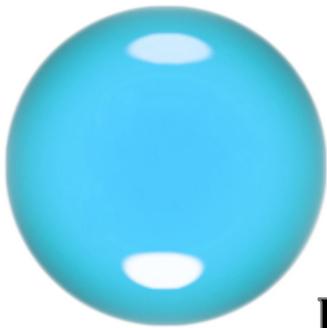


Problem of the Week

Grade 5/6 (B)

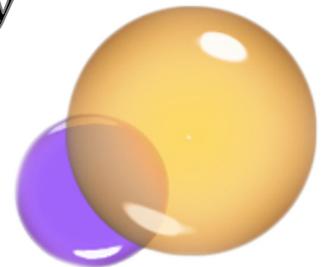
Problem and Solutions

2018 - 2019



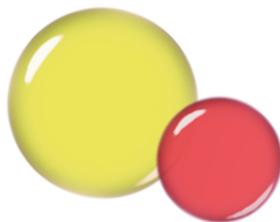
Strands

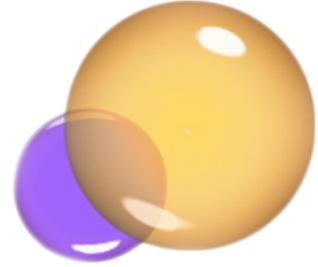
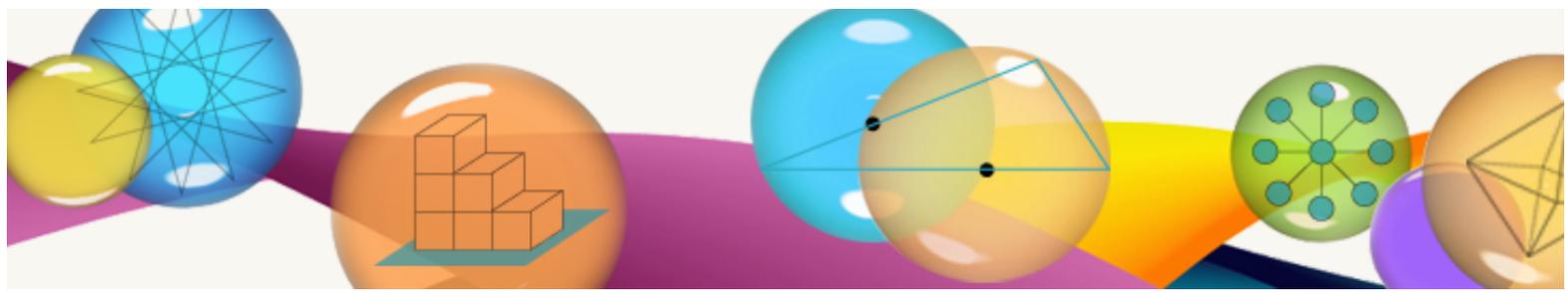
Data Management & Probability
Geometry & Spatial Sense
Measurement
Number Sense & Numeration
Patterning & Algebra



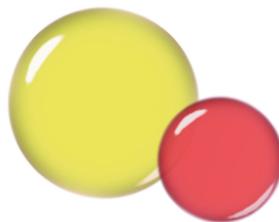
(Click the strand name above to jump to that section)

The problems in this booklet are organized into strands. A problem often appears in multiple strands. The problems are suitable for most students in Grade 5 or higher.





Data Management & Probability





Problem of the Week

Problem B

Game On ...

Many sports are divided into a fixed number of “playing periods” or “parts”, with each part having a fixed length of playing time. The following chart outlines how many parts are in one game for various sports, the playing time for one part, and the total playing time for one game without any stoppages.

- a) Complete the chart using the information provided.

Sport	Number of “parts” in one game	Playing time for one “part”	Total playing time for one game
NFL football	4 quarters	15 minutes	
NBA basketball	4 quarters		48 minutes
Hockey	___ periods	20 minutes	60 minutes
Soccer	2 halves	45 minutes	
Lacrosse	4 quarters		60 minutes

- b) A NFL football game between Buffalo and Detroit started at 1:00 p.m. The game took 2 hours and 12 minutes longer than the playing time with no stoppages. At what time did the game end?
- c) A basketball game between Toronto and Boston started at 7:00 p.m. and ended at 9:15 p.m. How much of this time was during stoppages in play?





Problem of the Week

Problem B and Solution

Game On ...



Problem

Many sports are divided into a fixed number of “playing periods” or “parts”, with each part having a fixed length of playing time. The following chart outlines how many parts are in one game for various sports, the playing time for one part, and the total playing time for one game without any stoppages.

- a) Complete the chart using the information provided.

Sport	Number of “parts” in one game	Playing time for one “part”	Total playing time for one game
NFL football	4 quarters	15 minutes	60 minutes
NBA basketball	4 quarters	12 minutes	48 minutes
Hockey	3 periods	20 minutes	60 minutes
Soccer	2 halves	45 minutes	90 minutes
Lacrosse	4 quarters	15 minutes	60 minutes

- b) A NFL football game between Buffalo and Detroit started at 1:00 p.m. The game took 2 hours and 12 minutes longer than the playing time with no stoppages. At what time did the game end?
- c) A basketball game between Toronto and Boston started at 7:00 p.m. and ended at 9:15 p.m. How much of this time was during stoppages in play?

Solution

- a) See the completed chart in the problem statement above.
- b) Since the game took 2 hours and 12 minutes longer than the time with no stoppages, there were 2 hours and 12 minutes of stoppages. The time from start to finish for the football game was 60 minutes of playing time plus 2 hours and 12 minutes of stoppages, or 3 hours and 12 minutes. Thus, the game finished at 4:12 p.m.
- c) From start to finish, the time for the basketball game was 2 hours and 15 minutes, or $2 \times 60 + 15 = 135$ minutes. This includes playing time and stoppage time. Thus, the stoppage time is 135 minutes minus playing time, or $135 - 48 = 87$ minutes.





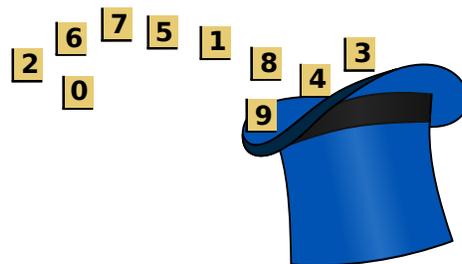
Problem of the Week

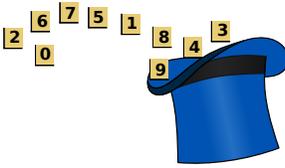
Problem B

Luck of the Draw

James' classroom is organizing a draw to give out a prize which is a "MATHIE" T-shirt. Possible ways to organize the draw are presented below.

- Tiles with the whole numbers from 0 through 9 are thrown into a hat. Each tile has a single digit on it. Each digit is represented exactly once. Without looking, you reach in and draw out one tile. You will win the draw if you pull out a 9. What is the probability of winning this draw?
- Tiles with the letters A through Z are thrown into a hat. Each tile has a single letter on it. Each letter is represented exactly once. Without looking, you reach in and draw out one tile. You will win the draw if you pull out an X or a Z. What is the probability of winning this draw?
- Tiles with the whole numbers from 1 through 4 are thrown into a hat. Each tile has a single digit on it. Each digit is represented exactly once. Without looking, you reach in and draw out two tiles. You will win if your two tiles, placed side-by-side, can form the number 13. What is the probability of winning this draw?
- Which of the above draws has the greatest probability of winning the T-shirt?





Problem of the Week

Problem B and Solution

Luck of the Draw

Problem

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- Which of the above draws has the greatest probability of winning the T-shirt?

Solution

- There are 10 tiles (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) so the probability of drawing a 9 and winning is 1 in 10, or $\frac{1}{10}$.
- There are 26 tiles, each with one letter of the alphabet. You are hoping to draw an X or Z. Since there are two possible successful outcomes, the probability of winning is 2 in 26, or $\frac{2}{26} = \frac{1}{13}$.
- There are six possible combinations of two numbers that you could draw. They are; 1 and 2, 1 and 3, 1 and 4, 2 and 3, 2 and 4, or 3 and 4. To form the number 13 you need to draw the 1 and the 3. Thus the probability of winning is 1 in 6, or $\frac{1}{6}$.
- If you take a whole number and divide it in equal sections, then the size of each section gets smaller as the number of sections increases. (For example: $\frac{1}{2} > \frac{1}{3}$, and $\frac{1}{5} > \frac{1}{10}$.) Therefore, using the probabilities $\frac{1}{10}$, $\frac{1}{13}$ and $\frac{1}{6}$, the fraction with the smallest denominator is the greatest. Therefore the draw in c) will have the greatest probability of winning with a probability of $\frac{1}{6}$.





Problem of the Week

Problem B

A 'Prime' Trip

November 11, 2018 marks the 100th anniversary of the end of World War I. In the city of Versailles, France, people gathered to watch the signing of the treaty which ended the war.

The Prime Minister of Canada wanted to go to the 100th anniversary ceremony. He asked grade 5 students to suggest a creative way for him to get from his home in Ottawa to the ceremony in Versailles. Of the suggestions received, the Prime Minister considered the following itinerary:

- take a limousine from his temporary residence at 1 Sussex Drive in Ottawa to the Ottawa Central Station;
- take a bus from there to the Greyhound bus station at 610 Bay Street in Toronto;
- then take a taxi to Union Station in Toronto;
- then take the 27.3 km trip on the Union Pearson Express train to Toronto Pearson International Airport;
- then fly from Toronto to Charles de Gaulle Airport in Paris, France;
- then fly by helicopter to Versailles (landing at Forêt Domaniale de Louveciennes); and finally
- take a taxi to the monument at the Palace of Versailles where the ceremony is to be held.

If the Prime Minister actually followed this plan, how many kilometres did he travel in total to get to the ceremony?

For information on the distances, consider using the websites

<https://maps.google.com> ; <https://www.timeanddate.com/worldclock/distance.html>



The Treaty of Versailles is signed (Source: wikipedia)





The Treaty of Versailles is signed
(Source: wikipedia)

Problem of the Week

Problem B and Solution

A 'Prime' Trip

Problem

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Solution

Answers will vary, depending on the sources used for the different segments of the trip. Remember, this total distance will be approximate. The Google maps website was used for all distances except for the flight from Toronto, Ontario to Paris, France.

According to our sources, the distances covered by the Prime Minister will be as follows:

- a limousine from his home at 1 Sussex Drive in Ottawa to the Ottawa Central Station, 5.8 km;
- a bus to the Greyhound bus station on 610 Bay Street in Toronto, 448 km;
- a taxi to Union Station in Toronto, 1.8 km;
- the Union Pearson Express train to Toronto Pearson International Airport, 27.3 km;
- flight from Toronto to Charles de Gaulle Airport in Paris, France, 6 018 km;
- flight by helicopter to Versailles, 43 km;
- a taxi to the monument where the ceremony is to be held, 10 km.

Therefore, the Prime Minister will travel the sum of these distances, approximately 6 554 km, in total. It would probably be reasonable to say that the trip is between 6 500 km and 6 600 km.





Problem of the Week

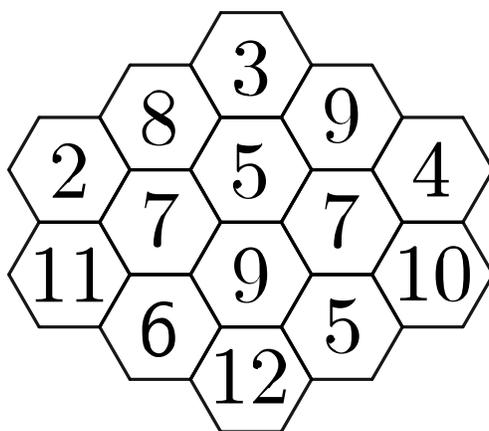
Problem B

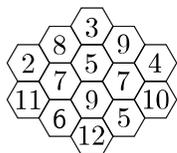
Don't Get Vexed by the Wrong Vertex!

A standard six-sided die has its faces marked with the numbers 1, 2, 3, 4, 5, and 6. The die is “fair” and each number is used exactly once. When a mathematician says a die is “fair”, they mean that on any roll there is an equally likely chance of landing on any face of the die.

A game board is made up of fourteen hexagons, as shown in the diagram below. The numbers on the hexagons are arranged randomly. You may place your game piece on any vertex shared by three hexagons. Two standard six-sided dice are then rolled and the two top numbers are added together. If this sum is equal to the number on any of the three hexagons sharing the vertex where your game piece is placed, you win the roll.

Which vertices give the best chances to win the roll? Explain your reasoning.





Problem of the Week

Problem B and Solution

Don't Get Vexed by the Wrong Vertex!

Problem

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Solution

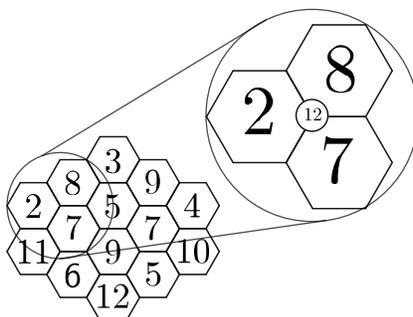
You want to place your game piece at a vertex where the adjacent numbers have the highest probability of being the sums on a roll.

Here are the number of ways each possible sum could occur on a double roll.

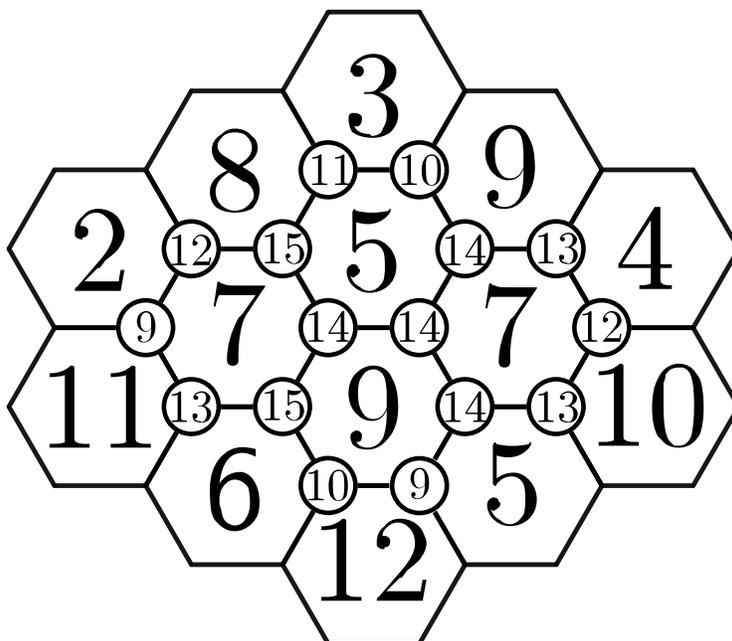
Table of Roll Sums

First Die \ Second Die	Second Die					
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We will need to look at each possible vertex and count the number of times that each of the numbers on the three adjacent hexagons occur in the table above. First, consider when we look at the vertex adjacent to the hexagons with the numbers 2, 8 and 7. The 2 occurs once in the table, the 8 occurs 5 times and the 7 occurs 6 times. Therefore, 2, 8 or 7 occur $1 + 5 + 6 = 12$ times. We will place the 12 on the vertex as shown in the diagram.



We do this for each of the possible vertices and record the numbers on each vertex. The results are shown below.



The number on each vertex is the number of possible rolls out of 36. Therefore, the larger the number the greater the probability of winning the roll. The largest number on any vertex is 15, which occurs on the vertex adjacent to the hexagons with numbers 5, 7, 8 and on the vertex adjacent to the hexagons with numbers 6, 7, 9. Therefore, each of these two vertices has a probability of winning of $\frac{15}{36}$.

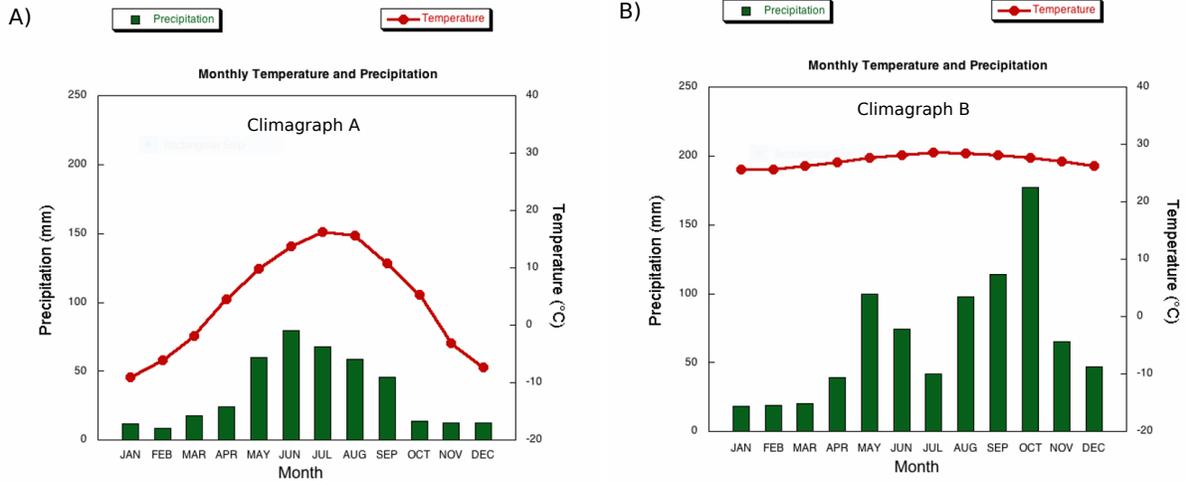
Thus, in order to have the best chance to win, you want to place your game piece on the vertex that connects the hexagons with numbers 5, 7, 8, or on the vertex that connects the hexagons with numbers 6, 7, 9.





Problem of the Week Problem B A Tale of Two Cities

Here are two climagraphs for two cities in the world.



Courtesy of: National Drought Mitigation Center, University of Nebraska - Lincoln

Study them carefully and then answer the following questions.

- a) If you wanted to spend your time in July at a nearby beach, which city would you choose, A or B?
- b) If you wanted to spend your time skiing in January, to which city would you go? Is there anything different you'd like to see on the climagraph you choose?
- c) Describe the weather in January for each city.
- d) Describe the weather in July for each city.



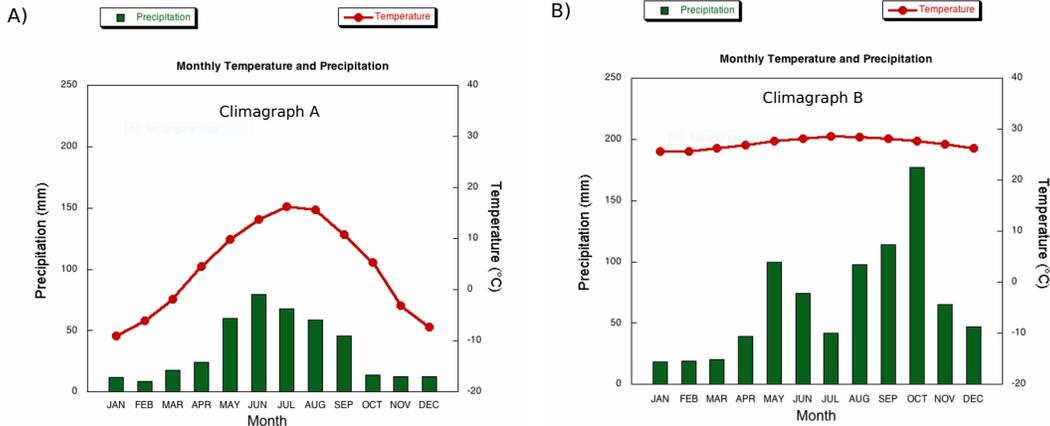


Problem of the Week Problem B and Solution A Tale of Two Cities



Problem

Here are two climagraphs for two cities in the world.



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- If you wanted to spend your time in July at a nearby beach, which city would you choose, A or B?
- If you wanted to spend your time skiing in January, to which city would you go? Is there anything different you'd like to see on the climagraph you choose?
- Describe the weather in January for each city.
- Describe the weather in July for each city.

Solution

- We would choose city B. City A has a July temperature of less than 20°C while city B has a July temperature of almost 30°C , a much nicer temperature for beach fun. City B also has less precipitation in July.
- The January temperature in city A is about -10°C , while city B sits closer to 30°C . Snow is much more likely in city A and is highly unlikely in city B. So City A would be best for skiing in January. It would even better if there were more precipitation, and thus more snow.
- City A: Cold $\approx -8^{\circ}\text{C}$, Dry ≈ 10 mm of precipitation
City B: Hot $\approx 26^{\circ}\text{C}$, Dry ≈ 20 mm of precipitation
- City A: Cool $\approx 17^{\circ}\text{C}$, Dry ≈ 70 mm of precipitation
City B: Hot $\approx 28^{\circ}\text{C}$, Dry ≈ 40 mm of precipitation

Answers may vary as students estimate temperature and precipitation from the climagraphs.



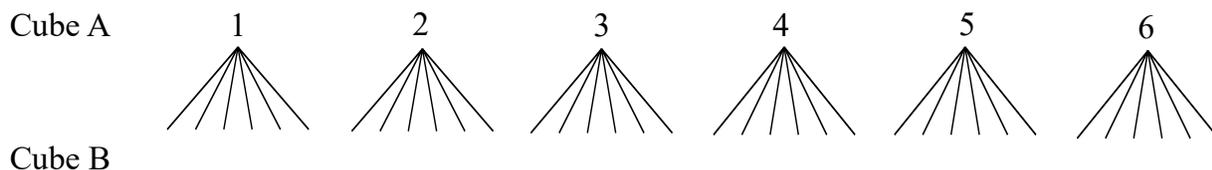
Problem of the Week

Problem B

Single, Double, Triple Trouble

In Monopoly™, you get to roll again if you roll doubles (the same number on two six-sided number cubes).

- a) What is the theoretical probability of rolling doubles? (You may find it helpful to complete the tree diagram below.)



- b) What is the theoretical probability of rolling two doubles in a row? (Think about what you would have to add to your tree diagram in part a) in order to determine the number of possible outcomes. Then deduce an easy way to get the desired probability as a simple product.)
- c) If you get three doubles in a row, you go to the “Jail” space on the board. What are the chances of rolling three doubles in a row?



Source: Creative Commons: flickr.com





Problem of the Week

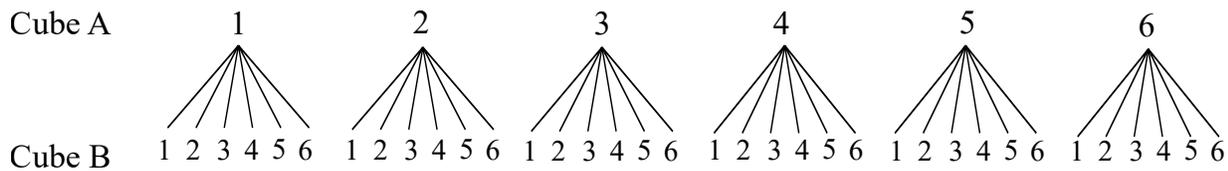
Problem B and Solution

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- c) If you get three doubles in a row, you go to the “Jail” space on the board. What are the chances of rolling three doubles in a row?

Solution

- a) From the completed tree diagram shown above, we see that there are 36 possible pairs which could occur in a roll of two number cubes. Of these, only 6 are doubles. Thus, the theoretical probability of rolling doubles is $\frac{1}{6}$.
- b) To adapt the tree diagram to a second roll would require adding six branches to EACH of the 36 numbers in the second row of part a) to represent the value on the first cube of the second roll, resulting in $6 \times 36 = 216$ branches and thus 216 numbers in the third row. Then, 6 branches would need to be added to EACH of the 216 numbers on the third row to represent the value on the second cube of the second roll, resulting in $6 \times 216 = 1296$ numbers in the fourth row. Thus, there are 1296 possible pair combinations for the two rolls.

Now, for each of the six possible doubles on the first roll, there are 6 possible doubles for the second roll (1 and 1, 2 and 2, 3 and 3, 4 and 4, 5 and 5, and 6 and 6). Thus, there are $6 \times 6 = 36$ ways to get doubles on the first and second rolls, out of a total of 1296 possible pairs. Thus, the probability of rolling two doubles in a row is $\frac{36}{1296} = \frac{1}{36}$.

The easy way to calculate this is to note that it is equal to the product of the probability of getting a double in each of the two rolls, namely $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

- c) The chances of rolling three doubles in a row is similarly found to be $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$.





Problem of the Week

Problem B

Gone Fishing...

Adam and Malene went fishing for three days and caught a lot of fish in Lake Wabanonka. Adam caught 9 pike and 5 lake trout. Malene caught 7 pike and 3 lake trout.

Assume that their total catch is representative of the populations of pike and lake trout in the lake.

- What is the theoretical probability that the next fish they catch is a pike?
- What is the theoretical probability that Malene will catch the next fish?
- If they kept fishing and caught 75 fish, how many of these fish could they expect to be pike?





Problem of the Week

Problem B and Solution

Gone Fishing...

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Solution

- Adam caught 9 pike and 5 lake trout.
Malene caught 7 pike and 3 lake trout.
Since $9 + 7 = 16$ pike were caught out of $9 + 5 + 7 + 3 = 24$ fish, the theoretical probability that the next fish is a pike is $\frac{16}{24} = \frac{2}{3}$, approximately a 67% chance.
- Malene has caught 10 of the 24 fish so far. So the probability that she will catch the next fish is $\frac{10}{24} = \frac{5}{12}$, approximately a 42% chance.
- If they kept fishing until they had 75 fish, they would expect $\frac{2}{3}$ of them to be pike, i.e., they would expect to catch $\frac{2}{3} \times 75 = 50$ pike.





Problem of the Week

Problem B

Not So Phunee...

Phunee Ghai is a struggling stand-up comedian who has comedy gigs around town on five nights out of seven. He tries to wear different outfits, just in case some audience members may want to see him more than once. Because Phunee doesn't have a lot of money, he tries to strategically coordinate his outfits so that he does not have to repeat his outfits. Here's what Phunee has in his closet.

- 1 flannel yellow and green shirt
- 1 pair Spikee running shoes
- 1 pair blue jeans
- 1 pair cowboy boots
- 1 pair khaki chinos (pants)
- 1 pair orange shorts
- 1 retro cable-knit baby blue sweater
- 2 long-sleeved T-shirts (one navy blue, one light green)
- 1 pair velcro-strapped loafers
- 1 pair black jeans



- a) Assuming he works every night, for how many weeks can Phunee do his stand-up comedy routine, wearing a different combination of clothing each time, without repeating an outfit?
- b) What is the probability of him wearing a pair of jeans and a sweater?
- c) Actually, Phunee does not work on Sundays and Mondays. If he started wearing his combinations on a Thursday, on what day would he have to make sure he has a new article of clothing to add to his wardrobe?

STRAND DATA MANAGEMENT AND PROBABILITY





Problem of the Week

Problem B and Solution

Not So Phunee...

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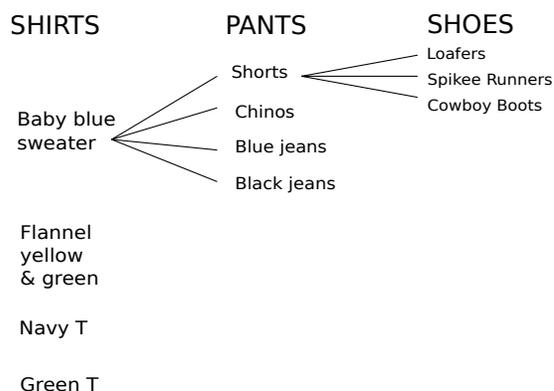




Solution

- a) The diagram shows the start of a tree structure for enumerating Phunee's possible outfits.

For each shirt, there are four possible pairs of pants. For each of those, there are three possible pairs of shoes. Thus, for each shirt, there are $4 \times 3 = 12$ possibilities. Since there are 4 different shirts, in total there are $12 \times 4 = 48$ possible different outfits.



One could also multiply the number of shirts by the number of shoes and by the number of pants, giving $4 \times 3 \times 4 = 48$ outfits.

Therefore, Phunee could work 6 weeks and 6 days before having to wear the same outfit twice.

- b) Phunee has 1 sweater, 2 pairs of jeans (blue jeans and black jeans), and 3 different pairs of shoes (loafers, runners, boots). We list all possible combinations:

Sweater	Blue Jeans	Loafers
Sweater	Blue Jeans	Runners
Sweater	Blue Jeans	Boots
Sweater	Black Jeans	Loafers
Sweater	Black Jeans	Runners
Sweater	Black Jeans	Boots

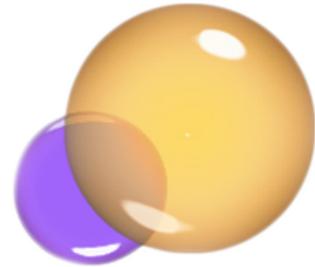
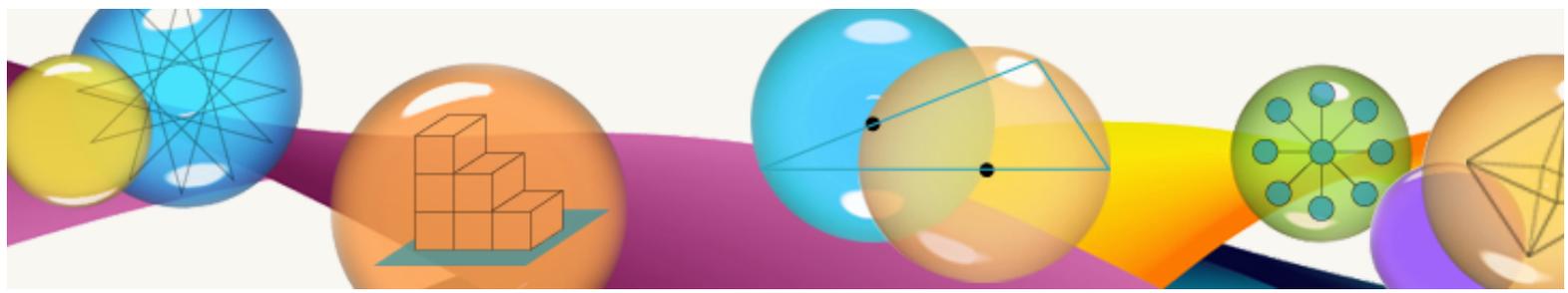
There are 6 combinations out of a total of 48 possible combinations, so the probability of wearing a sweater and jeans is $\frac{6}{48} = \frac{1}{8}$.

Another way to look at this would be to calculate the probability of wearing a sweater and the probability of wearing jeans. Phunee has only 1 sweater out of 4 shirts, so his chances of wearing a sweater are 1 in 4. He has 2 pairs of jeans out of 4 pairs of pants, so his chances of wearing jeans are 2 in 4. Since these are independent events, the probability of him wearing a pair of jeans and a sweater is $\frac{1}{4} \times \frac{2}{4} = \frac{2}{16} = \frac{1}{8}$.

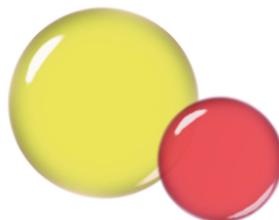
Some of the terminology and theory used in this second approach to part (b) will be developed in later mathematics courses.

- c) Since Phunee works 5 days out of 7, 9 weeks of 5 working days would be 45 days. So three days into his 10th week of work, Phunee will have worn each of his 48 outfits once, and will need to buy something else to complement his wardrobe. Since he started on a Thursday, this would normally occur on a Sunday. Since Phunee doesn't work Sunday or Monday, he will need a new article of clothing on his 49th day of work, which is a Tuesday.





Geometry & Spatial Sense





Problem of the Week

Problem B

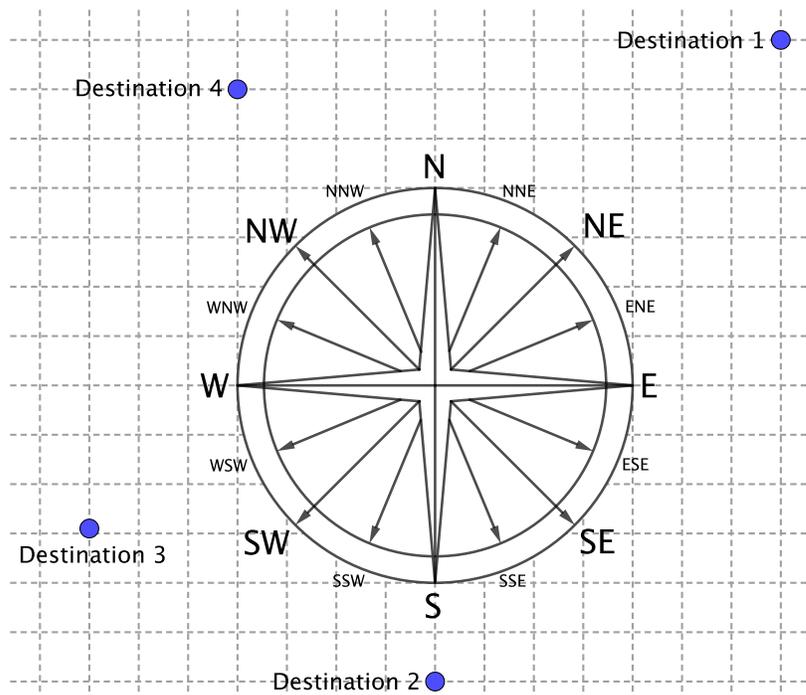
A Directional Challenge

Prior to the 10th century, sailors travelled solely by using the known positions of planets and stars. The magnetic compass was first used for navigation during the Song dynasty in China, about a thousand years ago. It provided a way to determine direction even on foggy days and at night.

- a) On the compass represented on the grid below, the range of possible directions is divided into sixteen segments, each consisting of $360^\circ \div 16 = 22.5^\circ$. Each direction is named, e.g., N, NE, SSW, etcetera. Calculate the (smaller) angle between each pair of directions given in the table. Verify your answers by measuring the angles with a protractor.

Angle	Number of Degrees
N and E	
N and NE	
N and S	
N and SE	
SW and WSW	

- b) Suppose that a schooner starts from the location at the centre of the compass. Using a protractor with 0° along North, measure the angle (clockwise) of the direction the schooner should travel to each of the four destinations indicated by the solid dots. If the direction is one of the labelled directions, state the name of that direction.
- c) Use a ruler to measure the distances from the centre of the compass to each of the four destinations. What is the ratio of the distance to Destination 1 to the distance to Destination 3? What is the ratio of the distance to Destination 4 to the distance to Destination 2?



STRANDS GEOMETRY AND SPATIAL SENSE, MEASUREMENT





Problem of the Week Problem B and Solution A Directional Challenge

Problem

The magnetic compass was first used for navigation during the Song dynasty in China, about a thousand years ago. It provided a way to determine direction even on foggy days and at night.

- a) On the compass represented on the grid below, the range of possible directions is divided into sixteen segments, each consisting of $360^\circ \div 16 = 22.5^\circ$. Each direction is named, e.g., N, NE, SSW, etcetera. Calculate the (smaller) angle between each pair of directions given in the table. Verify your answers by measuring the angles with a protractor.

Angle	Number of Degrees
N and E	90°
N and NE	45°
N and S	180°
N and SE	135°
SW and WSW	22.5°

- b) Suppose that a schooner starts from the location at the centre of the compass. Using a protractor with 0° along North, measure the angle (clockwise) of the direction the schooner should travel to each of the four destinations indicated by the solid dots. If the direction is one of the labelled directions, state the name of that direction.
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Solution

- a) The solution to part a) is shown in the chart above.

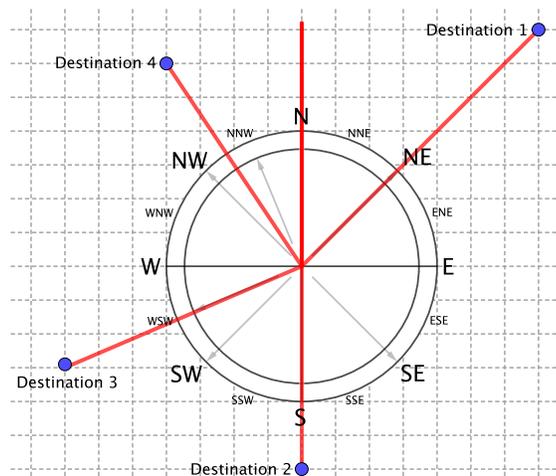
- b) For Destination 1, the schooner should travel at an angle of 45° , i.e., in the NE direction.

For Destination 2, the schooner should travel at an angle of 180° , i.e., in the S direction.

For Destination 3, the schooner should travel at an angle of $180^\circ + 45^\circ + 22.5^\circ = 247.5^\circ$, i.e., in the WSW direction.

For Destination 4, the schooner should travel at an angle of $270^\circ + 45^\circ + \frac{1}{2} \times 22.5^\circ = 326.25^\circ$, i.e., in a direction that is halfway between NW and NNW.

- c) The ratio of the distance to Destination 1 to that for Destination 3 is about 1.3. The ratio of the distance to Destination 4 to that for Destination 2 is about 1.2.



Answers to b) and c) may vary, depending on the accuracy of the measurements.



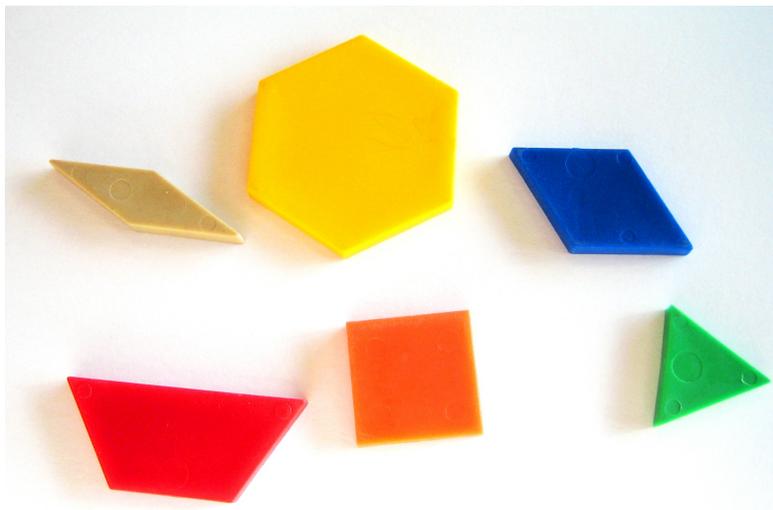


Problem of the Week

Problem B

Fraction Action

For this activity, you will need a set of pattern blocks, as shown below.



- a)(i) Suppose that the large yellow hexagonal pattern block represent one whole. Use your pattern blocks to discover what fraction is represented by each of the other three shapes listed below.

Hexagon = 1; Blue Rhombus = ___; Green Triangle = ___;

Red Trapezoid = ___.

- (ii) What is the total value of the collection of four shapes listed in part i)?
- (iii) Using the values from part i) and as many of each of those four shapes as needed, make a collection of shapes with total value $2\frac{5}{6}$.
- b)(i) Suppose that the Blue Rhombus is now worth only $\frac{1}{6}$. What fraction would each of the shapes now represent?

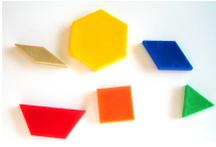
Hexagon = ___; Blue Rhombus = $\frac{1}{6}$; Green Triangle = ___;

Red Trapezoid = ___.

- (ii) Using these new values, make a collection of shapes with total value $2\frac{2}{3}$.

STRANDS NUMBER SENSE AND NUMERATION, GEOMETRY AND SPATIAL SENSE





Problem of the Week

Problem B and Solution

Fraction Action

Problem

For this activity, you will need a set of pattern blocks, as shown above.

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Hexagon = $\underline{\quad}$; Blue Rhombus = $\underline{\frac{1}{6}}$; Green Triangle = $\underline{\quad}$; Red Trapezoid = $\underline{\quad}$.
- (ii) Using these new values, make a collection of shapes with total value $2\frac{2}{3}$.

Solution

- a) i) Hexagon = 1, Blue Rhombus = $\frac{1}{3}$, Green Triangle = $\frac{1}{6}$, Red Trapezoid = $\frac{1}{2}$
ii) The total value of the 4 shapes in part i) is

$$1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = 1 + \frac{2}{6} + \frac{1}{6} + \frac{3}{6} = 1 + \frac{6}{6} = 2,$$

i.e., the shapes assembled together would make 2 complete hexagons.

- iii) Answers may vary. To make $2\frac{5}{6}$ we could use:

- 2 Hexagons + 5 Triangles
- 2 Hexagons + 2 Rhombi + 1 Triangle
- 2 Hexagons + 1 Rhombus + 3 Triangles
- 2 Hexagons + 1 Trapezoid + 2 Triangles
- 2 Hexagons + 1 Trapezoid + 1 Rhombus

etcetera...

- b) i) If the blue rhombus is now only worth $\frac{1}{6}$, i.e., half its original value, then so is each other piece. So the hexagon is worth $\frac{1}{2}$, the green triangle is worth $\frac{1}{12}$ and the red trapezoid is worth $\frac{1}{4}$.
- ii) Answers will vary but when rearranged would cover 5 complete hexagons and a blue rhombus. Here are some possibilities:
- 5 Hexagons + 1 Rhombus
 - 4 Hexagons + 4 Rhombi
 - 4 Hexagons + 2 Trapezoids + 1 Rhombus
 - 3 Hexagons + 4 Trapezoids + 1 Rhombus
 - 16 Rhombi
 - 32 Triangles
- etcetera...





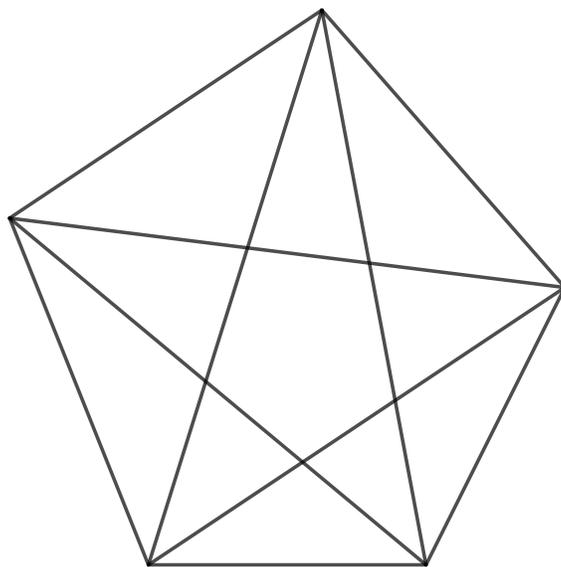
Problem of the Week

Problem B

There Are More Than Ten...

The pentagon shown contains many triangles. Suppose you were asked to count all the triangles in this pentagon.

- Think about how you would do this, and write down what your strategy would be.
- Now share your strategy with a classmate. Work together using one of your proposed strategies to count all the triangles you can find. Then try the other person's strategy. Did you get the same result?
- Compare answers with all your classmates.





Problem of the Week

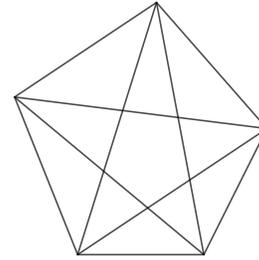
Problem B and Solution

There Are More Than Ten...

Problem

The pentagon shown contains many triangles. Suppose you were asked to count all the triangles in this pentagon.

- a) Think about how you would do this, and write down what your strategy would be.
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- c) Compare answers with all your classmates.

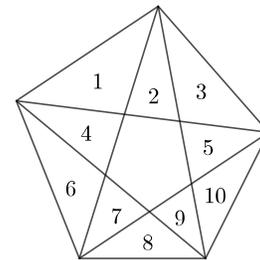


Solution

There are several possible strategies for counting all the triangles in the given pentagonal figure. We have chosen to look at four possible cases.

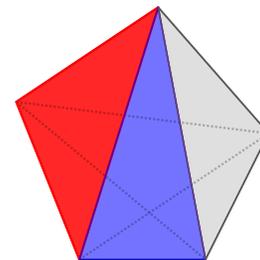
- 1: Count the smallest triangles.

Each triangle has either a side or a vertex on the outer pentagon. There are 10 such triangles.



- 2: Count the triangles that have their three vertices on the pentagon.

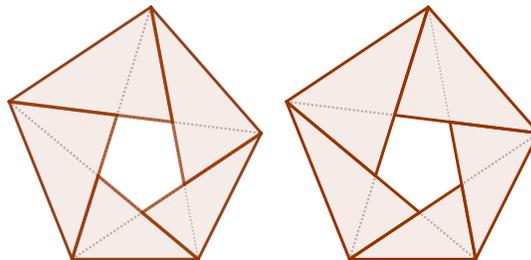
Since the pentagon has five vertices, there are five sets of three triangles. One such set of three triangles is shown to the right. You might think that there are $5 \times 3 = 15$ triangles but five of the triangles would be counted twice each. (Draw the individual diagrams to verify this for yourself.) So, there are 10 triangles in this case. It is important not to count the same triangles more than once.





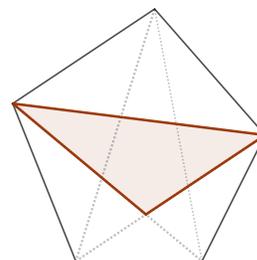
- 3: Count the triangles that have one side on the outer pentagon that contain two of the smallest triangles that were counted in the first case.

There are two ways to do these combinations, each giving five triangles. So there are $2 \times 5 = 10$ such triangles. These are illustrated in diagrams to the right.



- 4: Count the interior triangles which have a diagonal as the longest side and a point inside the pentagon as the third vertex.

Since there are five diagonals, there are five such triangles. One of these triangles is shown to the right.



Thus, in total there are $10 + 10 + 10 + 5 = 35$ triangles in total in the given figure.



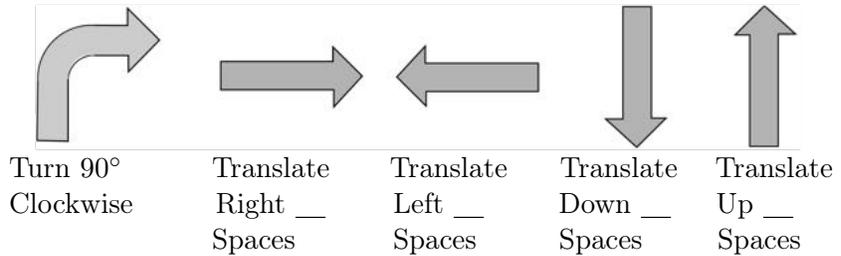


Problem of the Week

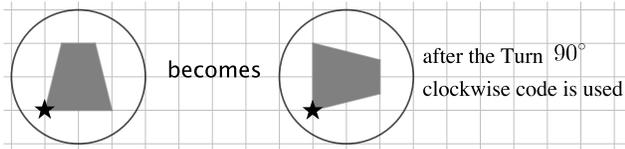
Problem B

Don't Catch a Code!

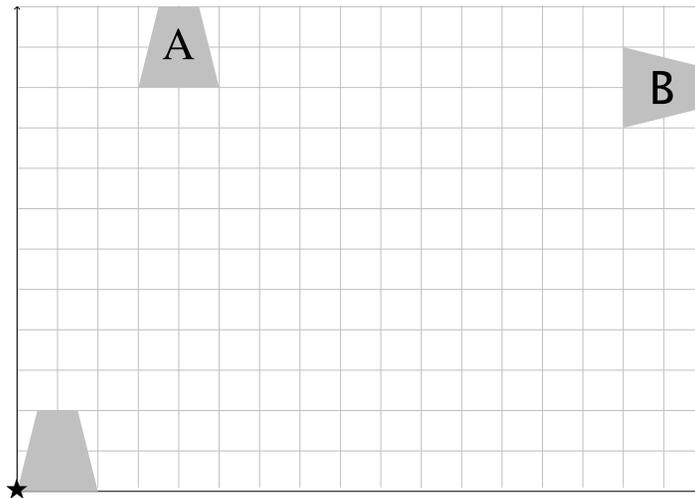
A robot moves on a grid using the codes shown to the right.



NOTE: When the “Turn 90° Clockwise” code is used, the robot turns inside the four squares.



- a) Using the trapezoid shown on the following grid and starting from the star in the lower left-hand corner, write a suitable set of code which will move the trapezoid to the final position A in the fewest lines of code. Note that when translating the robot, the instruction is carried out from whichever orientation the robot is currently in.
- b) Repeat part a) for final position B. Start from the star again.
- c) Using the trapezoid shown on the grid, again starting from the star at the lower left-hand corner, write a suitable set of code which will create a shape of your choice. In our solution, we create an Inukshuk shape. You will need to add a “Print” command to the list of codes so you can print a copy of the trapezoid at a desired position. When one of the other commands is executed after the “Print” command, the instruction is carried out from the present location of the trapezoid.
- d) Challenge a classmate to write suitable code for the shape you created in the previous part. Then discuss your codes together to see whether improvements to the code can be made.



STRAND GEOMETRY AND SPATIAL SENSE





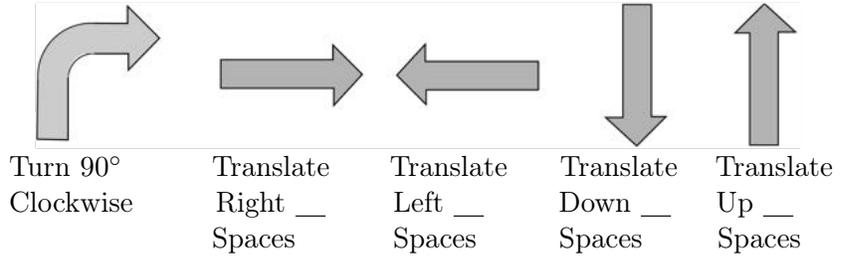
Problem of the Week

Problem B and Solution

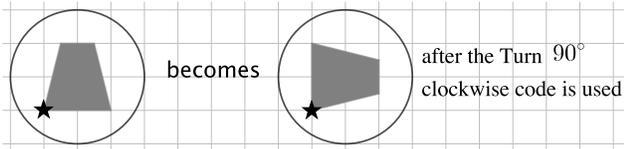
Don't Catch a Code!

Problem

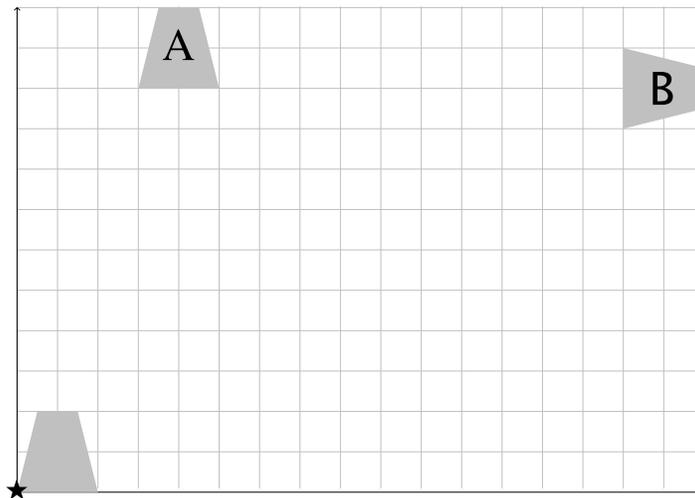
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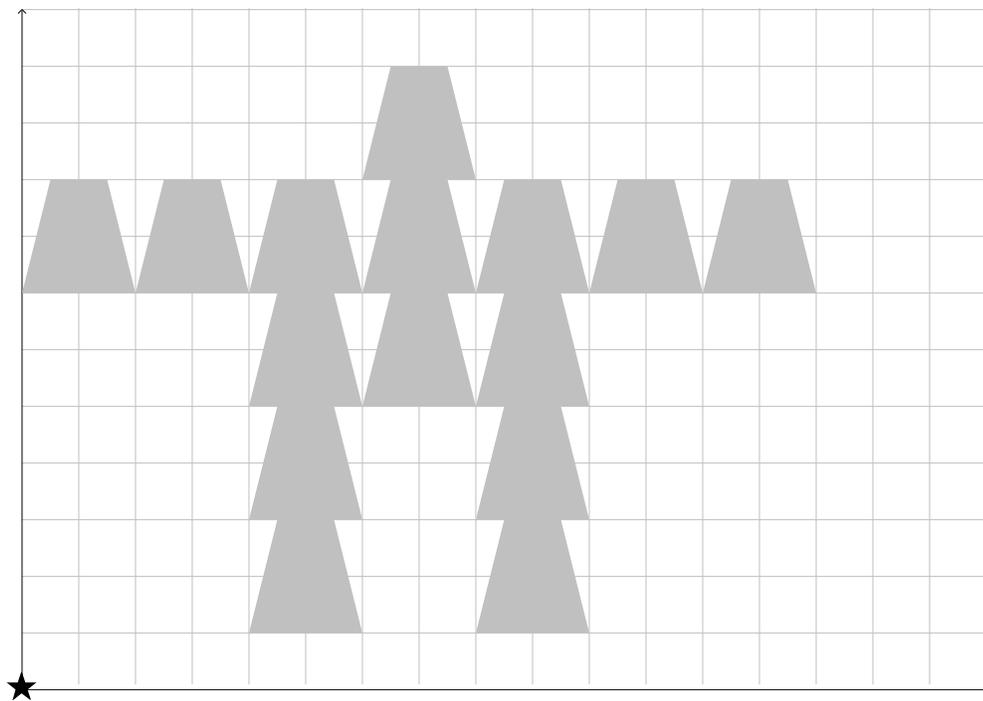




Solution

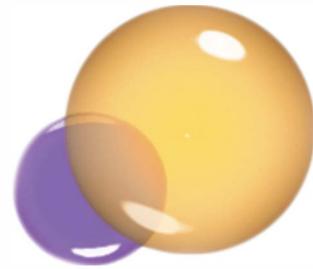
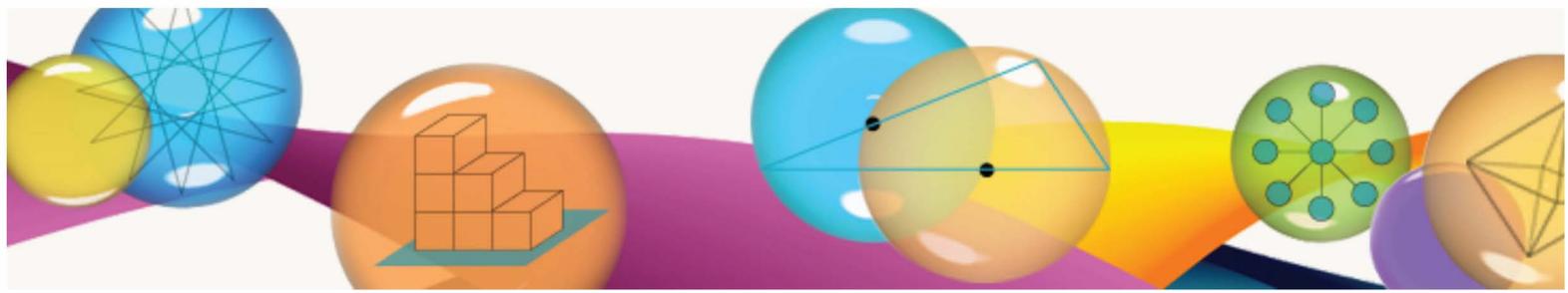
- a) To reach position A by the fewest lines of code, do either “Translate right 3 spaces” then “Translate up 10 spaces”, or “Translate up 10 spaces” then “Translate right 3 spaces”.
- b) To reach position B by the fewest lines of code, do any of “Translate right 15 spaces” then “Translate up 9 spaces” then “Turn 90° Clockwise”, or “Translate up 9 spaces” then “Translate right 15 spaces” then “Turn 90° Clockwise”, or “Turn 90° Clockwise” then “Translate right 15 spaces” then “Translate up 9 spaces”, etcetera.
- c) The following sequence of commands will generate the Inukshuk shape shown below. In our sequence of commands, we abbreviate “Translate right _” to “Right _” and “Translate up _” to “Up _”.

Up 1, Right 4, Print; Right 4, Print; Up 2, Print; Left 4, Print; Up 2, Print; Right 2, Print; Right 2, Print; Up 2, Right 4, Print; Left 2, Print; Right 6, Up 2, Print

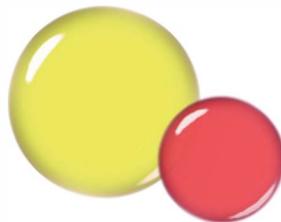


- d) Examining your code with a classmate may create some interesting discussion.





Measurement





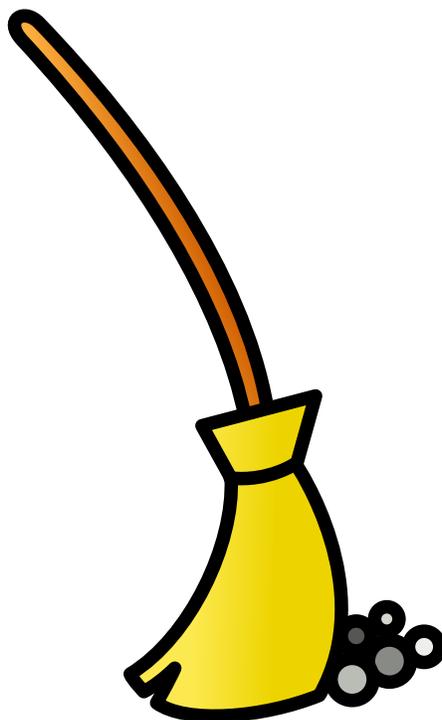
Problem of the Week

Problem B

An Almost Clean Sweep

In July, Minh's family spends the month at their cottage. On July 1st, Minh swept up 80 g of sand.

- a) If Minh continues sweeping up 80 g of sand each day, how many kg of sand will Minh sweep up after 7 days?
- b) How much sand, in kg, will Minh sweep up by the end of July?
- c) Actually, Minh's broom is faulty. Alas, about a tenth of the sand ends up unswept. How much sand in total will Minh sweep up over the family's month at the cottage?





Problem of the Week

Problem B and Solution

An Almost Clean Sweep

Problem

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- Actually, Minh's broom is faulty. Alas, about a tenth of the sand ends up unswept. How much sand in total will Minh sweep up over the family's month at the cottage?

Solution

- In 7 days, Minh will sweep $80 \text{ g} \times 7 \text{ days} = 560 \text{ g}$, or $560 \div 1\,000 = 0.56 \text{ kg}$ of sand.
- By the end of July, Minh will sweep up $80 \text{ g} \times 31 \text{ days} = 2\,480 \text{ g}$, or 2.48 kg of sand.
- Since about a tenth of the sand is not swept up, the actual amount of sand Minh sweeps up will be nine tenths of the amount found in part b), or $0.9 \times 2\,480 = 2\,232 \text{ g}$, or 2.232 kg of sand.





Problem of the Week

Problem B

Can You Canoe?

Chris and Wes decided to go fishing at their favourite fishing hole but needed to go around several islands. From west to east they are; Little Island, Kettle Island and Big Island.

Starting at the dock on the west side of Lake Numero, they paddled east 600 m (past Little Island), then turned south and went 400 m between Little Island and Kettle Island (to go below Kettle Island).

Next, they turned east again and paddled 800 m (past Kettle Island), and then north 400 m between Kettle Island and Big Island.

Finally, they turned west and went 100 m to get to their fishing hole north of Kettle Island.

- Using grid paper, an appropriate legend, and a suitable scale, plot a possible route that Chris and Wes used, with approximate locations of the three islands.
- Using the shortest route possible, how far is their fishing hole from where they started?
- Assuming Kettle Island is a rectangle, what is the greatest possible area of Kettle Island?





Problem of the Week

Problem B and Solution

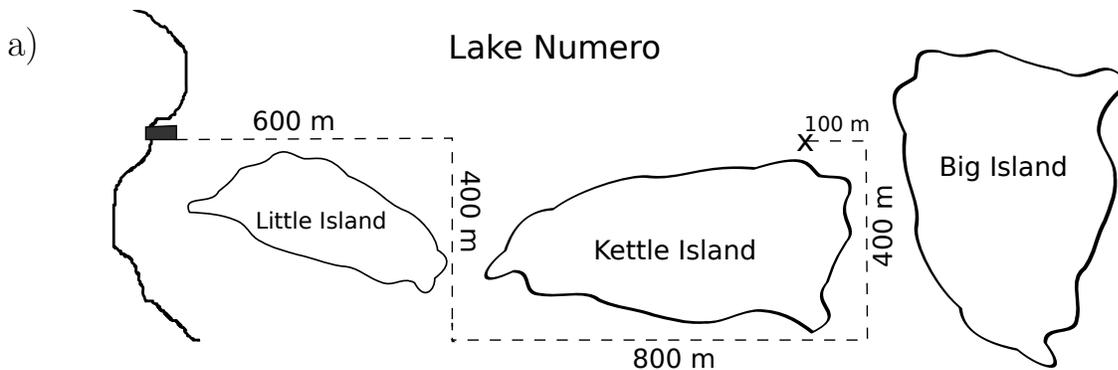
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Solution



(NOTE: The shapes of the islands are hypothetical.)

- Since they went equal distances south and north, their fishing hole is $600 + 800 - 100 = 1300$ m due east from the dock.
- Given the distances Chris and Wes paddled, a rectangular Kettle Island would have to have area less than $800 \times 400 = 320\,000$ m² or 0.32 km².

(Note: In the solution to part c) we assume that the sides of Kettle Island are parallel to the path that the canoe takes. On the next page, we show a diagram where they are not parallel.)





Problem of the Week

Problem B

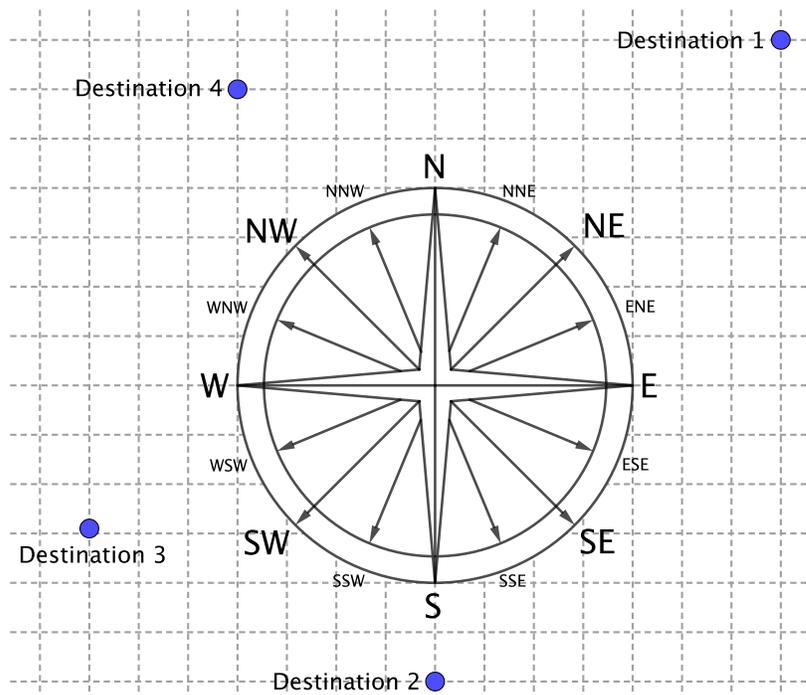
A Directional Challenge

Prior to the 10th century, sailors travelled solely by using the known positions of planets and stars. The magnetic compass was first used for navigation during the Song dynasty in China, about a thousand years ago. It provided a way to determine direction even on foggy days and at night.

- a) On the compass represented on the grid below, the range of possible directions is divided into sixteen segments, each consisting of $360^\circ \div 16 = 22.5^\circ$. Each direction is named, e.g., N, NE, SSW, etcetera. Calculate the (smaller) angle between each pair of directions given in the table. Verify your answers by measuring the angles with a protractor.

Angle	Number of Degrees
N and E	
N and NE	
N and S	
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SW and WSW	

- b) Suppose that a schooner starts from the location at the centre of the compass. Using a protractor with 0° along North, measure the angle (clockwise) of the direction the schooner should travel to each of the four destinations indicated by the solid dots. If the direction is one of the labelled directions, state the name of that direction.
- c) Use a ruler to measure the distances from the centre of the compass to each of the four destinations. What is the ratio of the distance to Destination 1 to the distance to Destination 3? What is the ratio of the distance to Destination 4 to the distance to Destination 2?



STRANDS GEOMETRY AND SPATIAL SENSE, MEASUREMENT





Problem of the Week

Problem B and Solution

A Directional Challenge

Problem

The magnetic compass was first used for navigation during the Song dynasty in China, about a thousand years ago. It provided a way to determine direction even on foggy days and at night.

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Solution

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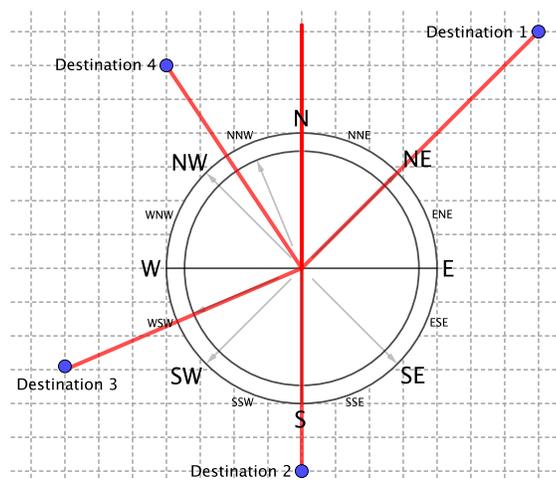
- b) For Destination 1, the schooner should travel at an angle of 45° , i.e., in the NE direction.

For Destination 2, the schooner should travel at an angle of 180° , i.e., in the S direction.

For Destination 3, the schooner should travel at an angle of $180^\circ + 45^\circ + 22.5^\circ = 247.5^\circ$, i.e., in the WSW direction.

For Destination 4, the schooner should travel at an angle of $270^\circ + 45^\circ + \frac{1}{2} \times 22.5^\circ = 326.25^\circ$, i.e., in a direction that is halfway between NW and NNW.

- c) The ratio of the distance to Destination 1 to that for Destination 3 is about 1.3. The ratio of the distance to Destination 4 to that for Destination 2 is about 1.2.



Answers to b) and c) may vary, depending on the accuracy of the measurements.





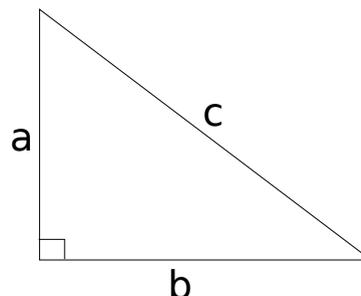
Problem of the Week

Problem B

Right On, Dude!

Let's explore some right angled triangles.

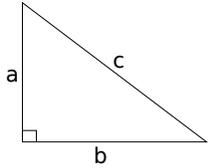
- a) On centimetre graph paper, construct a right angle with sides $a = 3$ cm and $b = 4$ cm which meet at the right angle. Measure the length of the third side c . Enter your answer in the table below.



- b) Next, construct a right angled triangle with sides $a = 6$ cm and $b = 8$ cm which meet at the right angle. How long do you think the third side c will be? Measure it to find out, and enter its length in the table.
- c) Look at the patterns for sides a , b , and c in your table to try to predict the side lengths b and c for the next triangle in the table. Draw this triangle to confirm your predictions, and enter the lengths in the table.
- d) Repeat part c) to complete the table.
- e) Write a pattern rule for the length of each side in the table.

Triangle	Side a (cm)	Side b (cm)	Side c (cm)
Part a)	3	4	
Part b)	6	8	
Part c)	9		
Part d)	12		





Problem of the Week

Problem B and Solution

Right On, Dude!

Problem

- On centimetre graph paper, construct a right angle with sides $a = 3$ cm and $b = 4$ cm which meet at the right angle. Measure the length of the third side c . Enter your answer in the table below.
- Next, construct a right angled triangle with sides $a = 6$ cm and $b = 8$ cm which meet at the right angle. How long do you think the third side c will be? Measure it to find out, and enter its length in the table.
- Look at the patterns for sides a , b , and c in your table to try to predict the side lengths b and c for the next triangle. Draw this triangle to confirm your predictions, and enter the lengths in the table.
- Repeat part c) for the last row to complete the table.
- Write a pattern rule for the length of each side in the table.

Solution

Triangle	Side a (cm)	Side b (cm)	Side c (cm)
Part a)	3	4	5
Part b)	6	8	10
Part c)	9	12	15
Part d)	12	16	20

- The third side is 5 cm long.
- Since sides a and b are $2 \times$ those of part a), we expect $c = 2 \times 5 = 10$ cm, which is what it measures to be.
- , d) Predicted sides are, respectively, $3 \times$ and $4 \times$ those in part a). The completed table above shows the resulting lengths.
- The pattern rules for the side lengths are:
 - Side a : multiples of 3, i.e., 3, 6, 9, 12, ... ;
 - Side b : multiples of 4, i.e., 4, 8, 12, 16, ... ;
 - Side c : multiples of 5, i.e., 5, 10, 15, 20,

Alternatively, the rules could be written as:

- "Starting with 3 add 3 to the previous side length" for Side a ;
- "Starting with 4 add 4 to the previous side length" for Side b ;
- "Starting with 5 add 5 to the previous side length" for Side c .





Problem of the Week

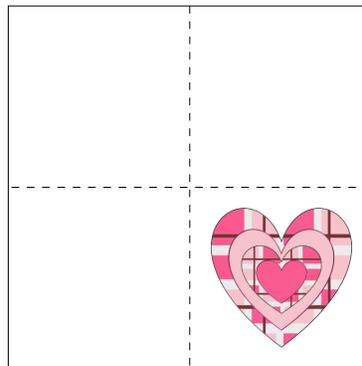
Problem B

Welcome to the Fold

Henry is folding paper to make Valentines.

- a) He folds a square of paper along the vertical dashed line, and then along the horizontal dashed line.

If the resulting smaller square (which is four layers thick) has a perimeter of 30 cm, what was the area of the original square of paper?



- b) Henry wants to make a very special valentine for his Nana, by folding a rectangular piece of paper twice, as in part a). If he wants the folded paper to be a 10 cm by 15 cm rectangle, what shape and size of paper should he start with?





Problem of the Week

Problem B and Solution

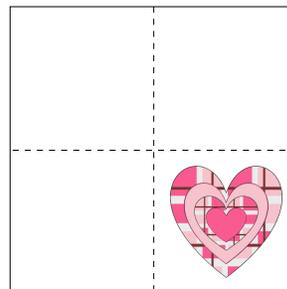
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Solution

- a) Since the perimeter of a square is four times its side length, the side length of the smaller square is $30 \div 4 = 7.5$ cm. The original square had side length twice that of the smaller square, i.e., $7.5 \times 2 = 15$ cm. Thus the area of the original square was $15 \times 15 = 225$ cm².

Another way to determine the total area is to determine the area of the small square that was created by folding and then multiply this area by 4, since there are four congruent smaller squares. The smaller square has area $7.5 \times 7.5 = 56.25$ cm². Therefore the total area is $4 \times 56.25 = 225$ cm², as before.

- b) Henry can only obtain a 10 cm by 15 cm rectangle by folding another, larger rectangle of paper. Since both the length and the width of the original rectangle will be halved by the two folds, he must start with a rectangle which has twice the length and width of the folded rectangle, i.e., a 20 cm by 30 cm rectangle of paper.





Problem of the Week

Problem B

Go with the Flow

Springtime is when sap runs from maple trees, and can be used to make yummy maple syrup.

The sap runs best when the temperature goes below freezing during the night and above freezing during the day. A very good flow rate would be 2 drips per second. Each drip contains $\frac{1}{4}$ mL of sap.



- At this rate, how many mL of sap would be collected in one minute?
- How many mL would be collected in an hour? How many litres is this?
- If the sap ran consistently for 8 hours, how many L of sap would be collected?
- If you collected sap from 28 trees for 8 hours, and each tree gave the same amount of sap as in part c), how much sap would you get in total?

STRANDS NUMBER SENSE AND NUMERATION, MEASUREMENT





Problem of the Week

Problem B and Solution

Go with the Flow

Problem

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- If you collected sap from 28 trees for 8 hours, and each tree gave the same amount of sap as in part c), how much sap would you get in total?

Solution

- Since $\frac{1}{4}$ mL/drip \times 2 drips/sec = $\frac{1}{2}$ mL/sec, we see that in one minute $\frac{1}{2}$ mL/sec \times 60 sec = 30 mL would be collected.
- In 1 hour, 30 mL/min \times 60 min = 1800 mL would be collected, which is equivalent to 1.8 L in one hour.
- In 8 hours, 1.8 L/hour \times 8 hr = 14.4 L would be collected.
- From 28 similar trees, you would get 14.4 L \times 28 trees = 403.2 L of sap.





Problem of the Week

Problem B

Bill O. Coins

Would you rather have eleven crisp \$50 bills, or four stacks of dimes, each as tall as the length of your wingspan, fingertip to fingertip?

Make your choice, and then, using the fact that a dime is approximately 1 mm thick, determine which option is worth more.





Problem of the Week

Problem B and Solution

Bill O. Coins

Problem

Would you rather have eleven crisp \$50 bills, or four stacks of dimes, each as tall as the length of your wingspan, fingertip to fingertip?

Make your choice, and then, using the fact that a dime is approximately 1 mm thick, determine which option is worth more.



Solution

The first option has you take eleven \$50 bills, which has value $11 \times \$50 = \550 .

The second option has you take 4 stacks of dimes, each the same height as your wingspan. The thickness of a dime is 1 mm, so a 10 mm = 1 cm stack is worth \$1.00. You can measure your wingspan, or, since wingspan \approx height, you can just use your height.

Let's consider two people, Person A of wingspan/height 150 cm and Person B of wingspan/height 130 cm.

Person A

Each stack of height 150 cm has value \$150. So four such stacks have value $\$150 \times 4 \text{ stacks} = \600 .

Person B

Each stack of height 130 cm has value \$130. So four such stacks have value $\$130 \times 4 \text{ stacks} = \520 .

The 'break-even' height is when four stacks of dimes has value \$550. So, one stack has value $\$550 \div 4 = \137.50 , which is the value of the dimes if the person's wingspan/height is 137.5 cm.

Therefore if your wingspan/height is less than 137.5 cm, take the bills; otherwise, you should take the dimes.

For further thought: A Canadian dime is actually 1.22 mm thick. How does this change your answer?





Problem of the Week

Problem B

A Round Trunk in a Square Hole

Rhab's mother and father decided that they wanted him to spend more time outdoors, so they built him a tree fort. A red maple tree goes right through the middle of the fort, like in the tree fort shown in the photo below.

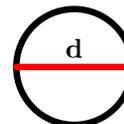


Source: Creative Commons

Useful things to know:

The distance around a circle is about 3.14 times the distance from one edge of the circle to another, through the middle of the circle.

This is called the diameter d of the circle.



- Rhab's mom found two pieces of plywood with dimensions 1.2 m (120 cm) by 2.4 m (240 cm) to use for the floor. What will be the total floor area of the tree fort if they use both sheets of plywood to form a square floor for the tree fort?
- Rhab's parents thought about how they should make the hole in the plywood floor so that the tree would fit. They decided that a square would be the easiest solution. Rhab's dad measured the distance around the tree (the circumference) and found that it was 140 cm. He was about to cut a square with side length 35 cm when Rhab, remembering what he learned in school, stopped his dad before it was too late. Why did Rhab stop his dad?
- Rhab found the diameter of the tree to be about 45 cm. If they decide to cut a square hole through which the trunk will fit, what are the dimensions and area of the square hole that his dad should cut?
- What is the remaining area of the floor of the tree fort after the square hole has been cut out?
- Where Rhab's family lives, red maples grow to around 60 cm in diameter. How might this affect the design of the tree fort?





Problem of the Week

Problem B and Solution

A Round Trunk in a Square Hole

Problem

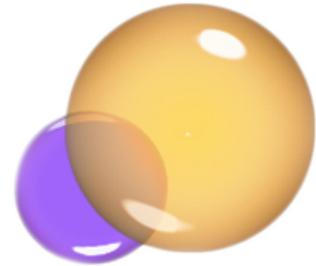
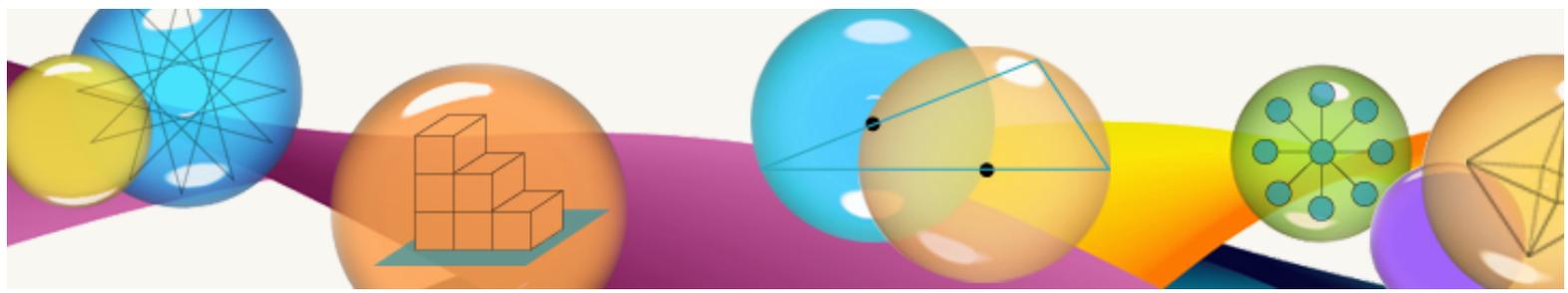
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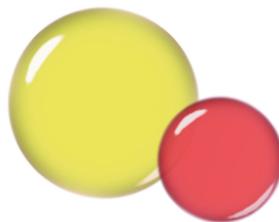
Solution

- By putting the two pieces of plywood together, the floor of the treehouse will be $2.4 \text{ m} \times 2.4 \text{ m} = 5.76 \text{ m}^2$.
- The distance around the tree does not help you determine the area taken up by the tree or the dimensions of the sides of the square to be cut. Rhab's father would need to find the radius or diameter of the tree. The diameter of the tree can be found by dividing 140 cm (the known circumference) by π (approximately 3.14).
- If they decide to cut the square with side lengths of 45 cm, then the area will be $45 \text{ cm} \times 45 \text{ cm} = 2025 \text{ cm}^2 = 0.2025 \text{ m}^2$. If they decide to cut the square slightly larger, say with side lengths of 50 cm, then the area will be $50 \text{ cm} \times 50 \text{ cm} = 2500 \text{ cm}^2 = 0.25 \text{ m}^2$.
COMMENT: If they made the hole $50 \text{ cm} \times 50 \text{ cm}$, that would allow for 5 cm of growth in the diameter of the tree. A maple tree's circumference grows by approximately 1.5 cm each year. By the time the tree has grown an extra 5 cm in diameter, Rhab may be too old to be hanging around in his tree fort.
- After removing a $50 \text{ cm} \times 50 \text{ cm}$ square, the remaining area would be $5.76 \text{ m}^2 - 0.25 \text{ m}^2 = 5.51 \text{ m}^2$.
- If the diameter of the red maple tree can grow to 60 cm, then they should probably make the side lengths of the square about 65 cm. Then $65 \text{ cm} \times 65 \text{ cm} = 4225 \text{ cm}^2 = 0.4225 \text{ m}^2$ of space is taken from the fort, leaving $5.76 \text{ m}^2 - 0.4225 \text{ m}^2 = 5.3375 \text{ m}^2$ of space for frolicking. The area would only be reduced by about 0.2 m^2 .





Number Sense & Numeration





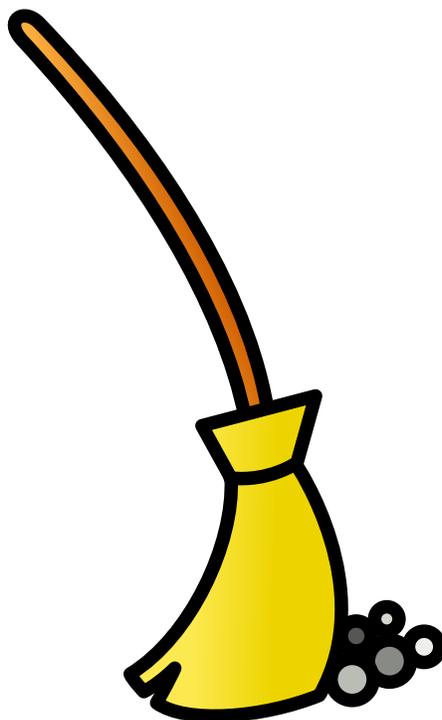
Problem of the Week

Problem B

An Almost Clean Sweep

In July, Minh's family spends the month at their cottage. On July 1st, Minh swept up 80 g of sand.

- a) If Minh continues sweeping up 80 g of sand each day, how many kg of sand will Minh sweep up after 7 days?
- b) How much sand, in kg, will Minh sweep up by the end of July?
- c) Actually, Minh's broom is faulty. Alas, about a tenth of the sand ends up unswept. How much sand in total will Minh sweep up over the family's month at the cottage?





Problem of the Week

Problem B and Solution

An Almost Clean Sweep

Problem

In July, Minh's family spends the month at their cottage. On July 1st, Minh swept up 80 g of sand.

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- Actually, Minh's broom is faulty. Alas, about a tenth of the sand ends up unswept. How much sand in total will Minh sweep up over the family's month at the cottage?

Solution

- In 7 days, Minh will sweep $80 \text{ g} \times 7 \text{ days} = 560 \text{ g}$, or $560 \div 1\,000 = 0.56 \text{ kg}$ of sand.
- By the end of July, Minh will sweep up $80 \text{ g} \times 31 \text{ days} = 2\,480 \text{ g}$, or 2.48 kg of sand.
- Since about a tenth of the sand is not swept up, the actual amount of sand Minh sweeps up will be nine tenths of the amount found in part b), or $0.9 \times 2\,480 = 2\,232 \text{ g}$, or 2.232 kg of sand.





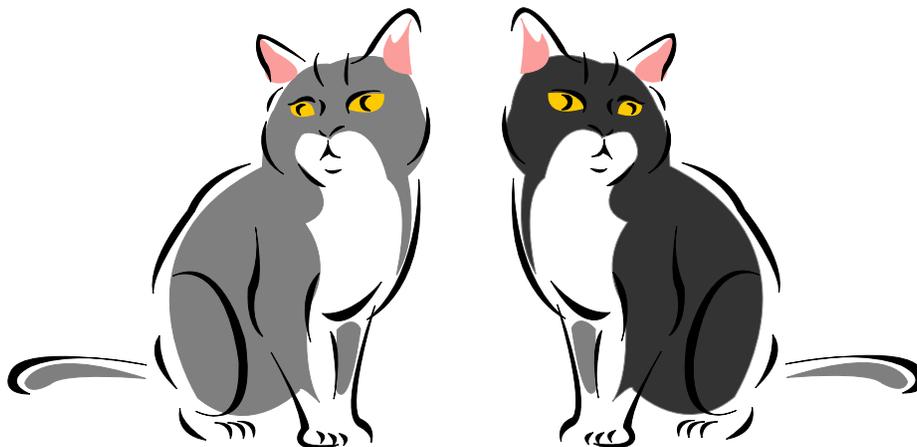
Problem of the Week

Problem B

A Feline Puzzle

Mrs. Murphy takes her two cats to the veterinarian in a pet carrier which weighs 2.1 kg. Her older cat, Chuckles, weighs 1 kg more than her younger cat, Shorty. When the vet weighs both cats in their carrier, the scale registers 11.3 kg.

How much does each cat weigh?





Problem of the Week

Problem B and Solution

A Feline Puzzle

Problem

Mrs. Murphy takes her two cats to the veterinarian in a pet carrier which weighs 2.1 kg. Her older cat, Chuckles, weighs 1 kg more than her younger cat, Shorty. When the vet weighs both cats in their carrier, the scale registers 11.3 kg.

How much does each cat weigh?

Solution

We know that the combined weight of the two cats is equal to the total weight of the two cats and the carrier minus the weight of the carrier. That is, the combined weight of the two cats is $11.3 - 2.1 = 9.2$ kg.

The 9.2 kg is made up of Shorty's weight plus Chuckles' weight. If Chuckles were 1 kg lighter, he would be the same weight as Shorty. So two times the weight of Shorty would be $9.2 - 1 = 8.2$ kg.

Then, Shorty's weight would be $8.2 \div 2 = 4.1$ kg.

Since Chuckles weighs 1 kg more than Shorty, Chuckles weighs $4.1 + 1 = 5.1$ kg.

We can verify our answer by adding Shorty's weight, Chuckles' weight and the weight of the carrier. Then, $4.1 + 5.1 + 2.1 = 11.3$ kg, the given total weight.

Therefore, Shorty weighs 4.1 kg and Chuckles weighs 5.1 kg.





Problem of the Week

Problem B

Write On!

Victoria's university English teacher Mr. McTuffie has told her she will have 27 assignments this semester. Victoria has discovered that she needs to spend 3.5 hours on each assignment, and has decided to alternate between doing two assignments the first week and three assignments the next week.

- How many hours will Victoria spend on her English assignments in the first four weeks?
- What is the total time she will spend on her assignments?
- How many weeks will it take Victoria to complete her assignments?





Problem of the Week

Problem B and Solution

Write On!

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- How many hours will Victoria spend on her English assignments in the first four weeks?
- What is the total time she will spend on her assignments?
- How many weeks will it take Victoria to complete her assignments?

Solution

- a) Since Victoria does 2 assignments one week and 3 the next, in the first four weeks she will have done $2 + 3 + 2 + 3 = 10$ assignments.

If each assignment takes 3.5 hours, she will have spent $3.5 \times 10 = 35$ hours on her English assignments in the first four weeks.

An alternate solution

For the first week she will spend $2 \times 3.5 = 7$ hours. The second week she will spend $3 \times 3.5 = 10.5$ hours. Therefore the total time spent in the first four weeks would be $7 + 10.5 + 7 + 10.5 = 35$ hours.

- b) On 27 assignments, she will spend a total of $27 \times 3.5 = 94.5$ hours.
- c) Since Victoria does 5 assignments every two weeks, the following table shows that she will do 25 assignments in 10 weeks. The next week, she will do 2 assignments, completing the 27. So, it will take her a total of 11 weeks.

Number of Weeks	Number of Assignments
2	5
4	10
6	15
8	20
10	25

An alternate solution

Since Victoria does 5 assignments every two weeks, after 10 weeks, she will have done $5 \times 5 = 25$ assignments. The next week, she will do 2 assignments, completing the 27. So, it will take her a total of 11 weeks.





Problem of the Week

Problem B

Game On ...

Many sports are divided into a fixed number of “playing periods” or “parts”, with each part having a fixed length of playing time. The following chart outlines how many parts are in one game for various sports, the playing time for one part, and the total playing time for one game without any stoppages.

- a) Complete the chart using the information provided.

Sport	Number of “parts” in one game	Playing time for one “part”	Total playing time for one game
NFL football	4 quarters	15 minutes	
NBA basketball	4 quarters		48 minutes
Hockey	___ periods	20 minutes	60 minutes
Soccer	2 halves	45 minutes	
Lacrosse	4 quarters		60 minutes

- b) A NFL football game between Buffalo and Detroit started at 1:00 p.m. The game took 2 hours and 12 minutes longer than the playing time with no stoppages. At what time did the game end?
- c) A basketball game between Toronto and Boston started at 7:00 p.m. and ended at 9:15 p.m. How much of this time was during stoppages in play?





Problem of the Week

Problem B and Solution

Game On ...



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- c) A basketball game between Toronto and Boston started at 7:00 p.m. and ended at 9:15 p.m. How much of this time was during stoppages in play?

Solution

- a) See the completed chart in the problem statement above.
- b) Since the game took 2 hours and 12 minutes longer than the time with no stoppages, there were 2 hours and 12 minutes of stoppages. The time from start to finish for the football game was 60 minutes of playing time plus 2 hours and 12 minutes of stoppages, or 3 hours and 12 minutes. Thus, the game finished at 4:12 p.m.
- c) From start to finish, the time for the basketball game was 2 hours and 15 minutes, or $2 \times 60 + 15 = 135$ minutes. This includes playing time and stoppage time. Thus, the stoppage time is 135 minutes minus playing time, or $135 - 48 = 87$ minutes.





Problem of the Week

Problem B

An 'Annual' Problem

I am a 4-digit number, identity unknown.

1. My thousands digit is twice my tens digit.
2. My hundreds digit is 1 less than my tens digit.
3. My ones digit is not 0.
4. The sum of my digits is 11.
5. All of my digits are different.



What number am I?

CHALLENGE: Make up another riddle with a unique solution.



Problem of the Week

Problem B and Solution

An ‘Annual’ Problem

Problem

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CHALLENGE: Make up another riddle with a unique solution.

Solution

Only the doubles of digits 1, 2, 3, and 4 are single digits. Thus the first clue permits only numbers of the form

$$2_1_ , \quad 4_2_ , \quad 6_3_ , \quad \text{and} \quad 8_4_$$

The second clue tells us that the hundreds digit must be 1 less than the tens digit. That limits the choices further to

$$201_ , \quad 412_ , \quad 623_ , \quad \text{and} \quad 834_$$

But clues 3 and 4 eliminate $623_$, since the ones digit would have to be 0 to achieve a sum of 11.

Also, any number of the form $834_$ already has a digit sum of 15, and hence violates clue 4.

So we are left with numbers of the form $201_$ and $412_$.

But for a sum of 11, since $4 + 1 + 2 + 4 = 11$, the last choice would need the ones digit to equal 4, violating clue 5.

Since $2 + 0 + 1 + 8 = 11$, the only solution is 2018.





Problem of the Week

Problem B

What A Sale!

The window of a store advertises that it is holding a sale. The new salesperson puts up the sign in the window on clear plastic, so it can be seen from both sides of the glass. From inside the store it reads:

SALE. ENJOY

18

% OFF

Amanda, a very smart shopper, selects a sweater with a regular price of \$50.

- What does the store expect Amanda to pay for the sweater?
- Amanda was looking at the sign from outside the window. What does Amanda expect to pay for the sweater?
- If the store grants Amanda the discount she saw from the outside, how much more will Amanda save than the store expected her to save?





Problem of the Week

Problem B and Solution

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Solution

- The store expects to give a discount equal to 18% of \$50 = $0.18 \times 50 = \$9$.
So the store expects Amanda to pay $\$50 - \$9 = \$41$.
- Amanda sees the sign as 81% off. Amanda expects to get a discount equal to 81% of \$50 = $0.81 \times 50 = \$40.50$.
So Amanda expects to pay $\$50 - \$40.50 = \$9.50$
- Since $\$41 - \$9.50 = \$31.50$, Amanda would save an additional \$31.50 compared to the store's intended sale price.





Problem of the Week

Problem B

A Cool Trip

In 1911, the Norwegian explorer Roald Amundsen and his four companions were the first persons to travel to the South Pole. They spent 3 days at the Pole, plus 99 days travelling from their base camp to the Pole and back.

- a) The trip from the Pole back to the base camp took 39 days. How many days did the trip from the base camp **to** the Pole take?
- b) On the trip from the Pole back to the base camp, they averaged 30.4 km per day. How far was the base camp from the Pole?
- c) What was their average speed (in km per day) on the trip from the base camp **to** the Pole?



Source: Wikipedia





Problem of the Week

Problem B and Solution

A Cool Trip

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Source: Wikipedia

Solution

- The total travel time was 99 days, so the trip to the pole took $99 - 39 = 60$ days (much longer than the trip back).
- If they averaged 30.4 km per day for 39 days, they travelled a total distance of $39 \times 30.4 = 1185.6$ km.
- Since the distance to the pole was also 1185.6 km and they travelled for 60 days, their average speed was $1185.6 \div 60 = 19.76$ km per day.





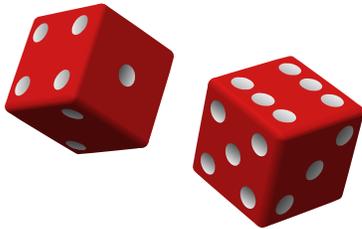
Problem of the Week

Problem B

These are Sum Products

Ahmed has two number cubes which he rolls ten times. Each of the 10 rolls gives a **different** combination of the two numbers on the top face (Note, for example, that 5 on the first cube and 1 on the second cube is the same combination as 1 on the first cube and 5 on the second cube). Ahmed finds the product of each pair of numbers and then sums the ten resulting products.

- Complete the chart of products.
- What is the set of ten rolls with the lowest possible sum of the products?
- What is the set of ten rolls with the highest possible sum of the products?
- Determine 2 different possible sets of ten rolls such that the sum of products is equal to 100. The first set must include every possible roll involving a 3 on at least one of the cubes. The second set must not include any roll with a 3 on either cube.



	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3					
4	4					
5	5					
6	6					





Problem of the Week

Problem B and Solution

These are Sum Products

Problem

Ahmed has two number cubes which he rolls ten times. Each of the 10 rolls gives a **different** combination of the two numbers on the top face (Note, for example, that 5 on the first cube and 1 on the second cube is the same combination as 1 on the first cube and 5 on the second cube). Ahmed finds the product of each pair of numbers and then sums the ten resulting products.

- Complete the chart of products.
- What is the set of ten rolls with the lowest possible sum of the products?
- What is the set of ten rolls with the highest possible sum of the products?
- Determine 2 different possible sets of ten rolls such that the sum of products is equal to 100. The first set must include every possible roll involving a 3 on at least one of the cubes. The second set must not include any roll with a 3 on either cube.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Solution

- The completed chart is shown in the problem statement above.
- The set of ten rolls would be $1 \times 1 = 1$, $1 \times 2 = 2$, $1 \times 3 = 3$, $1 \times 4 = 4$, $2 \times 2 = 4$, $1 \times 5 = 5$, $1 \times 6 = 6$, $2 \times 3 = 6$, $2 \times 4 = 8$, $3 \times 3 = 9$.
Their sum is $1 + 2 + 3 + 4 + 4 + 5 + 6 + 6 + 8 + 9 = 48$.
These are the ten lowest products in the completed table.
- The set of ten rolls would be $6 \times 6 = 36$, $5 \times 6 = 30$, $5 \times 5 = 25$, $4 \times 6 = 24$, $4 \times 5 = 20$, $3 \times 6 = 18$, $4 \times 4 = 16$, $3 \times 5 = 15$, $3 \times 4 = 12$, $2 \times 6 = 12$.
Their sum is $36 + 30 + 25 + 24 + 20 + 18 + 16 + 15 + 12 + 12 = 208$.
These are the ten highest products in the completed table.
- Answers will vary.

One possible set of ten rolls which includes every possible roll involving a 3 on at least one of the cubes is $1 \times 3 = 3$, $2 \times 3 = 6$, $3 \times 3 = 9$, $3 \times 4 = 12$, $3 \times 5 = 15$, $3 \times 6 = 18$, $1 \times 1 = 1$, $1 \times 2 = 2$, $2 \times 5 = 10$, $4 \times 6 = 24$.

In this set, the first 6 rolls in the list must be included and the sum of the products of these rolls is 63. The remaining 4 rolls must add to $100 - 63 = 37$.

One possible set of ten rolls which does not include any roll containing a 3 is

$1 \times 1 = 1$, $1 \times 2 = 2$, $1 \times 4 = 4$, $1 \times 6 = 6$,
 $2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 5 = 10$, $4 \times 4 = 16$, $4 \times 6 = 24$, $5 \times 5 = 25$.

There are many combinations that work.

For Further Investigation: Try putting restrictions on the problem to create different sets of 10 rolls for which the sum of the products is 100.



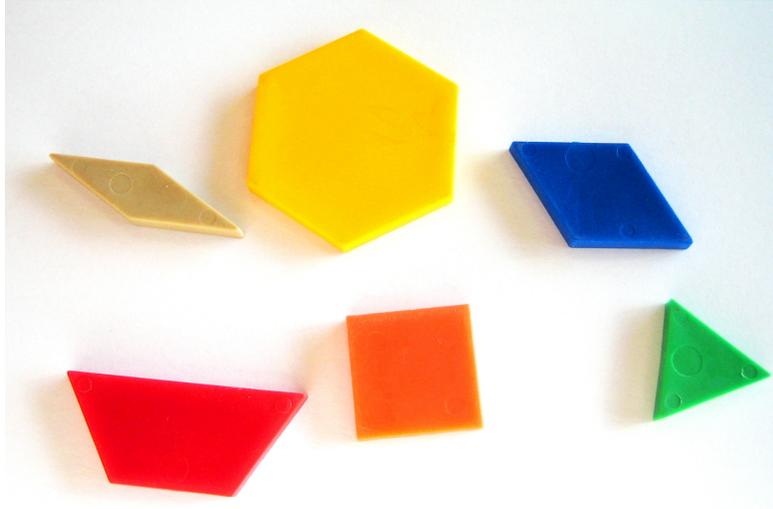


Problem of the Week

Problem B

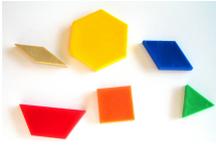
Fraction Action

For this activity, you will need a set of pattern blocks, as shown below.



- a)(i) Suppose that the large yellow hexagonal pattern block represent one whole. Use your pattern blocks to discover what fraction is represented by each of the other three shapes listed below.
- Hexagon = 1; Blue Rhombus = ___; Green Triangle = ___;
Red Trapezoid = ___.
- (ii) What is the total value of the collection of four shapes listed in part i)?
- (iii) Using the values from part i) and as many of each of those four shapes as needed, make a collection of shapes with total value $2\frac{5}{6}$.
- b)(i) Suppose that the Blue Rhombus is now worth only $\frac{1}{6}$. What fraction would each of the shapes now represent?
- Hexagon = ___; Blue Rhombus = $\frac{1}{6}$; Green Triangle = ___;
Red Trapezoid = ___.
- (ii) Using these new values, make a collection of shapes with total value $2\frac{2}{3}$.





Problem of the Week

Problem B and Solution

Fraction Action

Problem

For this activity, you will need a set of pattern blocks, as shown above.

- a)(i) Suppose that the large yellow hexagonal pattern block represent one whole. Use your pattern blocks to discover what fraction is represented by each of the other three shapes listed below.
Hexagon = $\underline{1}$; Blue Rhombus = $\underline{\quad}$; Green Triangle = $\underline{\quad}$; Red Trapezoid = $\underline{\quad}$.
- (ii) What is the total value of the collection of four shapes listed in part i)?
- (iii) Using the values from part i) and as many of each of those four shapes as needed, make a collection of shapes with total value $2\frac{5}{6}$.
- b)(i) Suppose that the Blue Rhombus is now worth only $\frac{1}{6}$. What fraction would each of the shapes now represent?
Hexagon = $\underline{\quad}$; Blue Rhombus = $\underline{\frac{1}{6}}$; Green Triangle = $\underline{\quad}$; Red Trapezoid = $\underline{\quad}$.
- (ii) Using these new values, make a collection of shapes with total value $2\frac{2}{3}$.

Solution

- a) i) Hexagon = 1, Blue Rhombus = $\frac{1}{3}$, Green Triangle = $\frac{1}{6}$, Red Trapezoid = $\frac{1}{2}$
ii) The total value of the 4 shapes in part i) is

$$1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = 1 + \frac{2}{6} + \frac{1}{6} + \frac{3}{6} = 1 + \frac{6}{6} = 2,$$

i.e., the shapes assembled together would make 2 complete hexagons.

- iii) Answers may vary. To make $2\frac{5}{6}$ we could use:

- 2 Hexagons + 5 Triangles
- 2 Hexagons + 2 Rhombi + 1 Triangle
- 2 Hexagons + 1 Rhombus + 3 Triangles
- 2 Hexagons + 1 Trapezoid + 2 Triangles
- 2 Hexagons + 1 Trapezoid + 1 Rhombus
- etcetera...

- b) i) If the blue rhombus is now only worth $\frac{1}{6}$, i.e., half its original value, then so is each other piece. So the hexagon is worth $\frac{1}{2}$, the green triangle is worth $\frac{1}{12}$ and the red trapezoid is worth $\frac{1}{4}$.
- ii) Answers will vary but when rearranged would cover 5 complete hexagons and a blue rhombus. Here are some possibilities:
- 5 Hexagons + 1 Rhombus
 - 4 Hexagons + 4 Rhombi
 - 4 Hexagons + 2 Trapezoids + 1 Rhombus
 - 3 Hexagons + 4 Trapezoids + 1 Rhombus
 - 16 Rhombi
 - 32 Triangles
 - etcetera...





Problem of the Week

Problem B

Go with the Flow

Springtime is when sap runs from maple trees, and can be used to make yummy maple syrup.

The sap runs best when the temperature goes below freezing during the night and above freezing during the day. A very good flow rate would be 2 drips per second. Each drip contains $\frac{1}{4}$ mL of sap.



- At this rate, how many mL of sap would be collected in one minute?
- How many mL would be collected in an hour? How many litres is this?
- If the sap ran consistently for 8 hours, how many L of sap would be collected?
- If you collected sap from 28 trees for 8 hours, and each tree gave the same amount of sap as in part c), how much sap would you get in total?

STRANDS NUMBER SENSE AND NUMERATION, MEASUREMENT





Problem of the Week

Problem B and Solution

Go with the Flow

Problem

Springtime is when sap runs from maple trees, and can be used to make yummy maple syrup. The sap runs best when the temperature goes below freezing during the night and above freezing during the day. A very good flow rate would be 2 drips per second. Each drip contains $\frac{1}{4}$ mL of sap.

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- If the sap ran consistently for 8 hours, how many L of sap would be collected?
- If you collected sap from 28 trees for 8 hours, and each tree gave the same amount of sap as in part c), how much sap would you get in total?

Solution

- Since $\frac{1}{4}$ mL/drip \times 2 drips/sec = $\frac{1}{2}$ mL/sec, we see that in one minute $\frac{1}{2}$ mL/sec \times 60 sec = 30 mL would be collected.
- In 1 hour, 30 mL/min \times 60 min = 1800 mL would be collected, which is equivalent to 1.8 L in one hour.
- In 8 hours, 1.8 L/hour \times 8 hr = 14.4 L would be collected.
- From 28 similar trees, you would get 14.4 L \times 28 trees = 403.2 L of sap.





Problem of the Week

Problem B

Sap to Syrup

In order to make sap into maple syrup, the sap must be boiled down to $\frac{1}{40}$ of its original volume. (This means that 40 L of sap will make only 1 L of maple syrup.)

- a) How much maple syrup would you get from boiling down 160 L of sap?
- b) In a previous Problem of the Week, we found that 403.2 L of sap can be collected from 28 trees. Approximately how many litres of maple syrup would result from this amount of sap?



- c) You have four 1 L bottles and six 500 mL bottles for the maple syrup from part b). If you fill those bottles first and then put the remaining maple syrup into 125 mL bottles, how many 125 mL bottles can you completely fill?





Problem of the Week

Problem B and Solution

Sap to Syrup

Problem

In order to make sap into maple syrup, the sap must be boiled down to $\frac{1}{40}$ of its original volume. (This means that 40 L of sap will make only 1 L of maple syrup.)

- How much maple syrup would you get from boiling down 160 L of sap?
- In a previous Problem of the Week, we found that 403.2 L of sap can be collected from 28 trees.
Approximately how many litres of maple syrup would result from this amount of sap?



- You have four 1 L bottles and six 500 mL bottles for the maple syrup from part b). If you fill those bottles first and then put the remaining maple syrup into 125 mL bottles, how many 125 mL bottles can you completely fill?

Solution

- The sap to maple syrup ratio is 40 : 1. Therefore, 160 L of sap would yield $\frac{1}{40} \times 160 = 4$ L of maple syrup.
- Given the 403.2 L of sap collected, and noting that $403.2 \approx 10 \times 40$, then with a ratio of 40 : 1, we see that about 10 L of maple syrup would result. More precisely, if we do the division, we get $403.2 \text{ L} \div 40 = 10.08$ L of maple syrup.
- The six 500 mL bottles will hold $6 \times 500 = 3\,000$ mL or 3 L of maple syrup. So, combined with the four 1 L bottles, there would be enough capacity to bottle 7 of the 10 L of maple syrup from part b), leaving 3 L to be bottled in the 125 mL bottles. Since $8 \times 125 \text{ mL} = 1\,000 \text{ mL} = 1 \text{ L}$, you will need $3 \times 8 = 24$ of the 125 mL bottles for the 3 L of maple syrup.





Problem of the Week

Problem B

Powerful Patterns

Consider the first few powers of 2:

$$2, 2 \times 2 = 4, 2 \times 2 \times 2 = 8, 2 \times 2 \times 2 \times 2 = 16, \text{ etcetera.}$$

Powers are often written in a more efficient way, with a superscript which tells how many multiples of 2. For example, 2^3 means $2 \times 2 \times 2$, while 2^5 means $2 \times 2 \times 2 \times 2 \times 2$. The first nine powers of 2 are

$$2, 4, 8, 16, 32, 64, 128, 256, 512$$

Observe that there is a distinct pattern in the ones digits of these powers, namely they follow a repeating sequence

$$\{2, 4, 8, 6, 2, 4, 8, 6, \dots\}$$

Thus, we can predict the ones digit for other powers of 2. For example, the ones digit of 2^{14} is the 14th number in this sequence, which is 4.

- Complete the left table below by finding the first five powers of each of the numbers from 1 to 10. What patterns do you observe in the ones digits of these powers?
- Complete the right table by finding the first five powers of each of the numbers from 11 to 14. Are there new patterns in the ones digits?
- Try to explain why the ones digits of powers of 4 and of powers of 9 follow a slightly different pattern than those for 2, 3, 7, and 8. What do you notice about the patterns?

1	1	1	1	1	1
2	2	4	8	16	32
3					
4					
5					
6					
7					
8					
9					
10					

11					
12					
13					
14					

STRAND PATTERNING AND ALGEBRA





Problem of the Week

Problem B and Solution

Powerful Patterns

Problem

- a) Construct a table containing the first five powers of each of the numbers from 1 to 10. What patterns do you observe in the ones digits of these powers?
- b) Construct a second table containing the first five powers of each of the numbers from 11 to 14. Are there new patterns in the ones digits?
- c) Try to explain why the ones digits of powers of 4 and of powers of 9 follow a slightly different pattern than those for 2, 3, 7, and 8. What do you notice about the patterns?

Solution

1	1	1	1	1	1
2	2	4	8	16	32
3	3	9	27	81	243
4	4	16	64	256	1024
5	5	25	125	625	3125
6	6	36	216	1296	7776
7	7	49	343	2401	16 807
8	8	64	512	4096	32 768
9	9	81	729	6561	59 049
10	10	100	1000	10 000	100 000

11	11	121	1331	14 641	161 051
12	12	144	1728	20 736	248 832
13	13	169	2197	28 561	371 293
14	14	196	2744	38 416	537 824

- a) These are the patterns observed (reading from left to right in the table) in the table to the left above:

1: 1, 1, 1, 1, 1, ...

2: 2, 4, 6, 8, 2, ...

3: 3, 9, 7, 1, 3, ...

4: 4, 6, 4, 6, 4, ...

5: 5, 5, 5, 5, 5, ...

6: 6, 6, 6, 6, 6, ...

7: 7, 9, 3, 1, 7, ...

8: 8, 4, 2, 6, 8, ...

9: 9, 1, 9, 1, 9, ...

10: 0, 0, 0, 0, 0, ...

- b) These are the patterns observed in the table to the left above::

11: 1, 1, 1, 1, 1, ... **12:** 2, 4, 8, 6, 2, ... **13:** 3, 9, 7, 1, 3, ...

14: 4, 6, 4, 6, 4, ...

For a two-digit number, the pattern of the ones digit is the same as the corresponding single-digit pattern found in part a).

- c) The patterns of the ones digits of powers of 4 and of powers of 9 alternate between two numbers, whereas for powers of 2, 3, 7 and 8, there is a four-number pattern. This is because $4 \times 4 = 16$, and $6 \times 4 = 24$, so the ones digit is back to 4. Similarly, $9 \times 9 = 81$, and $1 \times 9 = 9$, so the ones digit is back to 9.



Problem of the Week

Problem B

Bill O. Coins

Would you rather have eleven crisp \$50 bills, or four stacks of dimes, each as tall as the length of your wingspan, fingertip to fingertip?

Make your choice, and then, using the fact that a dime is approximately 1 mm thick, determine which option is worth more.





Problem of the Week

Problem B and Solution

Bill O. Coins

Problem

Would you rather have eleven crisp \$50 bills, or four stacks of dimes, each as tall as the length of your wingspan, fingertip to fingertip?

Make your choice, and then, using the fact that a dime is approximately 1 mm thick, determine which option is worth more.



Solution

The first option has you take eleven \$50 bills, which has value $11 \times \$50 = \550 .

The second option has you take 4 stacks of dimes, each the same height as your wingspan. The thickness of a dime is 1 mm, so a 10 mm = 1 cm stack is worth \$1.00. You can measure your wingspan, or, since wingspan \approx height, you can just use your height.

Let's consider two people, Person A of wingspan/height 150 cm and Person B of wingspan/height 130 cm.

Person A

Each stack of height 150 cm has value \$150. So four such stacks have value $\$150 \times 4 \text{ stacks} = \600 .

Person B

Each stack of height 130 cm has value \$130. So four such stacks have value $\$130 \times 4 \text{ stacks} = \520 .

The 'break-even' height is when four stacks of dimes has value \$550. So, one stack has value $\$550 \div 4 = \137.50 , which is the value of the dimes if the person's wingspan/height is 137.5 cm.

Therefore if your wingspan/height is less than 137.5 cm, take the bills; otherwise, you should take the dimes.

For further thought: A Canadian dime is actually 1.22 mm thick. How does this change your answer?





Problem of the Week

Problem B

Qwerty Words

Use the given keyboard to solve the riddles below. Each letter in each row of letters has the point value indicated at the end of the row. Shaded letters are letters which are usually typed with the left hand. In your answer, letters may be used more than once.

NOTE: Each riddle will have more than one answer. You may wish to compare with your classmates.

Row 1	Q	W	E	R	T	Y	U	I	O	P	1 point
Row 2	A	S	D	F	G	H	J	K	L	2 points	
Row 3	Z	X	C	V	B	N	M	3 points			

Find a word that:

- has three letters, is typed with the left hand only, and is worth 5 points;
- has six letters and uses only rows 1 and 3;
- is typed with the right hand only, and is worth 6 points;
- is four letters long, and uses only row 2;
- is worth exactly 9 points;
- has three syllables and is worth either 10, 11, 12, 13, 14 or 15 points;
- is worth fewer than 5 points and is typed with both hands;
- is a three-letter word worth either 6, 7, 8 or 9 points;
- is worth 20 points with the letter values changed to 6 points each for Row 1;
- is worth exactly 20 or 21 points and the letter values are the same as the ones originally given. Hyphenated words are acceptable. See how many different words your class can come up with.





Problem of the Week

Problem B and Solution

Qwerty Words

Problem

Use the given keyboard to solve the riddles below. Each letter in each row of letters has the point value indicated at the end of the row. Shaded letters are letters which are usually typed with the left hand. In your answer, letters may be used more than once.

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Find a word that:

- a) has three letters, is typed with the left hand only, and is worth 5 points;
- b) has six letters and uses only rows 1 and 3;
- c) is typed with the right hand only, and is worth 6 points;
- d) is four letters long, and uses only row 2;
- e) is worth exactly 9 points;
- f) has three syllables and is worth either 10, 11, 12, 13, 14 or 15 points;
- g) is worth fewer than 5 points and is typed with both hands;
- h) is a three-letter word worth either 6, 7, 8 or 9 points;
- i) is worth 20 points with the letter values changed to 6 points each for Row 1;
- j) is worth exactly 20 or 21 points and the letter values are the same as the ones originally given. Hyphenated words are acceptable. See how many different words your class can come up with.

Solution

NOTE: Many, many answers are possible. Students could verify each other's answers. In part (h), a three-letter word worth 9 points is not possible.

- | | |
|---|---|
| <ol style="list-style-type: none"> a) Bee, vet, sat, saw, was c) Nil, him, hum e) Chum, buns, nuns, mums, cane g) Ire, toy, pat, let, putt, pity, me i) Toys, rows, ladder, caber, slacken | <ol style="list-style-type: none"> b) Myopic, exotic, poetic, better d) Dash, gash, lads, sash f) Syllable, insulate, larceny, position h) Ace, big, lad, nib, cub, van, nab j) pomegranates |
|---|---|





Problem of the Week

Problem B

SumBuddy's Wrong

Amelia and Benji are discussing the sums of a number N and its two nearest neighbours (adjacent whole numbers). For example, if $N = 3$, then the sum is $2 + 3 + 4 = 9$.

Benji claims that he has discovered a whole number N such that this sum is 25. Amelia says that it is not possible.

a) Who is correct, Benji or Amelia?
Give a reason for your answer.

b) Complete the table and describe the pattern of such sums. Explain why this pattern occurs.

Triple	Sum
$1 + 2 + 3$	6
$2 + 3 + 4$	9

CHALLENGE: For each sum in the table, find as many *different* triples as you can of distinct whole numbers (not necessarily adjacent) which have that same sum.





Problem of the Week

Problem B and Solution

SumBuddy's Wrong

Problem

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Benji claims that he has discovered a whole number N such that this sum is 25. Amelia says that it is not possible.

- Who is correct, Benji or Amelia? Give a reason for your answer.
- Complete the table and describe the pattern of such sums. Explain why this pattern occurs.

Triple	Sum
$1 + 2 + 3$	6
$2 + 3 + 4$	9
$3 + 4 + 5$	12
$4 + 5 + 6$	15
$5 + 6 + 7$	18
$6 + 7 + 8$	21
$7 + 8 + 9$	24

CHALLENGE: For each sum in the table, find as many *different* triples as you can of distinct whole numbers (not necessarily adjacent) which have that same sum.

Solution

- For a sum of 25, the three numbers would need to be around 8. For example, $7 + 8 + 9 = 24$. But the next such sum is $8 + 9 + 10 = 27$, so there could not be three adjacent whole numbers which sum to 25. So Amelia is correct.
- Completing the table reveals that the sums of three consecutive numbers follow the pattern 6, 9, 12, 15, 18, 21, 24 (i.e., multiples of 3).

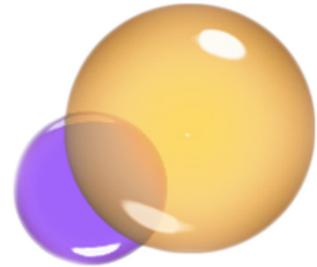
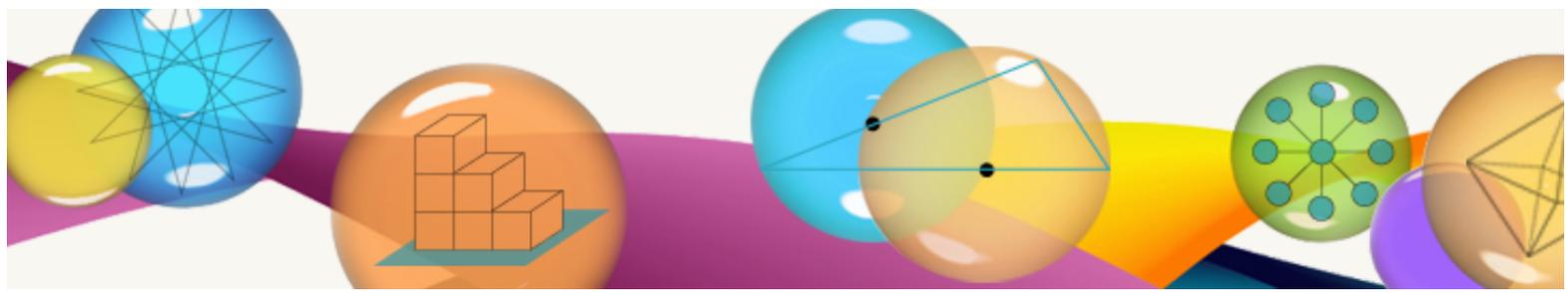
The average of the three numbers forming the sum is the middle number. So three times the middle number should equal the sum. It follows that the sum is a multiple of 3.

For the challenge a complete solution will not be provided. Whole numbers are the counting numbers zero and larger. The number 9 can be expressed as follows using three distinct whole numbers:

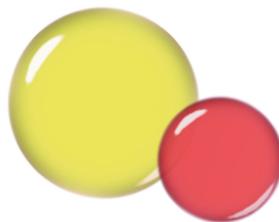
$$0 + 1 + 8 = 0 + 2 + 7 = 0 + 3 + 6 = 0 + 4 + 5 = 1 + 2 + 6 = 1 + 3 + 5 = 2 + 3 + 4$$

When finding all of the possibilities for a particular sum, it is important to be very systematic.





Patterning & Algebra





Problem of the Week

Problem B

Planes in Formations

At an airshow, a World War II Lancaster bomber flies in formation with four Spitfire fighter planes, as shown in the picture on the left below. On the second pass, two Lancaster bombers fly in formation with six Spitfires, as shown in the picture on the right below.



- a) If there were more Lancasters, the pattern would continue. Therefore, there would be 3 Lancasters and 8 Spitfires in the next formation. Draw this formation and the next two formations and fill in the remainder of Table 1.
Suggestion: Use a small symbol to represent each of the two planes for easy drawing.

Lancasters	Spitfires
1	4
2	6
3	
4	
5	

Table 1

- b) What if the Spitfires change their formations in two different ways, as shown in Tables 2 and 3? Fill in the remainder of each table, and then draw a possible formation to match each pattern.

Lancasters	Spitfires
1	6
2	10
3	14
4	
5	

Table 2

Lancasters	Spitfires
1	6
2	8
3	10
4	
5	

Table 3

- c) Draw another formation which follows a new pattern, and make a table of values for your pattern.

STRAND PATTERNING AND ALGEBRA





Problem of the Week

Problem B and Solution

Planes in Formations

Problem

At an airshow, a World War II Lancaster bomber flies in formation with four Spitfire fighter planes, as shown in the picture on the left below. On the second pass, two Lancaster bombers fly in formation with six Spitfires, as shown in the picture on the right below.



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Suggestion: Use a small symbol to represent each of the two planes for easy drawing.

Lancasters	Spitfires
1	4
2	6
3	8
4	10
5	12

Table 1

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Lancasters	Spitfires
1	6
2	10
3	14
4	18
5	22

Table 2

Lancasters	Spitfires
1	6
2	8
3	10
4	12
5	14

Table 3

- c) Draw another formation which follows a new pattern, and make a table of values for your pattern.





Solution

a) Table 1 is completed above. The diagram below shows the first four formations in this pattern. Two Spitfires are added for each added Lancaster.



b) Table 2 is completed above. The diagram below shows a possible solution for the first three formations in this pattern. Four Spitfires are added for each added Lancaster.

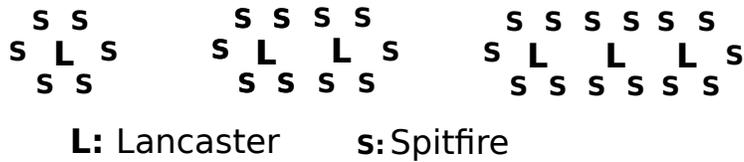


Table 3 is completed above. The diagram below shows a possible solution for the first four formations in this pattern. In this case, since only two Spitfires are added for each added Lancaster, there could be two Spitfires leading and two Spitfires following the Lancasters.



c) Answers will vary. Students should be prepared to explain how the drawings of their formations match the patterns they describe.





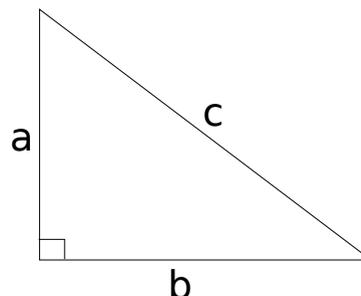
Problem of the Week

Problem B

Right On, Dude!

Let's explore some right angled triangles.

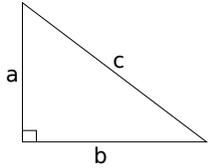
- a) On centimetre graph paper, construct a right angle with sides $a = 3$ cm and $b = 4$ cm which meet at the right angle. Measure the length of the third side c . Enter your answer in the table below.



- b) Next, construct a right angled triangle with sides $a = 6$ cm and $b = 8$ cm which meet at the right angle. How long do you think the third side c will be? Measure it to find out, and enter its length in the table.
- c) Look at the patterns for sides a , b , and c in your table to try to predict the side lengths b and c for the next triangle in the table. Draw this triangle to confirm your predictions, and enter the lengths in the table.
- d) Repeat part c) to complete the table.
- e) Write a pattern rule for the length of each side in the table.

Triangle	Side a (cm)	Side b (cm)	Side c (cm)
Part a)	3	4	
Part b)	6	8	
Part c)	9		
Part d)	12		





Problem of the Week

Problem B and Solution

Right On, Dude!

Problem

- a) On centimetre graph paper, construct a right angle with sides $\mathbf{a} = 3$ cm and $\mathbf{b} = 4$ cm which meet at the right angle. Measure the length of the third side \mathbf{c} . Enter your answer in the table below.
- b) Next, construct a right angled triangle with sides $\mathbf{a} = 6$ cm and $\mathbf{b} = 8$ cm which meet at the right angle. How long do you think the third side \mathbf{c} will be? Measure it to find out, and enter its length in the table.
- c) Look at the patterns for sides \mathbf{a} , \mathbf{b} , and \mathbf{c} in your table to try to predict the side lengths \mathbf{b} and \mathbf{c} for the next triangle. Draw this triangle to confirm your predictions, and enter the lengths in the table.
- d) Repeat part c) for the last row to complete the table.
- e) Write a pattern rule for the length of each side in the table.

Solution

Triangle	Side a (cm)	Side b (cm)	Side c (cm)
Part a)	3	4	5
Part b)	6	8	10
Part c)	9	12	15
Part d)	12	16	20

- a) The third side is 5 cm long.
- b) Since sides \mathbf{a} and \mathbf{b} are $2 \times$ those of part a), we expect $\mathbf{c} = 2 \times 5 = 10$ cm, which is what it measures to be.
- c), d) Predicted sides are, respectively, $3 \times$ and $4 \times$ those in part a). The completed table above shows the resulting lengths.
- e) The pattern rules for the side lengths are:
 - Side \mathbf{a} : multiples of 3, i.e., 3, 6, 9, 12, ... ;
 - Side \mathbf{b} : multiples of 4, i.e., 4, 8, 12, 16, ... ;
 - Side \mathbf{c} : multiples of 5, i.e., 5, 10, 15, 20,

Alternatively, the rules could be written as:

- "Starting with 3 add 3 to the previous side length" for Side \mathbf{a} ;
- "Starting with 4 add 4 to the previous side length" for Side \mathbf{b} ;
- "Starting with 5 add 5 to the previous side length" for Side \mathbf{c} .





Problem of the Week

Problem B

Powerful Patterns

Consider the first few powers of 2:

$$2, 2 \times 2 = 4, 2 \times 2 \times 2 = 8, 2 \times 2 \times 2 \times 2 = 16, \text{ etcetera.}$$

Powers are often written in a more efficient way, with a superscript which tells how many multiples of 2. For example, 2^3 means $2 \times 2 \times 2$, while 2^5 means $2 \times 2 \times 2 \times 2 \times 2$. The first nine powers of 2 are

$$2, 4, 8, 16, 32, 64, 128, 256, 512$$

Observe that there is a distinct pattern in the ones digits of these powers, namely they follow a repeating sequence

$$\{2, 4, 8, 6, 2, 4, 8, 6, \dots\}$$

Thus, we can predict the ones digit for other powers of 2. For example, the ones digit of 2^{14} is the 14th number in this sequence, which is 4.

- Complete the left table below by finding the first five powers of each of the numbers from 1 to 10. What patterns do you observe in the ones digits of these powers?
- Complete the right table by finding the first five powers of each of the numbers from 11 to 14. Are there new patterns in the ones digits?
- Try to explain why the ones digits of powers of 4 and of powers of 9 follow a slightly different pattern than those for 2, 3, 7, and 8. What do you notice about the patterns?

1	1	1	1	1	1
2	2	4	8	16	32
3					
4					
5					
6					
7					
8					
9					
10					

11					
12					
13					
14					

STRAND PATTERNING AND ALGEBRA





Problem of the Week Problem B and Solution Powerful Patterns

Problem

- a) Construct a table containing the first five powers of each of the numbers from 1 to 10. What patterns do you observe in the ones digits of these powers?
- b) Construct a second table containing the first five powers of each of the numbers from 11 to 14. Are there new patterns in the ones digits?
- c) Try to explain why the ones digits of powers of 4 and of powers of 9 follow a slightly different pattern than those for 2, 3, 7, and 8. What do you notice about the patterns?

Solution

1	1	1	1	1	1
2	2	4	8	16	32
3	3	9	27	81	243
4	4	16	64	256	1024
5	5	25	125	625	3125
6	6	36	216	1296	7776
7	7	49	343	2401	16 807
8	8	64	512	4096	32 768
9	9	81	729	6561	59 049
10	10	100	1000	10 000	100 000

11	11	121	1331	14 641	161 051
12	12	144	1728	20 736	248 832
13	13	169	2197	28 561	371 293
14	14	196	2744	38 416	537 824

- a) These are the patterns observed (reading from left to right in the table) in the table to the left above:

1: 1, 1, 1, 1, 1, ...

2: 2, 4, 6, 8, 2, ...

3: 3, 9, 7, 1, 3, ...

4: 4, 6, 4, 6, 4, ...

5: 5, 5, 5, 5, 5, ...

6: 6, 6, 6, 6, 6, ...

7: 7, 9, 3, 1, 7, ...

8: 8, 4, 2, 6, 8, ...

9: 9, 1, 9, 1, 9, ...

10: 0, 0, 0, 0, 0, ...

- b) These are the patterns observed in the table to the left above::

11: 1, 1, 1, 1, 1, ... **12:** 2, 4, 8, 6, 2, ... **13:** 3, 9, 7, 1, 3, ...

14: 4, 6, 4, 6, 4, ...

For a two-digit number, the pattern of the ones digit is the same as the corresponding single-digit pattern found in part a).

- c) The patterns of the ones digits of powers of 4 and of powers of 9 alternate between two numbers, whereas for powers of 2, 3, 7 and 8, there is a four-number pattern. This is because $4 \times 4 = 16$, and $6 \times 4 = 24$, so the ones digit is back to 4. Similarly, $9 \times 9 = 81$, and $1 \times 9 = 9$, so the ones digit is back to 9.

