Problem of the Month
Problem 4: January 2024

Hint

(a) Any list that compresses [1 : 9] must contain 1. Think about the largest possible number of integers in \( f(A) \) when \( A \) is a list of length \( k \).

(b) First, try to find a list that compresses [1 : 63] that is as short as possible. It might help to read about the binary representation of positive integers.

(c) Work out a few more examples like the one in (b). It is possible to compress [1 : \( n \)] using a list \( A \) that consists entirely or almost entirely of powers of 2.

(d) For \( k \geq 3 \) and \( m \geq 2 \), if \( A \) compresses \([m : m + k - 1]\), then \( A \) must contain \( m \) and \( m + 1 \).

(e) The answer is 39. Do not worry about trying to compress [5 : \( k \)] using as short a list as possible. As well, inductive thinking could be useful here. Suppose you can show that there is some \( k \) with the property that [5 : \( k \)], [5 : \( k + 1 \)], [5 : \( k + 2 \)], [5 : \( k + 3 \)], and [5 : \( k + 4 \)] are all compressible. Can you deduce that [5 : \( n \)] is compressible for all \( n \geq k \)?