's opposite side to the triangle's hypotenuse.

11. An equilateral triangle is any triangle that has all three sides of equal length. Use the Cosine Law to show that each angle inside any equilateral triangle is 60°.

12. * Solve for x using only the Pythagorean Theorem and/or SOH CAH TOA. Verify your answer with either the Sine or Cosine Law. Which method is simpler?



13. ** The perimeter of the following triangle is 56cm. Solve for each side length and angle. Do the angles change for different values of x?