



The **CENTRE** for **EDUCATION**
in **MATHEMATICS** and **COMPUTING**

Problem of the Week
Problems and Solutions
2019-2020

Problem B (Grade 5/6)

Themes

Number Sense (N)

Geometry (G)

Algebra (A)

Data Management (D)

Computational Thinking (C)

(Click on a theme name above to jump to that section)

*The problems in this booklet are organized into themes.
A problem often appears in multiple themes.

Number Sense (N)





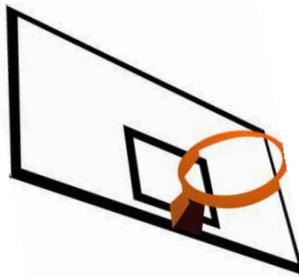
Problem of the Week

Problem B

More Practice

When Nick shoots a basketball, he either sinks the shot or misses. For each shot Nick sinks, he is given 5 points by his father. For each missed shot, Nick’s Dad takes 2 points away.

Nick attempts a total of 28 shots and ends up with zero points (i.e., he breaks even). How many shots did Nick sink?



Shots Sunk	Points Gained	Shots Missed	Points Lost	Points Gained – Points Lost
20	100	8	16	$100 - 16 = 84$



The above table may be useful. A sample trial of shots made and lost is shown.



Problem of the Week

Problem B and Solution

More Practice

Problem

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Solution

Solution 1

Shots Sunk	Points Gained	Shots Missed	Points Lost	Points Gained – Points Lost
20	100	8	16	$100 - 16 = 84$
16	80	12	24	$80 - 24 = 56$
14	70	14	28	$70 - 28 = 42$
12	60	16	32	$60 - 32 = 28$
10	50	18	36	$50 - 36 = 14$
8	40	20	40	$40 - 40 = 0$



By trying different combinations of shots sunk and shots missed, as shown in the table, we see that Nick breaks even if he sinks 8 shots and misses 20 shots.

Solution 2

We make a couple of observations. First, to break even, Nick must have sunk an even number of shots. If he sunk an odd number of shots, he would have earned an odd number of points, since multiplying any odd number by 5 produces an odd number. But he always loses an even number of points for missed shots since multiplying any number by 2 produces an even number. An odd number minus an even number will never equal zero. Thus, Nick could not have broken even by sinking an odd number of shots.

Second, Nick sunk less than 14 shots (half of the 28 shots). If he sunk 14 shots, Nick would have gained $14 \times 5 = 70$ points, but only lost $2 \times 14 = 28$ points. He would not have broken even.

We have quickly reduced the number of possibilities for successful shots to $\{0, 2, 4, 6, 8, 10, 12\}$. It does not take much time to narrow down to the correct solution (shown above). That is, Nick broke even by sinking 8 shots and missing 20 shots.

An algebraic solution is possible and is presented on the next page.

**Solution 3**

Let a represent the number of shots sunk and b represent the number of shots missed.

Then $5 \times a$ represents the number of points Nick gained for successful shots and $2 \times b$ represents the number of points Nick lost for missed shots

We want Nick to break even, so $5 \times a = 2 \times b$. This is generally written $5a = 2b$.

Since the total number of shots made was 28, $a + b = 28$.

By dividing both sides of the first equation by 5, we obtain $a = \frac{2}{5}b = 0.4b$.

Substituting $0.4b$ for a in the second equation we get

$$\begin{aligned}0.4b + b &= 28 \\1.4b &= 28 \\b &= \frac{28}{1.4} \\b &= \frac{280}{14} \\b &= 20\end{aligned}$$

Since b represents the number of missed shots, Nick missed 20 shots. Since Nick attempted a total of 28 shots and missed 20 shots, he sunk $28 - 20 = 8$ shots.

Therefore, Nick sunk 8 shots and missed 20 shots.

Equation solving techniques are generally covered in grade 7 or higher. This solution is provided just for information of those who may wish to get a glimpse of what is coming in future Mathematics courses.



Problem of the Week

Problem B

Will That be Rye or Honey Oats?

Jagheet drives from his house to his camp, 75 km away, maintaining an average speed of 60 km/h.

- How long, in minutes, does it take him to drive to his camp?
- If he left his house at 11:37 a.m., at what time could he expect to arrive at his camp?
- Realizing that he forgot to get bread, he stops at the grocery store for 12 minutes. Including this time, what is his new average speed in km/h?
(Round your answer to the nearest tenth.)





Problem of the Week

Problem B and Solution

Will That be Rye or Honey Oats?

Problem

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Solution

- Since Jagheet travels 75 km at 60 km/h, his travel time will be $75 \div 60 = 1.25$ h, or $1.25 \times 60 = 75$ minutes.
ALTERNATIVELY: His speed is 60 km/h, or 1 km/min. So, 75 km takes 75 minutes.
- He would expect to arrive at his camp 1 h and 15 minutes after 11:37 a.m., which would be at 12:52 p.m.
- His total time is now $75 + 12 = 87$ minutes. Thus his average speed is now $75 \div 87 \approx 0.862$ km/min, or $0.862 \times 60 \approx 51.7$ km/h, to the nearest tenth.



Problem of the Week

Problem B

Ch-ch-ch-change!

Canada has the following coins in circulation: nickel (5 cents), dime (10 cents), quarter (25 cents), loonie (\$1), and toonie (\$2).

Australia, on the other hand, has coins with value 5 cents, 10 cents, 20 cents, 50 cents, \$1, and \$2.

- a) Using the least number of coins in each case, determine how to obtain the amounts in the left hand column of the table in each currency. Enter the required coins in the second two columns of the table, as shown for the examples 30 cents and 35 cents.
- b) For how many amounts did you use a different number of coins in Canadian currency than in Australian currency?

Amount	Canadian \$	Australian \$
5 cents		
10 cents		
15 cents		
20 cents		
25 cents		
30 cents	$0.25 + 0.05$	$0.20 + 0.10$
35 cents	$0.25 + 0.10$	$0.20 + 0.10 + 0.05$
40 cents		
45 cents		
50 cents		
55 cents		
60 cents		
65 cents		
70 cents		
75 cents		
80 cents		
85 cents		
90 cents		
95 cents		





Problem of the Week

Problem B and Solution

Ch-ch-ch-change!



Problem

Canada has the following coins in circulation: nickel (5 cents), dime (10 cents), quarter (25 cents), loonie (\$1), and toonie (\$2). Australia, on the other hand, has coins with value 5 cents, 10 cents, 20 cents, 50 cents, \$1, and \$2.

Using the least number of coins in each case, determine how to obtain multiples of 5 cents, from 5 cents to 95 cents, in each currency. For how many of the amounts did you use a different number of coins in Canadian currency than in Australian currency?

Solution

The combinations are shown in the following table for each currency. An asterisk (*) is in the "Amount" column for any amount where a different number of coins were used. The table reveals a different number of coins were used for 12 of the amounts.

Amount \$	Canadian \$	Australian \$
0.05	0.05	0.05
0.10	0.10	0.10
0.15	0.10 + 0.05	0.10 + 0.05
* 0.20	0.10 + 0.10	0.20
* 0.25	0.25	0.20 + 0.05
0.30	0.25 + 0.05	0.20 + 0.10
* 0.35	0.25 + 0.10	0.20 + 0.10 + 0.05
* 0.40	0.25 + 0.10 + 0.05	0.20 + 0.20
0.45	0.25 + 0.10 + 0.10	0.20 + 0.20 + 0.05
* 0.50	0.25 + 0.25	0.50
* 0.55	0.25 + 0.25 + 0.05	0.50 + 0.05
* 0.60	0.25 + 0.25 + 0.10	0.50 + 0.10
* 0.65	0.25 + 0.25 + 0.10 + 0.05	0.50 + 0.10 + 0.05
* 0.70	0.25 + 0.25 + 0.10 + 0.10	0.50 + 0.20
0.75	0.25 + 0.25 + 0.25	0.50 + 0.20 + 0.05
* 0.80	0.25 + 0.25 + 0.25 + 0.05	0.50 + 0.20 + 0.10
0.85	0.25 + 0.25 + 0.25 + 0.10	0.50 + 0.20 + 0.10 + 0.05
* 0.90	0.25 + 0.25 + 0.25 + 0.10 + 0.05	0.50 + 0.20 + 0.20
* 0.95	0.25 + 0.25 + 0.25 + 0.10 + 0.10	0.50 + 0.20 + 0.20 + 0.05



Problem of the Week

Problem B

Need for Speed?

Toonces was driving his 1992 Tabby Car down the road at an average speed of 60 km/h. When he was 10 km from the end of the road, he was passed by his friend, Hector Gonzalez. Toonces' cat-like senses told him that Hector's car was 10 km from the end of the road when he was.



Source: carcabin.com

- How many minutes will it take Toonces to go the last 10 km?
- How many minutes will it take Gonzalez to go the last 10 km?
- How much time will speedy guy Gonzalez save compared to Toonces over that last 10 km stretch of the road?



Problem of the Week

Problem B and Solution

Need for Speed?



Source: carcabin.com

Problem

Toonces was driving his 1992 Tabby Car down the road at an average speed of 60 km/h. When he was 10 km from the end of the road, he was passed by his friend, Hector Gonzalez. Toonces' cat-like senses told him that Hector's car was going 15 km/h faster than he was.

- How many minutes will it take Toonces to go the last 10 km?
- How many minutes will it take Gonzalez to go the last 10 km?
- How much time will speedy guy Gonzalez save compared to Toonces over that last 10 km stretch of the road?

Solution

- Toonces drives 60 km/h, or 1 km/min. Thus it will take him 10 minutes to drive 10 km.
- Hector Gonzalez drives $60 + 15 = 75$ km/h, or 1.25 km/min. Thus it will take him $10 \div 1.25 = 8$ minutes to go 10 km.
- Speedy guy Gonzales will save $10 - 8 = 2$ minutes compared to Toonces on that 10 km stretch of road.



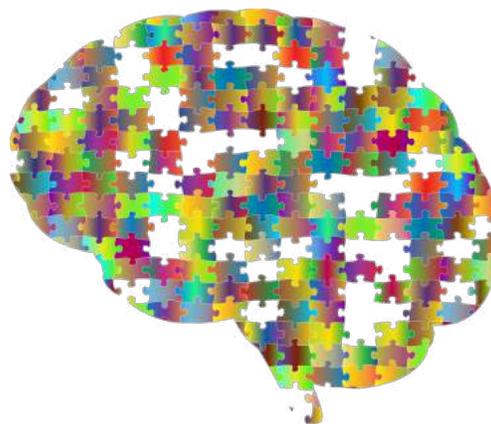
Problem of the Week

Problem B

'Tis a Puzzle

Joan is an experienced jigsaw puzzler. On average, she will correctly place a puzzle piece every 30 seconds.

- a) How long, in hours, should it take Joan to finish a 3000 piece puzzle?
- b) How long, in hours and minutes, should it take Joan to finish a 10 000 piece puzzle?
- c) Joan works on a puzzle from 7:00 p.m. to 9:00 p.m. every weekday. (She does not work on her puzzle on Saturday or Sunday.) If she started a new 10 000 piece puzzle on January 15, 2020, on which date would she finish?





Problem of the Week

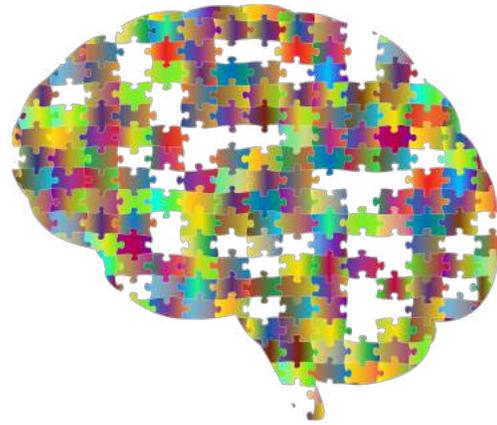
Problem B and Solution

'Tis a Puzzle

Problem

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- Joan works on a puzzle from 7:00 p.m. to 9:00 p.m. every weekday. (She does not work on her puzzle on Saturday or Sunday.) If she started a new 10 000 piece puzzle on January 15, 2020, on which date would she finish?



Solution

- If Joan places a puzzle piece every 30 seconds, then a 3000 piece puzzle will take $3000 \times 30 = 90\,000$ seconds, or $90\,000 \div 60 = 1500$ minutes, or $1500 \div 60 = 25$ hours.
- A 10 000 piece puzzle will take $10\,000 \times 30 = 300\,000$ seconds, or $300\,000 \div 60 = 5000$ minutes, or $5000 \div 60 = 83\frac{1}{3}$ hours, i.e., 83 hours and 20 minutes.
- Working for 2 hours per weekday (7 p.m. - 9 p.m.), it will take Joan $83\frac{1}{3} \text{ hours} \div 2 \text{ hours per day} = 41\frac{2}{3}$ days to complete the 10 000 piece puzzle. Starting on Wednesday, January 15, 2020, and consulting a calendar, we see that the 42nd weekday will occur on Thursday, March 12, 2020. So Joan will complete the puzzle on March 12, 2020.



Problem of the Week

Problem B

Hoops Data

At the time of writing, Kyle Lowry plays basketball with the NBA Champions, the Toronto Raptors. At one point he had played 805 career games. Here are some questions related to his performance in those games.

- If he started in 569 games, in what fraction of the games did he start?
- In a typical 48 minute game, he averaged playing 31.1 minutes. What fraction of each game did he play?
- He scored an average of 14.5 points per game. How many points, on average, has he scored per minute he played in a game? Express your answer as a decimal to the nearest hundredth (i.e., two decimal places).
- Over his career, he made, on average, 43 of 100 shots he took. Based on his past performance, if he took 11 shots in a particular game, how many of these 11 shots would you expect him to make?

For Further Thought:

Try comparing his performance to that of another NBA player by researching their data at the website <https://www.basketball-reference.com/>.





Problem of the Week

Problem B and Solution

Hoops Data

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For Further Thought:

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Solution

- He started in 569 of the 805 games, that is $\frac{569}{805}$ of the games. Expressed as a decimal to two decimal places, $\frac{569}{805} \approx 0.71$, so this represents 71% of the games he played.
- He played 31.1 minutes out of 48 minutes each game, that is $\frac{31.1}{48} = \frac{311}{480}$ of each game. Expressed as a decimal to two decimal places, $\frac{311}{480} \approx 0.65$, so this represents 65% of each game.
- He scored an average of 14.5 points per game played and played an average of 31.1 minutes in a game. Therefore, he scored $14.5 \div 31.1 \approx 0.47$ points per minute played.
- He made, on average, 43 of the 100 shots he took. If he took 11 shots, we would expect him to make $\frac{43}{100} \times 11 = 4.73$ shots (that is, we would expect him to make 4 or 5 shots).

For Further Thought: Answers will vary.



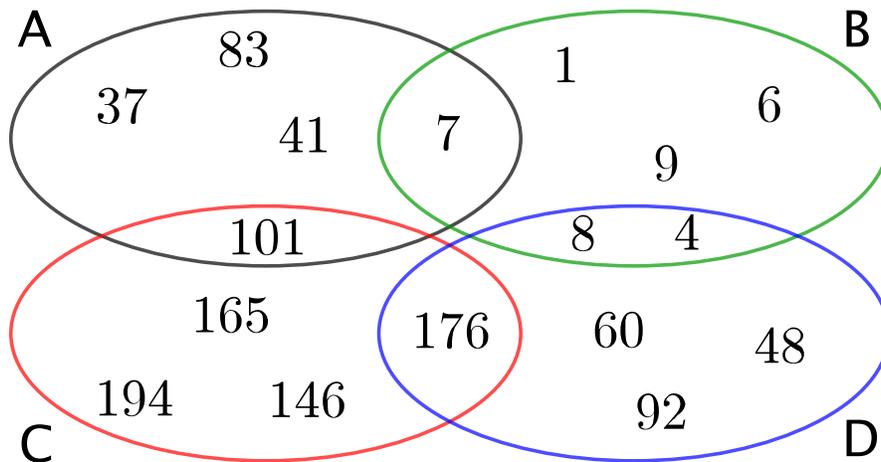
Problem of the Week

Problem B

Now and Venn

A *Venn Diagram* consists of overlapping shapes and is used to show what two or more sets have in common and do not have in common.

Consider the Venn diagram shown below.



- Think of possible reasons for putting the numbers in each section of this Venn diagram.
- Where would the number 144 belong? What about 2?
- Is there a number that would fit in an overlap of any three of these sections, according to your criteria from part a)?



Problem of the Week

Problem B and Solution

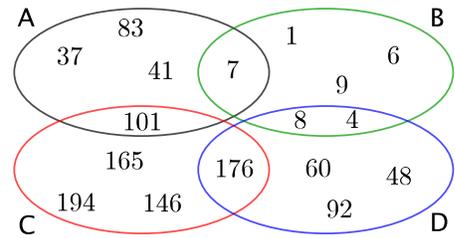
Now and Venn

Problem

A *Venn Diagram* consists of overlapping shapes and is used to show what two or more sets have in common and do not have in common.

Consider the Venn diagram shown to the right.

- a) Think of possible reasons for putting the numbers in each section of this Venn diagram.
- b) Where would the number 144 belong? What about 2?
- c) Is there a number that would fit in an overlap of any three of these sections, according to your criteria from part a)?



Solution

- a) Possible characteristics of the numbers in each section are:
 - A - prime numbers;
 - B - single digit numbers;
 - C - three-digit numbers (or numbers in the hundreds);
 - D - numbers which are multiples of 4.
- b) By the above definitions, the number 144 could go in the section shared by C and D, and the number 2 could go in the section shared by A and B.
- c) To fit in an overlap of three of these four conditions, a number would have to satisfy at least three of the four conditions.

To show that this is not possible, first notice that a number cannot be both a single digit number and three-digit number. That is, no overlap between sections B and C is possible.

Is it possible to have an overlap of the three sections labeled A, B and D? That is, can a number be a single digit number, prime, and a multiple of 4? The single digit numbers that are prime are 2, 3, 5, and 7, none of which are a multiple of 4. Therefore, a number cannot be a single digit and satisfy three of the four conditions.

Is it possible to have an overlap of the three sections labeled A, C and D? That is, can a number be a three-digit number, prime, and a multiple of 4? If a three-digit number is a multiple of 4, then 4 is a factor of that three-digit number, so the number is not prime. Therefore, a number cannot be a three-digit number and satisfy three of the four conditions.



Problem of the Week

Problem B

Paving the Way

Munit's driveway is 8 m wide and 16 m long. He wants to cover the entire rectangular area with pavers which are only available in sets of 10 pavers. Each set of 10 pavers covers 3 square metres in total.

- If each set of 10 pavers costs \$100, how much will it cost Munit to buy enough sets of pavers for the job? Add 1 extra set of pavers to your purchase to allow for possible breakage.
- Each paver in a set is $\frac{1}{2}$ m by $\frac{3}{5}$ m. Make a plan for placing the pavers so that they completely cover the driveway. In your design, how many pavers will need to be cut in order to fit the driveway? How much will be cut from each of these pavers?

You may assume the pavers can be laid either horizontally or vertically.

There are many ways to place the pavers. Compare your design to the design of others in your class.





Problem of the Week

Problem B and Solution

Paving the Way

Problem

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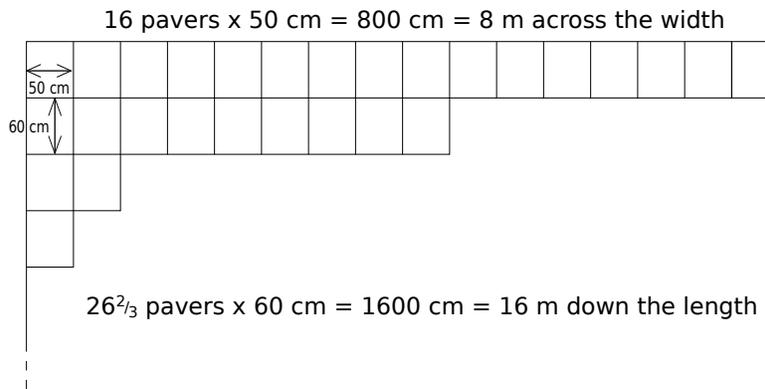
Solution

- a) The total area of pavers required is $8\text{ m} \times 16\text{ m} = 128\text{ m}^2$. Since each set covers 3 m^2 , Munit will need $128 \div 3 = 42\frac{2}{3} \approx 42.7$, so 43 sets of pavers. If an extra set of pavers is purchased, Munit will need 44 sets. At a cost of \$100 per set, his total cost is thus $\$100 \times 44 = \4400 .
- b) Each paver is 50 cm wide by 60 cm long ($\frac{1}{2}\text{ m}$ by $\frac{3}{5}\text{ m}$).

We present three of many possible plans.

PLAN 1:

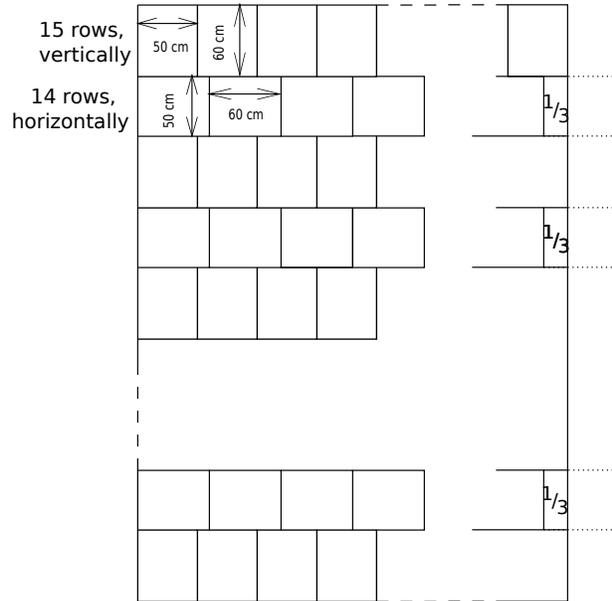
Set the pavers vertically across the $8\text{ m} = 800\text{ cm}$ width of the driveway. This requires $800 \div 50 = 16$ pavers. The $16\text{ m} = 1600\text{ cm}$ length of the driveway would need $1600 \div 60 = 26\frac{2}{3}$ pavers. So the final 27th row would need each of the 16 pavers cut to only $\frac{2}{3} \times 60 = 40\text{ cm}$, i.e., 16 cuts are required. That is, 20 cm is cut off of each of the 16 pavers in the 27th row. This method would use $27 \times 16 = 432$ pavers. Since we purchased an extra set of pavers, we have enough.





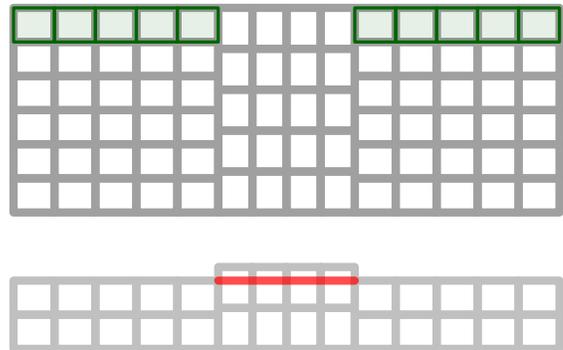
PLAN 2:

Alternate rows of pavers laid vertically and horizontally, starting with a vertical row. This will require exactly 15 vertical rows and 14 horizontal rows to cover the length of the driveway, since $15 \times 60 + 14 \times 50 = 900 + 700 = 1600$ cm. Further, for the vertical rows, the width will require 16 pavers, since $800 \div 50 = 16$. However, the horizontal rows will require 14 pavers, since $800 \div 60 = 13\frac{1}{3}$. The pavers at the ends of these 14 horizontal rows will need to be cut to $\frac{1}{3}$ of $60 = 20$ cm by 50 cm, requiring 14 cuts. That is, 40 cm is cut off of each of the pavers at the ends of the 14 horizontal rows. Each row with pavers laid vertically has 16 pavers and there are 15 such rows, so there are $15 \times 16 = 240$ pavers laid vertically. Each row with pavers laid horizontally has 14 pavers and there are 14 such rows, so there are $14 \times 14 = 196$ pavers laid horizontally. This plan uses $240 + 196 = 436$ pavers. Since we purchased an extra set of pavers, we have enough to complete the job.



PLAN 3:

Lay 5 pavers horizontally, followed by 4 pavers vertically, and then 5 more pavers horizontally. Do this for a total of 5 rows. This creates the unshaded part in the diagram to the upper right. Now, add 10 more horizontal pavers to complete the rectangle. These last 10 pavers are shaded in the diagram. The large rectangle is 8 m by 3 m. (You can verify this.) Do this process a total of 5 times to cover 8 m by 15 m of the driveway. Now, a strip 8 m by 1 m is needed to complete the coverage. This small strip is shown in the diagram to the bottom right.



In this small strip, 4 of the vertically placed pavers extend 0.2 m or 20 cm past the end of the driveway. Therefore, 4 cuts of 20 cm each are required. Each of the 8 m by 3 m sections has 80 pavers. There are 5 of these sections, so there are $5 \times 80 = 400$ pavers used. In the final two rows, 28 pavers are used. Therefore, the total number of pavers used is $400 + 28 = 428$. Since we have 440 pavers, we have 12 extra pavers. We could return the extra set to the store and reduce our cost to \$4300.



Problem of the Week

Problem B

A Terror on Ice

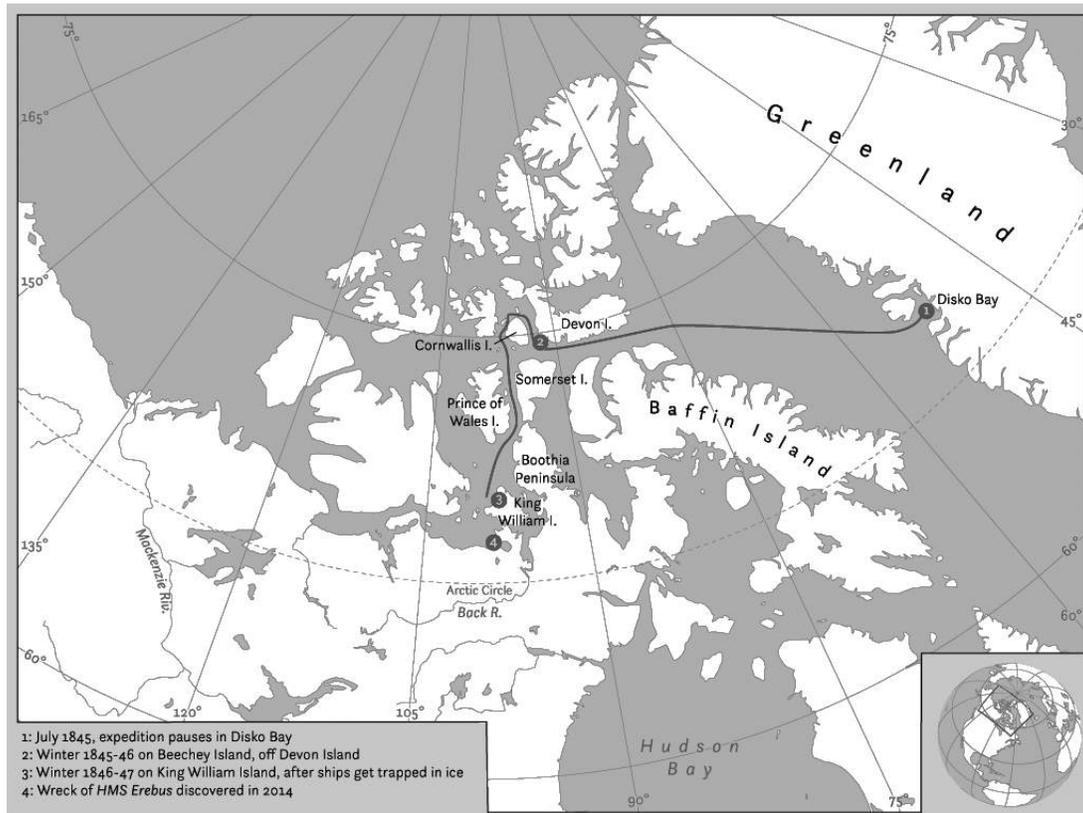
In 1845, Sir John Franklin left England on HMS Erebus, along with HMS Terror, in search of a northwest passage to India. He traveled approximately 12 000 km before getting stuck in the ice near King William Island, coming within about 300 km of Umingmaktok.

- What fraction of the distance he sailed in 1845 was the remaining distance to Umingmaktok (which would have revealed a complete northwest passage)?
- According to a scrawled note, the HMS Erebus was last seen by a European on April 22, 1848. The wreck of the ship was located on September 1, 2014.

How many years, months and days passed between these two sightings?

- The HMS Terror was originally launched on June 19, 1813, and its wreck was discovered in 2016. It participated in the War of 1812, and participated in a further Antarctic Expedition, from 1839 to 1843.

How old, in years, is the HMS Terror now (June 19, 2019)?



(wikimedia.commons)



Problem of the Week

Problem B and Solution

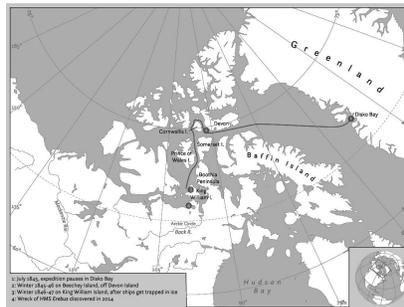
A Terror on Ice

Problem

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(wikimedia.commons)

Solution

- Since he sailed 12 000 km in 1845 and still had 300 km to go to Umingmaktok, the remaining distance was $\frac{300}{12000} = \frac{1}{40}$ of his trip.
- The number of years is $2014 - 1848 = 166$ years. The number of months from May to August is 4 months. The number of days is $8 + 1 = 9$ days. Thus a total of 166 years, 4 months, and 9 days passed between the two sightings.
- The age of the terror, in years as of June 19, 2019, is $2019 - 1813 = 206$ years.



Problem of the Week

Problem B

Friendly Towers

Friendly Towers is a high-rise apartment building. Dieter and three of his friends, Sanjay, Bethany, and Ivanka live on different floors of Friendly Towers.

- Dieter lives on the 9th floor.
- Friendly Towers has five times as many floors as Dieter's floor number.
- Bethany lives on the third floor below the top floor.
- Ivanka lives two floors above Sanjay, and Sanjay lives 5 floors below Bethany.
- On the floor which is $\frac{8}{9}$ ths of the way to the top of the building, you will find a pool used by residents of the building.

Which floor do each of Sanjay, Bethany, and Ivanka live on, and on which floor is the pool located?





Problem of the Week

Problem B and Solution

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Friendly Towers is a high-rise apartment building. Dieter and three of his friends, Sanjay, Bethany, and Ivanka live on different floors of Friendly Towers. Dieter lives on the 9th floor. Friendly Towers has five times as many floors as Dieter's floor number. Bethany lives on the third floor below the top floor. Ivanka lives two floors above Sanjay, and Sanjay lives 5 floors below Bethany. On the floor which is $\frac{8}{9}$ ths of the way to the top of the building, you will find a pool used by residents of the building. Which floor do each of Sanjay, Bethany, and Ivanka live on, and on which floor is the pool located?

Solution

To begin, we must determine how many floors there are in Friendly Towers. We are given that the number of floors is 5 times Dieter's floor number. Thus, Friendly Towers has $5 \times 9 = 45$ floors.

Bethany lives 3 floors from the top floor. Since $45 - 3 = 42$, she lives on the 42nd floor.

Sanjay lives 5 floors below Bethany. Since $42 - 5 = 37$, he lives on the 37th floor.

Ivanka lives 2 floors above Sanjay. Since $37 + 2 = 39$, she lives on the 39th floor.

The pool is $\frac{8}{9}$ of the way to the top. Since $\frac{8}{9} \times 45 = 40$, it is on the 40th floor.



Problem of the Week

Problem B

Go Outside and Play!

The Sony PlayStation2™ is the best selling video game console of all time.

Annual sales data (in millions of units) are displayed in the second column of the table below for the years 2000 to 2011.

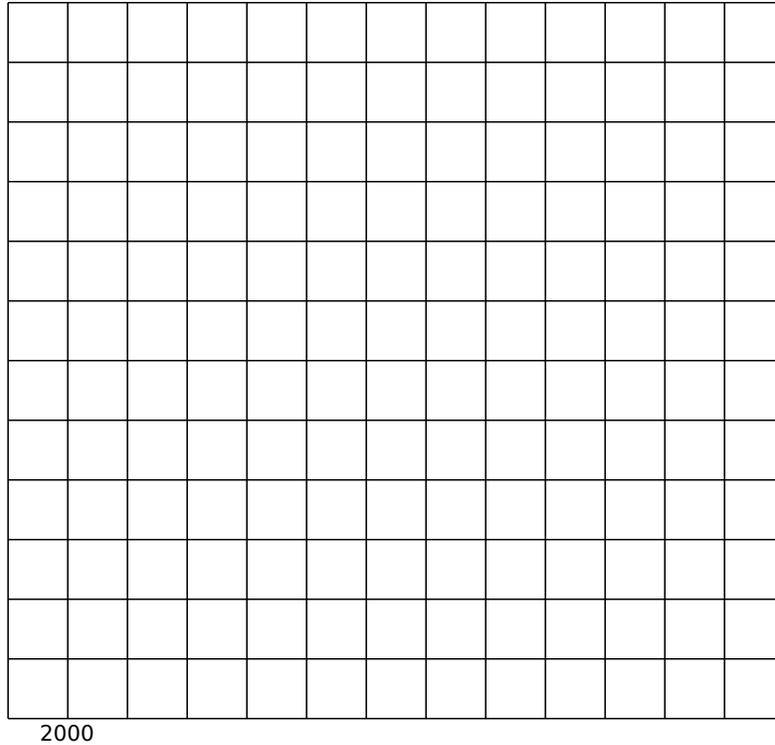


- a) Round each year's sales to the nearest million, and enter your answers in the table.
- b) Using the rounded numbers, approximately how many PS2s were sold from 2000 to 2011?
- c) Plot the rounded sales on a broken line graph. Include labels and units for each axis, and a title for your graph. A grid is provided for your use on the next page.
- d) Why is a broken line graph a good choice to display this data?
- e) Make up some questions for a friend to answer based on your graph.

Date of fiscal year end	Annual sales	Annual sales (to nearest million)
March 31, 2000	1.41	
March 31, 2001	9.20	
March 31, 2002	18.07	
March 31, 2003	22.52	
March 31, 2004	20.10	
March 31, 2005	16.17	
March 31, 2006	16.22	
March 31, 2007	14.20	
March 31, 2008	13.7	
March 31, 2009	7.90	
March 31, 2010	7.30	
March 31, 2011	6.4	

vgsales.fandom.com/wiki/PlayStation_2





- Remember that your graph needs a label for each axis, and units if necessary.
- Your graph also needs a title. A good title will contain something about each axis.
- Remember that the numbers on the vertical axis need to go up by the same amount, and they should be on the lines, not the spaces.



Problem of the Week

Problem B and Solution

Go Outside and Play!

Problem

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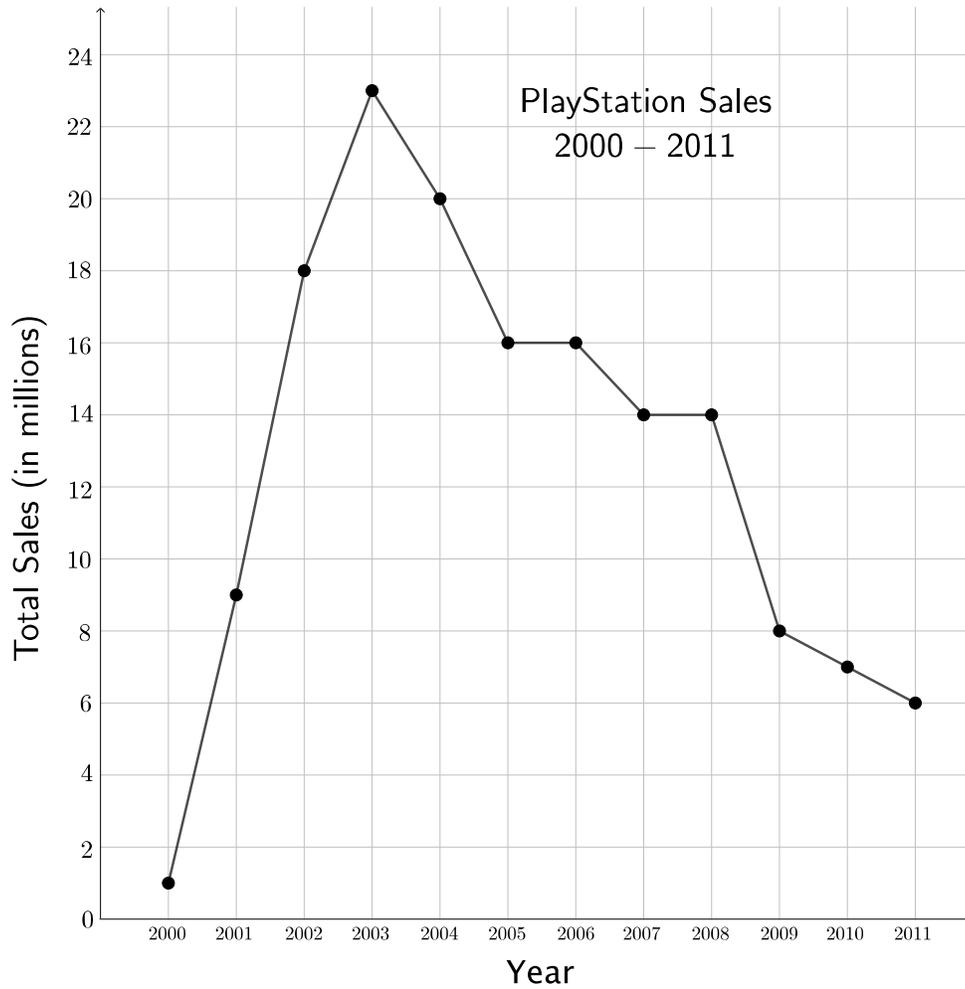


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Date of fiscal year end	Annual sales	Annual sales (to nearest million)
March 31, 2000	1.41	1
March 31, 2001	9.20	9
March 31, 2002	18.07	18
March 31, 2003	22.52	23
March 31, 2004	20.10	20
March 31, 2005	16.17	16
March 31, 2006	16.22	16
March 31, 2007	14.20	14
March 31, 2008	13.7	14
March 31, 2009	7.90	8
March 31, 2010	7.30	7
March 31, 2011	6.4	6

vgsales.fandom.com/wiki/PlayStation_2

- Remember that your graph needs a label for each axis, and units if necessary.
- Your graph also needs a title. A good title will contain something about each axis.
- Remember that the numbers on the vertical axis need to go up by the same amount, and they should be on the lines, not the spaces.



Solution

- a) The sales rounded to the nearest million are shown in the revised table within the question.
- b) The sum of the rounded sales from 2000 to 2011 is 152 million.
- c) See the graph above.
- d) A broken line graph is a good choice because it shows the general trend of the data over time.
- e) Answers will vary.



Problem of the Week

Problem B

Round and Round with Robbie

Cool Cole has been building his robot Robbie for the big 4 metre drag race. He has learned that his robot moves 75 mm in a straight line every time its tires do a full 360 degree rotation. He programs Robbie to do 50 tire rotations for the race.

- a) Robbie is 25 cm long. If the race starts with the nose of the robot touching the starting line, will the nose of the robot get to the end line of the drag race after 50 rotations?
- b) How many complete rotations will his robot's tires need to do in order for the entire robot to completely cross the finish line?





Problem of the Week

Problem B and Solution

Round and Round with Robbie

Problem

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- b) How many complete rotations will his robot's tires need to do in order for the entire robot to completely cross the finish line?



Solution

- a) In 50 rotations, Robbie will move

$$50 \times 75 = 3750 \text{ mm, or } 3.75 \text{ m.}$$

Since the race is 4 m, this is not enough for Robbie's nose to cross the finish line.

- b) To completely cross the finish line, Robbie must go the 4 m of the race, plus 25 cm, the length of the robot. Thus it must travel 4.25 m, or 4250 mm, which is $4250 \div 75 = 56\frac{2}{3}$ rotations.
So Robbie's back end will cross the finish line during the 57th rotation.



Problem of the Week

Problem B

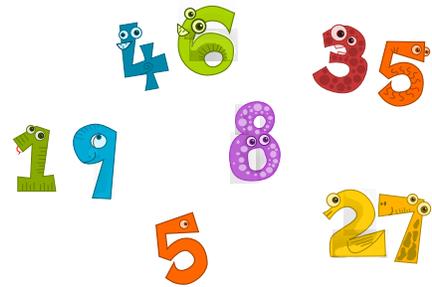
It's Hip to be Square

The whole number *factors* of any whole number N are the whole numbers which divide evenly into N .

For example, 12 has six whole number factors. They are 1, 2, 3, 4, 6, and 12.
The number 17 has two whole number factors. They are 1 and 17.

Notice that both 12 and 17 have an even number of whole number factors.

- List all the numbers from 1 to 10 which have an odd number of whole number factors.
- Why is there an odd number of factors for some of the numbers?
- Figure out which numbers between 10 and 50 have an odd number of whole number factors.
- What is the special name for the numbers that have an odd number of whole number factors?





Problem of the Week

Problem B and Solution

It's Hip to be Square

Problem

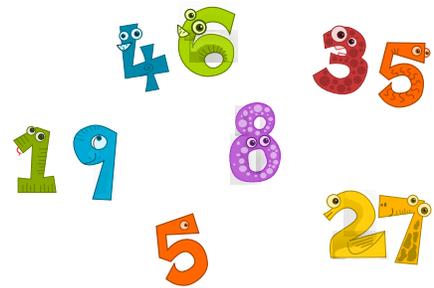
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- b) Why is there an odd number of factors for some of the numbers?
- c) Figure out which numbers between 10 and 50 have an odd number of whole number factors.
- d) What is the special name for the numbers that have an odd number of whole number factors?



Solution

a)

Number	1	2	3	4	5	6	7	8	9	10
Whole Number Factors	1	1,2	1,3	1,2,4	1,5	1,2,3,6	1,7	1,2,4,8	1,3,9	1,2,5,10
Number of Factors	1	2	2	3	2	4	2	4	3	4

We see that the numbers 1, 4, and 9 have an odd number of factors.

- b) For the numbers with an even number of whole number factors, when we place the factors in order from smallest to largest, the product of the middle pair of factors gives the number (e.g., for the number 8: $2 \times 4 = 8$). But in the numbers with an odd number of factors, the middle factor multiplied by itself gives the number. For example, for the number 9: $3 \times 3 = 9$, or $3^2 = 9$.
- c) If we use the idea that an odd number of factors occurs when the number equals the product of one factor and itself, then between 10 and 50, the numbers with an odd number of factors are
 $16 = 4 \times 4$, $25 = 5 \times 5$, $36 = 6 \times 6$, and $49 = 7 \times 7$.
 Note that: the factors of 16 are 1, 2, 4, 8, and 16; the factors of 25 are 1, 5, and 25; the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36; and the factors of 49 are 1, 7, and 49.
- d) Such numbers are called perfect squares, or square numbers.



Problem of the Week

Problem B

Years and Years of ‘Threepcats’

Find all years between 1000 BCE and 2020 CE, inclusive, which have exactly three digits the same. For example, the years 1011 and 1222 each have exactly three digits the same, but 1111 and 1123 do not.





Problem of the Week

Problem B and Solution

Years and Years of ‘Threeppeats’

Problem

Find all years between 1000 BCE and 2020 CE, inclusive, which have exactly three digits the same. For example, the years 1011 and 1222 each have exactly three digits the same, but 1111 and 1123 do not.



Solution

Between 1000 BCE and 2020, there are several different types of year labels which have exactly 3 digits the same. The table below enumerates and totals them.

Years	Years with triple digits	Total
1000 BCE to 1 BCE	1000, 999, 888, 777, 666, 555, 444, 333, 222, 111	10
1 CE to 1000 CE	111, 222, 333, 444, 555, 666, 777, 888, 999, 1000 (reverse order of 1000 BCE to 1 BCE)	10
1001 to 2020 CE	1011, 1211, 1311, 1411, 1511, 1611, 1711, 1811, 1911,	9
	1101, 1121, 1131, 1141, 1151, 1161, 1171, 1181, 1191,	9
	1110, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119,	9
	1222, 1333, 1444, 1555, 1666, 1777, 1888, 1999, 2000	9
Total		56



Problem of the Week

Problem B

Raise a Buck at Starstruck's

Last year, the ten employees at Starstruck's had an average annual income of \$30 000 each. This year, three of them received equal raises in pay, raising the total annual income for all ten employees to \$303 000.

- How much was the raise received by each of the three employees?
- What is the new average annual income of each of the ten employees this year?
- If the word "equal" is deleted from the given information, could you find the answer to part a)? Explain your reasoning.





Problem of the Week

Problem B and Solution

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- If the word "equal" is deleted from the given information, could you find the answer to part a)? Explain your reasoning.



Solution

- Last year, the total income for all 10 employees was $\$30\,000 \times 10 = \$300\,000$. Thus the raises totaled $\$303\,000 - \$300\,000 = \$3\,000$. So each of the three employees received a raise of $\$3\,000 \div 3 = \$1\,000$.
- After the new raises, the new average annual income was $\$303\,000 \div 10 = \$30\,300$.
- If the word 'equal' is deleted from part a), all we would know is the total of the raises, \$3 000. So we could not find individual raises, as any three amounts that sum to \$3 000 would be possible.



Problem of the Week

Problem B

Chip, Chip, Chooray!

At Biscuit Hill Elementary School, Chip and his sister, Charlene, have decided that they want to make cookies for all of the junior students in their school. The recipe that they found makes enough chocolate chip cookies of 7 cm diameter for 16 people.

Recipe		Junior Classes	
1 cup	butter	Mrs. Martin	25 students
1 cup	brown sugar	Mrs. Laing	26 students
$\frac{1}{2}$ cup	white sugar	Ms. Richmond	23 students
2	eggs	Mrs. Kelter	24 students
2 tsp	vanilla	Mr. Hallett	22 students
$2\frac{1}{4}$ cups	flour		
1 tsp	baking soda		
300 g	chocolate chips		



- How many batches should Chip and Charlene make so that they make the exact number of cookies needed for all of the students in the junior classes?
- They decide to make a whole number of batches so that they have some extra cookies to save for later and one cookie for each teacher. What quantity of each ingredient in the recipe will they need?



Problem of the Week
Problem B and Solution
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- They decide to make a whole number of batches so that they have some extra cookies to save for later and one cookie for each teacher. What quantity of each ingredient in the recipe will they need?

Solution

- There are $25 + 26 + 23 + 24 + 22 = 120$ students in total. Since one recipe makes enough cookies for 16 people, to make exactly enough, Chip and Charlene would need to make $120 \div 16 = 7.5$ batches.
- Eight batches (128 cookies) will leave 5 for the teachers and 3 to save for later. Thus they will need to multiply all the measurements by eight to get;
 $8 \times 1 = 8$ cups butter, $8 \times 1 = 8$ cups brown sugar,
 $8 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 4$ cups white sugar,
 $8 \times 2 = 16$ eggs, $8 \times 2 = 16$ tsp vanilla,
 $8 \times (2\frac{1}{4}) = 8 \times 2 + \frac{1}{4} = 16 + \frac{8}{4} = 16 + 2 = 18$ cups flour,
 $8 \times 1 = 8$ tsp baking soda, and $8 \times 300 = 2400$ g (2.4 kg) of chocolate chips.
 (If they want more cookies left over, they will need more batches.)



Problem of the Week

Problem B

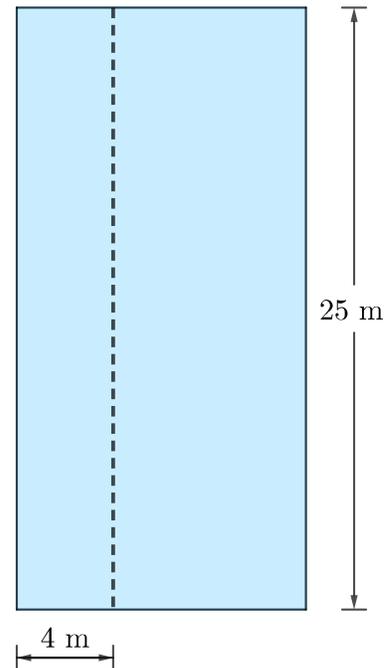
Laps and Lengths

A swimming pool is 25 m long and 12 m wide. A single 4 m wide lane is roped off.

Ismaeel swims in the 4 m wide lane. Swimming two lengths of the pool counts as one lap for Ismaeel.

Kosha swims on the other side of the roped off lane, in the larger rectangular portion of the pool. A lap for Kosha consists of swimming once around a circuit that is $\frac{1}{2}$ m away from each edge and $\frac{1}{2}$ m away from the rope divider.

- How much longer is Kosha's lap than Ismaeel's lap?
- Ismaeel and Kosha each complete ten laps. Determine how much farther than Ismaeel that Kosha swam.





Problem of the Week

Problem B and Solution

Laps and Lengths

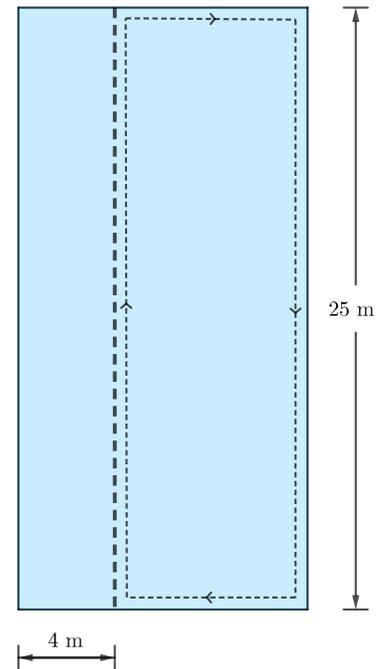
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Solution

- The large rectangular part of the pool is $12 - 4 = 8$ m wide. Since Kosha swims $\frac{1}{2}$ m from the 'edges', Kosha swims around a rectangle that is $8 - \frac{1}{2} - \frac{1}{2} = 7$ m wide and $25 - \frac{1}{2} - \frac{1}{2} = 24$ m long. The total distance Kosha swims in one lap is $24 + 7 + 24 + 7 = 62$ m. Ismaeel swims $25 + 25 = 50$ m in two lengths, or one lap. Thus Kosha's lap is $62 - 50 = 12$ m longer than Ismaeel's lap.
- In ten laps, Kosha swims $10 \times 62 = 620$ m, while Ismaeel swims $10 \times 50 = 500$ m. So Kosha swims $620 - 500 = 120$ m further.
ALTERNATIVELY: Since in each lap Kosha swims 12 m further, in 10 laps she swims $10 \times 12 = 120$ m further.



Problem of the Week

Problem B

And the Winner Is...

Winners in the Roll Down the Brim contest at Tom Hinton's have to answer skill testing questions in order to claim the grand prizes of cars, TVs, or \$5000 cash. You and a friend decide to try some of the questions, but you keep getting different answers.

For each question, decide which answer is correct, and explain why.

- Is the value of $6 + 8 \div 2 - 5$ equal to 5, or is it equal to 2?
- Is the value of $7 \times 4 + 12 \div 2$ equal to 20, or is it equal to 34?
- Is the value of $12 - 6 + 9 \div 3$ equal to 5, or is it equal to 9?
- Having calculated the value of $16 + 4 \times 3 \div 6 + 4 \times 3$, you got an answer of 30. But your friend got 42. Which answer is correct? Explain how each answer was obtained.





Problem of the Week

Problem B and Solution

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- Having calculated the value of $16 + 4 \times 3 \div 6 + 4 \times 3$, you got an answer of 30. But your friend got 42. Which answer is correct? Explain how each answer was obtained.

Solution

When evaluating expressions with a mixture of operations, multiplication and division are done before addition and subtraction, and in the order they appear from left to right.

- The correct order of operations does the division first so $6 + \underline{8 \div 2} - 5$ simplifies to $6 + \underline{4} - 5$. Then we do addition and subtraction in the order they appear producing an answer of 5. We can write this using brackets as follows:

$$6 + (8 \div 2) - 5 = 6 + 4 - 5 = 5.$$

From this point on in the solution, we will use round brackets to show the parts that will be completed next.

- The correct order of operations gives $(7 \times 4) + (12 \div 2) = 28 + 6 = 34$.
- The correct order of operations gives $12 - 6 + (9 \div 3) = 12 - 6 + 3 = 9$.
- If the correct order of operations is used, the answer is

$$16 + (4 \times 3) \div 6 + (4 \times 3) = 16 + (12 \div 6) + 12 = 16 + 2 + 12 = 30$$

The incorrect answer was done as

$$(16 + 4) \times (3 \div 6) + 4 \times 3 = 20 \times \frac{1}{2} + 4 \times 3 = (10 + 4) \times 3 = 14 \times 3 = 42$$

Geometry (G)





Problem of the Week

Problem B

Will That be Rye or Honey Oats?

Jagheet drives from his house to his camp, 75 km away, maintaining an average speed of 60 km/h.

- How long, in minutes, does it take him to drive to his camp?
- If he left his house at 11:37 a.m., at what time could he expect to arrive at his camp?
- Realizing that he forgot to get bread, he stops at the grocery store for 12 minutes. Including this time, what is his new average speed in km/h?
(Round your answer to the nearest tenth.)





Problem of the Week

Problem B and Solution

Will That be Rye or Honey Oats?

Problem

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- Realizing that he forgot to get bread, he stops at the grocery store for 12 minutes. Including this time, what is his new average speed in km/h? (Round your answer to the nearest tenth.)



Solution

- Since Jagheet travels 75 km at 60 km/h, his travel time will be $75 \div 60 = 1.25$ h, or $1.25 \times 60 = 75$ minutes.
ALTERNATIVELY: His speed is 60 km/h, or 1 km/min. So, 75 km takes 75 minutes.
- He would expect to arrive at his camp 1 h and 15 minutes after 11:37 a.m., which would be at 12:52 p.m.
- His total time is now $75 + 12 = 87$ minutes. Thus his average speed is now $75 \div 87 \approx 0.862$ km/min, or $0.862 \times 60 \approx 51.7$ km/h, to the nearest tenth.



Problem of the Week

Problem B

Ch-ch-ch-change!

Canada has the following coins in circulation: nickel (5 cents), dime (10 cents), quarter (25 cents), loonie (\$1), and toonie (\$2).

Australia, on the other hand, has coins with value 5 cents, 10 cents, 20 cents, 50 cents, \$1, and \$2.

- a) Using the least number of coins in each case, determine how to obtain the amounts in the left hand column of the table in each currency. Enter the required coins in the second two columns of the table, as shown for the examples 30 cents and 35 cents.
- b) For how many amounts did you use a different number of coins in Canadian currency than in Australian currency?

Amount	Canadian \$	Australian \$
5 cents		
10 cents		
15 cents		
20 cents		
25 cents		
30 cents	$0.25 + 0.05$	$0.20 + 0.10$
35 cents	$0.25 + 0.10$	$0.20 + 0.10 + 0.05$
40 cents		
45 cents		
50 cents		
55 cents		
60 cents		
65 cents		
70 cents		
75 cents		
80 cents		
85 cents		
90 cents		
95 cents		





Problem of the Week

Problem B and Solution

Ch-ch-ch-change!



Problem

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Using the least number of coins in each case, determine how to obtain multiples of 5 cents, from 5 cents to 95 cents, in each currency. For how many of the amounts did you use a different number of coins in Canadian currency than in Australian currency?

Solution

The combinations are shown in the following table for each currency. An asterisk (*) is in the "Amount" column for any amount where a different number of coins were used. The table reveals a different number of coins were used for 12 of the amounts.

Amount \$	Canadian \$	Australian \$
0.05	0.05	0.05
0.10	0.10	0.10
0.15	0.10 + 0.05	0.10 + 0.05
* 0.20	0.10 + 0.10	0.20
* 0.25	0.25	0.20 + 0.05
0.30	0.25 + 0.05	0.20 + 0.10
* 0.35	0.25 + 0.10	0.20 + 0.10 + 0.05
* 0.40	0.25 + 0.10 + 0.05	0.20 + 0.20
0.45	0.25 + 0.10 + 0.10	0.20 + 0.20 + 0.05
* 0.50	0.25 + 0.25	0.50
* 0.55	0.25 + 0.25 + 0.05	0.50 + 0.05
* 0.60	0.25 + 0.25 + 0.10	0.50 + 0.10
* 0.65	0.25 + 0.25 + 0.10 + 0.05	0.50 + 0.10 + 0.05
* 0.70	0.25 + 0.25 + 0.10 + 0.10	0.50 + 0.20
0.75	0.25 + 0.25 + 0.25	0.50 + 0.20 + 0.05
* 0.80	0.25 + 0.25 + 0.25 + 0.05	0.50 + 0.20 + 0.10
0.85	0.25 + 0.25 + 0.25 + 0.10	0.50 + 0.20 + 0.10 + 0.05
* 0.90	0.25 + 0.25 + 0.25 + 0.10 + 0.05	0.50 + 0.20 + 0.20
* 0.95	0.25 + 0.25 + 0.25 + 0.10 + 0.10	0.50 + 0.20 + 0.20 + 0.05



Problem of the Week

Problem B

Need for Speed?

Toonces was driving his 1992 Tabby Car down the road at an average speed of 60 km/h. When he was 10 km from the end of the road, he was passed by his friend, Hector Gonzalez. Toonces' cat-like senses told him that Hector's car was faster than he was.



Source: carcabin.com

- How many minutes will it take Toonces to go the last 10 km?
- How many minutes will it take Gonzalez to go the last 10 km?
- How much time will speedy guy Gonzalez save compared to Toonces over that last 10 km stretch of the road?



Problem of the Week

Problem B and Solution

Need for Speed?



Source: carcabin.com

Problem

Toonces was driving his 1992 Tabby Car down the road at an average speed of 60 km/h. When he was 10 km from the end of the road, he was passed by his friend, Hector Gonzalez. Toonces' cat-like senses told him that Hector's car was going 15 km/h faster than he was.

- How many minutes will it take Toonces to go the last 10 km?
- How many minutes will it take Gonzalez to go the last 10 km?
- How much time will speedy guy Gonzalez save compared to Toonces over that last 10 km stretch of the road?

Solution

- Toonces drives 60 km/h, or 1 km/min. Thus it will take him 10 minutes to drive 10 km.
- Hector Gonzalez drives $60 + 15 = 75$ km/h, or 1.25 km/min. Thus it will take him $10 \div 1.25 = 8$ minutes to go 10 km.
- Speedy guy Gonzales will save $10 - 8 = 2$ minutes compared to Toonces on that 10 km stretch of road.



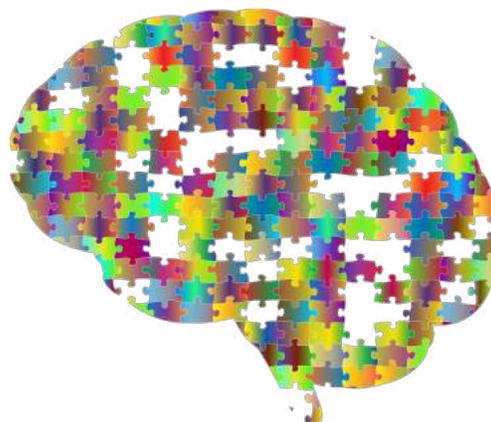
Problem of the Week

Problem B

'Tis a Puzzle

Joan is an experienced jigsaw puzzler. On average, she will correctly place a puzzle piece every 30 seconds.

- a) How long, in hours, should it take Joan to finish a 3000 piece puzzle?
- b) How long, in hours and minutes, should it take Joan to finish a 10 000 piece puzzle?
- c) Joan works on a puzzle from 7:00 p.m. to 9:00 p.m. every weekday. (She does not work on her puzzle on Saturday or Sunday.) If she started a new 10 000 piece puzzle on January 15, 2020, on which date would she finish?





Problem of the Week

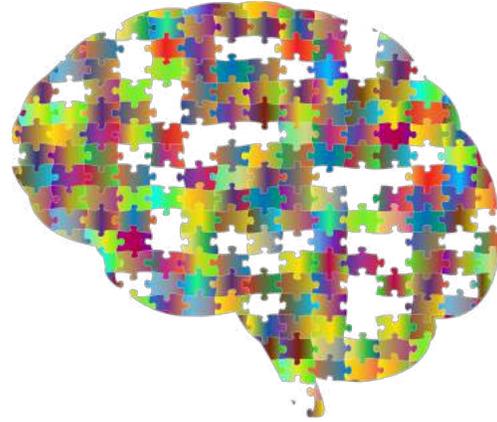
Problem B and Solution

'Tis a Puzzle

Problem

Joan is an experienced jigsaw puzzler. On average, she will correctly place a puzzle piece every 30 seconds.

- How long, in hours, should it take Joan to finish a 3000 piece puzzle?
- How long, in hours and minutes, should it take Joan to finish a 10 000 piece puzzle?
- Joan works on a puzzle from 7:00 p.m. to 9:00 p.m. every weekday. (She does not work on her puzzle on Saturday or Sunday.) If she started a new 10 000 piece puzzle on January 15, 2020, on which date would she finish?



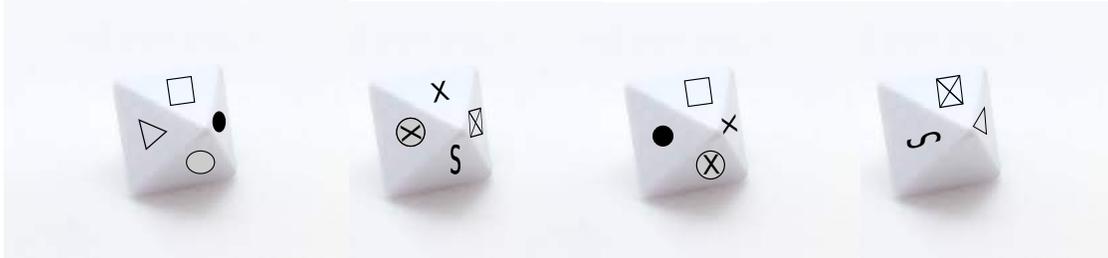
Solution

- If Joan places a puzzle piece every 30 seconds, then a 3000 piece puzzle will take $3000 \times 30 = 90\,000$ seconds, or $90\,000 \div 60 = 1500$ minutes, or $1500 \div 60 = 25$ hours.
- A 10 000 piece puzzle will take $10\,000 \times 30 = 300\,000$ seconds, or $300\,000 \div 60 = 5000$ minutes, or $5000 \div 60 = 83\frac{1}{3}$ hours, i.e., 83 hours and 20 minutes.
- Working for 2 hours per weekday (7 p.m. - 9 p.m.), it will take Joan $83\frac{1}{3} \text{ hours} \div 2 \text{ hours per day} = 41\frac{2}{3}$ days to complete the 10 000 piece puzzle. Starting on Wednesday, January 15, 2020, and consulting a calendar, we see that the 42nd weekday will occur on Thursday, March 12, 2020. So Joan will complete the puzzle on March 12, 2020.

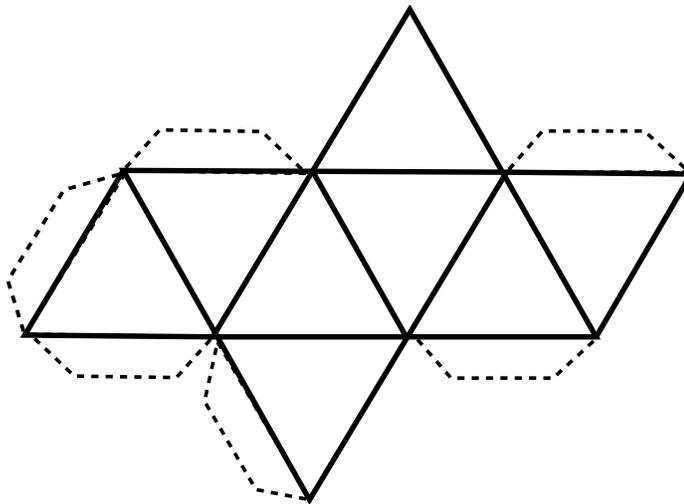


Problem of the Week Problem B The Die is Cast

An 8-sided die contains a different symbol on each side. The diagrams below are pictures of the die from different angles.



- a) Given the first three views of this die, determine what symbol would appear on the blank side shown in the fourth diagram. (You may find the net given below helpful.)



- b) Describe to a classmate how you solved the problem.



Problem of the Week

Problem B and Solution

The Die is Cast

Problem

An 8-sided die contains a different symbol on each side. The diagrams below are pictures of the die from different angles.



- Given the first three views of this die, determine what symbol would appear on the blank side shown in the fourth diagram.
- Describe to a classmate how you solved the problem.



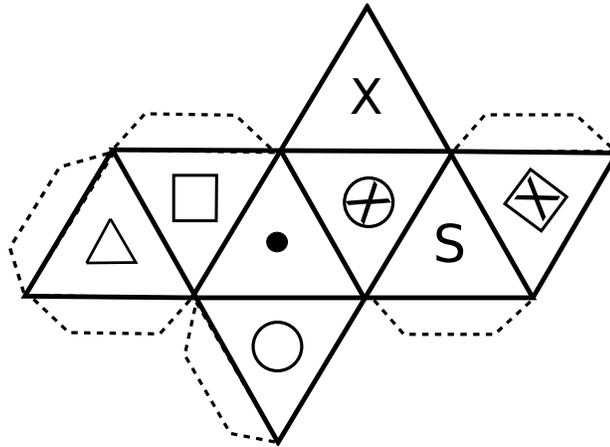
Solution

- a) A circle \bigcirc will appear in the blank space on the fourth die.

Reasoning:

Examining the first two views of the die, we see that the eight possible symbols on the octahedron are $\triangle \bigcirc \bullet \square \times \otimes S \boxtimes$. On the last two views, seven of those eight symbols appear, but on different 'halves' of the die than the first two views. Since the pair $\square \bullet$ is with the pair $\otimes \times$ on the third view, the fourth view must show the pairs $S \boxtimes$ and $\triangle \bigcirc$.

A completed version of the net for the die is shown below.



- b) Explanations will vary. The teacher may wish to choose students with several different versions to present to the class.



Problem of the Week

Problem B

Measure for Measure...

Some body measurements have interesting relationships to other body measurements.

Try the measurements below and then compare your results to those of your classmates.

Do you think their results will be similar?



- Measure your height in cm.
- Convert this measurement of your height into mm and m.
- Do you think your height is greater than, less than, or equal to 3 times the distance from the floor to the top of your kneecap? Measure and find out.
- Do you think your height is greater than, less than, or equal to 8 times the span of your hand? (Stretch your thumb and pinky finger as far as possible and measure between them to find the span of your hand.) Measure and find out.
- Do you think your height is greater than, less than, or equal to 3 times around your head? Measure the distance around and find out.



Problem of the Week

Problem B and Solution

Measure for Measure...

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- d) Do you think your height is greater than, less than, or equal to 8 times the span of your hand? (Stretch your thumb and pinky finger as far as possible and measure between them to find the span of your hand.) Measure and find out.
- e) Do you think your height is greater than, less than, or equal to 3 times around your head? Measure the distance around and find out.



Solution

Here are results for three different Grade 5/6 students for each part of the problem. They illustrate that the calculations in parts c), d) and e) give approximations for an individual's height, but that results will vary.

- a) Height in cm: 148, 146, 154.
- b) Height converted to mm: 1480, 1460, 1540;
height converted to m: 1.48, 1.46, 1.54.
- c) Distance from the floor to the top of kneecap, in cm: 52, 51, 51;
three times this distance compared to height: 156 (greater), 153 (greater), 153 (less).
- d) Span of hand from tip of thumb to tip of pinky finger: 17, 17, 19;
eight times span compared to height: 136 (less), 136 (less), 152 (less).
- e) Circumference of head: 55, 52, 60;
three times circumference compared to height: 165 (greater), 156 (greater), 180 (greater).

EXTENSION: For each of parts c), d), e), sum the differences for the whole class, adding differences for multiples that are greater than height, and subtracting differences for multiples that are less than height. Then divide by the number of students to find the average of these differences over the class, giving a measure of how accurate (or not) each of these approximations are.



Problem of the Week

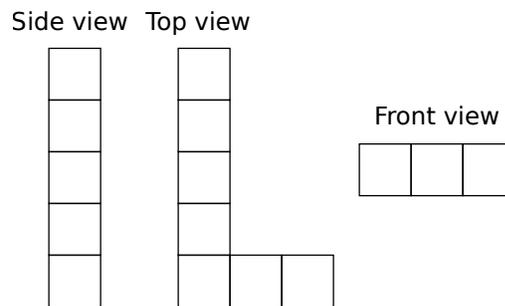
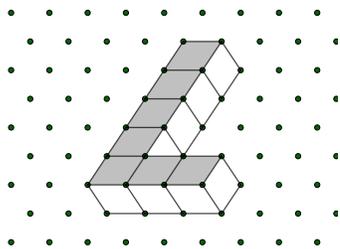
Problem B

You Know My Name

You can create letters of the alphabet that look three-dimensional by using isometric dot paper. From such a 3-dimensional image, you can create a front, side, and top view using centimetre square paper.

On the other hand, one can reverse this process, and use the three views (side, top, and front) to create the corresponding 3-dimensional letter.

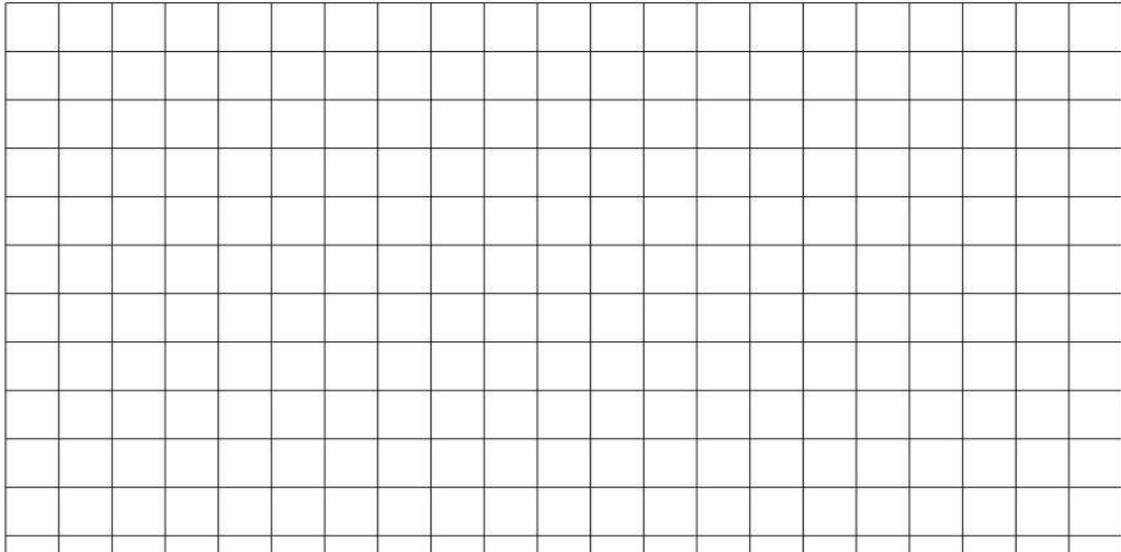
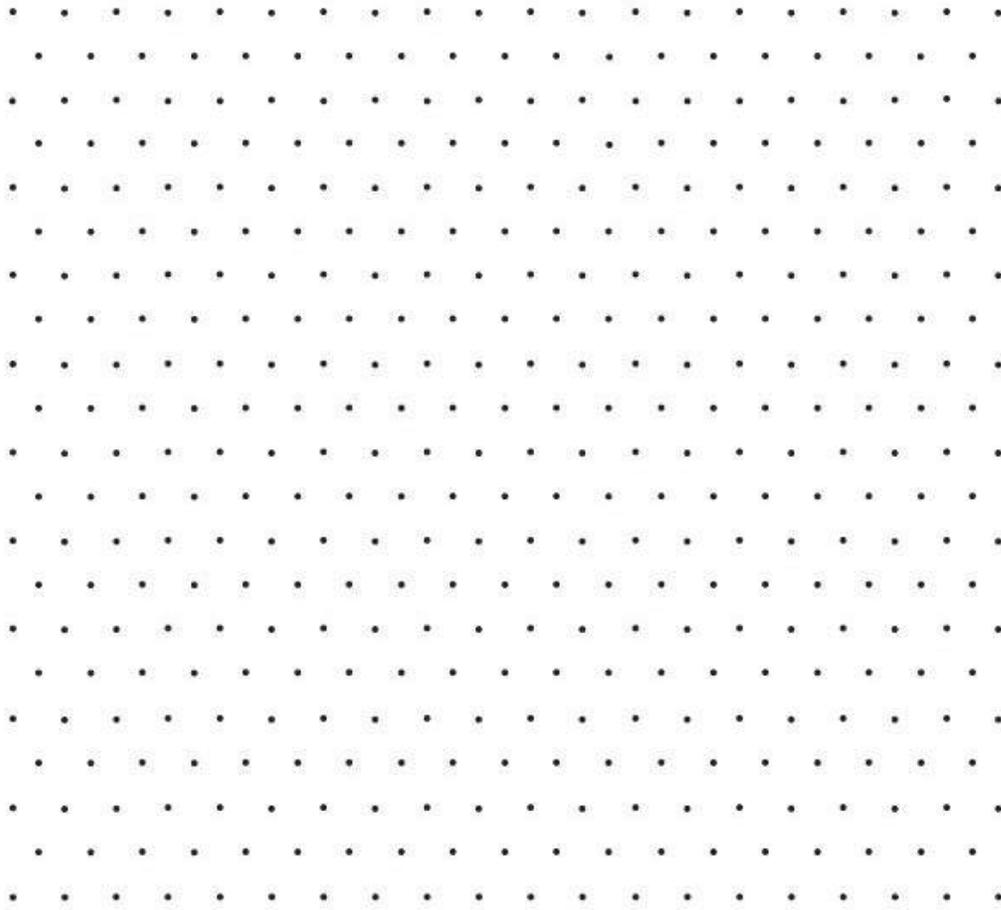
For example, the views shown at the right would lead to a 3-dimensional version of the letter **L**, shown below.



- a) Use connecting cubes to create a 3-dimensional construction of the letter **F**. Then draw the three views (side, front, and top) on grid paper, and the 3-dimensional image on isometric dot paper.
- b) Using isometric dot paper, draw your initials. Create a side, front, and top view of each letter on centimetre grid paper. Exchange your views with a partner and draw each other's 3-dimensional images on isometric dot paper (or create a 3D version using connecting cubes).

A sample of isometric dot paper and centimetre square paper is provided on the next page for your use.







Problem of the Week

Problem B and Solution

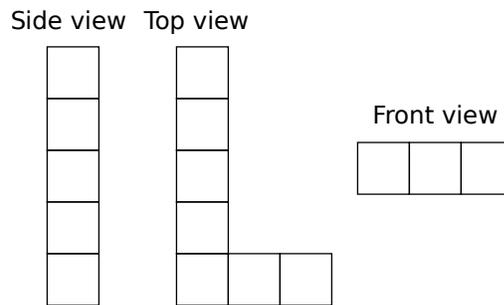
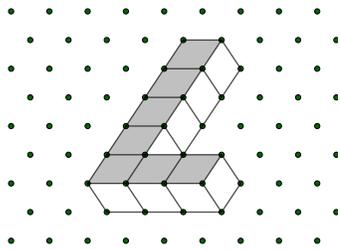
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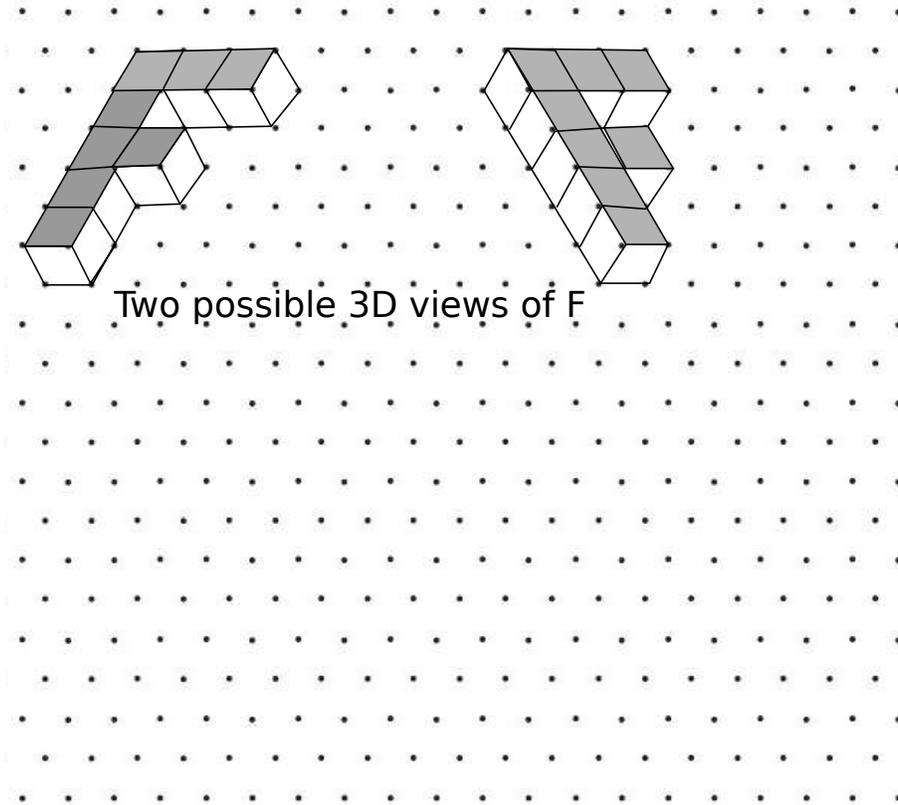


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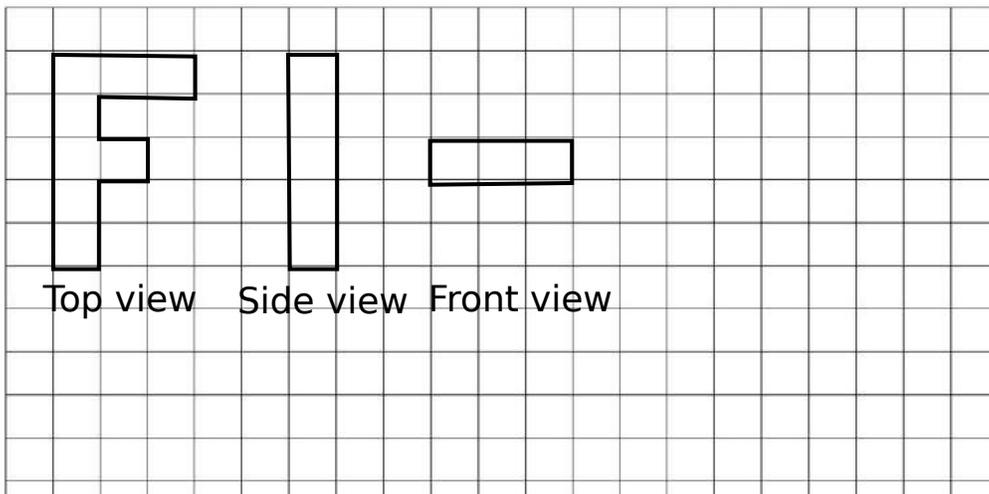


Solution

- a) See the diagrams on the next page for two possible 3-dimensional images on isometric dot paper and the three views of **F** on centimetre grid paper.
- b) Answers will vary. The teacher may wish to display some of the isometric diagrams.



Two possible 3D views of F





Problem of the Week

Problem B

Paving the Way

Munit's driveway is 8 m wide and 16 m long. He wants to cover the entire rectangular area with pavers which are only available in sets of 10 pavers. Each set of 10 pavers covers 3 square metres in total.

- a) If each set of 10 pavers costs \$100, how much will it cost Munit to buy enough sets of pavers for the job? Add 1 extra set of pavers to your purchase to allow for possible breakage.
- b) Each paver in a set is $\frac{1}{2}$ m by $\frac{3}{5}$ m. Make a plan for placing the pavers so that they completely cover the driveway. In your design, how many pavers will need to be cut in order to fit the driveway? How much will be cut from each of these pavers?

You may assume the pavers can be laid either horizontally or vertically.

There are many ways to place the pavers. Compare your design to the design of others in your class.





Problem of the Week

Problem B and Solution

Paving the Way

Problem

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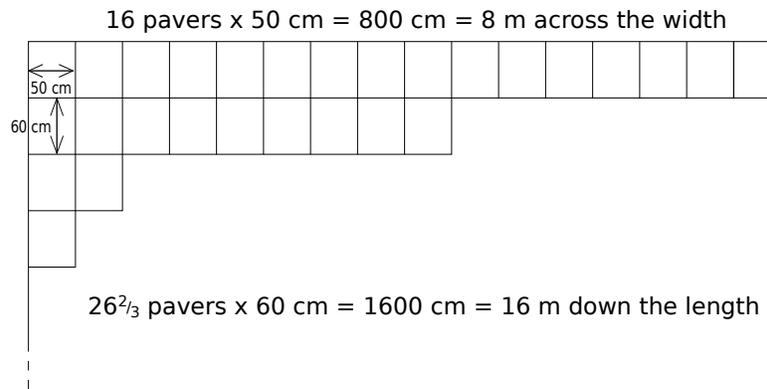
Solution

- a) The total area of pavers required is $8\text{ m} \times 16\text{ m} = 128\text{ m}^2$. Since each set covers 3 m^2 , Munit will need $128 \div 3 = 42\frac{2}{3} \approx 42.7$, so 43 sets of pavers. If an extra set of pavers is purchased, Munit will need 44 sets. At a cost of \$100 per set, his total cost is thus $\$100 \times 44 = \4400 .
- b) Each paver is 50 cm wide by 60 cm long ($\frac{1}{2}\text{ m}$ by $\frac{3}{5}\text{ m}$).

We present three of many possible plans.

PLAN 1:

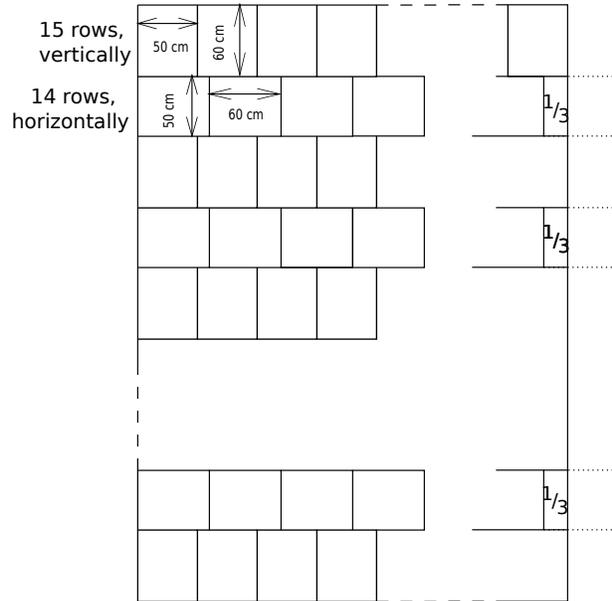
Set the pavers vertically across the $8\text{ m} = 800\text{ cm}$ width of the driveway. This requires $800 \div 50 = 16$ pavers. The $16\text{ m} = 1600\text{ cm}$ length of the driveway would need $1600 \div 60 = 26\frac{2}{3}$ pavers. So the final 27th row would need each of the 16 pavers cut to only $\frac{2}{3} \times 60 = 40\text{ cm}$, i.e., 16 cuts are required. That is, 20 cm is cut off of each of the 16 pavers in the 27th row. This method would use $27 \times 16 = 432$ pavers. Since we purchased an extra set of pavers, we have enough.





PLAN 2:

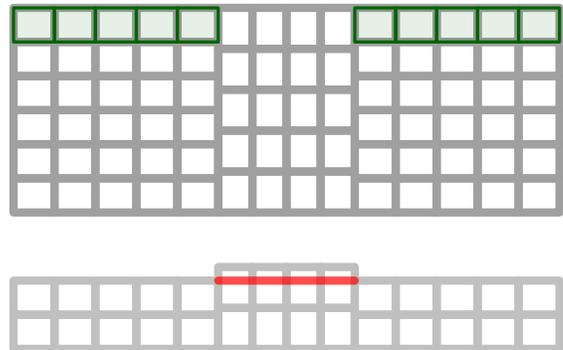
Alternate rows of pavers laid vertically and horizontally, starting with a vertical row. This will require exactly 15 vertical rows and 14 horizontal rows to cover the length of the driveway, since $15 \times 60 + 14 \times 50 = 900 + 700 = 1600$ cm. Further, for the vertical rows, the width will require 16 pavers, since $800 \div 50 = 16$. However, the horizontal rows will require 14 pavers, since $800 \div 60 = 13\frac{1}{3}$. The pavers at the ends of these 14 horizontal rows will need to be cut to $\frac{1}{3}$ of $60 = 20$ cm by 50 cm, requiring 14 cuts. That is, 40 cm is cut off of each of the pavers at the ends of the 14 horizontal rows. Each row with pavers laid vertically has 16 pavers and there are 15 such rows, so there are $15 \times 16 = 240$ pavers laid vertically. Each row with pavers laid horizontally has 14 pavers and there are 14 such rows, so there are $14 \times 14 = 196$ pavers laid horizontally. This plan uses $240 + 196 = 436$ pavers. Since we purchased an extra set of pavers, we have enough to complete the job.



PLAN 3:

Lay 5 pavers horizontally, followed by 4 pavers vertically, and then 5 more pavers horizontally. Do this for a total of 5 rows. This creates the unshaded part in the diagram to the upper right. Now, add 10 more horizontal pavers to complete the rectangle. These last 10 pavers are shaded in the diagram. The large rectangle is 8 m by 3 m. (You can verify this.) Do this process a total of 5 times to cover 8 m by 15 m of the driveway.

Now, a strip 8 m by 1 m is needed to complete the coverage. This small strip is shown in the diagram to the bottom right.



In this small strip, 4 of the vertically placed pavers extend 0.2 m or 20 cm past the end of the driveway. Therefore, 4 cuts of 20 cm each are required. Each of the 8 m by 3 m sections has 80 pavers. There are 5 of these sections, so there are $5 \times 80 = 400$ pavers used. In the final two rows, 28 pavers are used. Therefore, the total number of pavers used is $400 + 28 = 428$. Since we have 440 pavers, we have 12 extra pavers. We could return the extra set to the store and reduce our cost to \$4300.



Problem of the Week

Problem B

A Terror on Ice

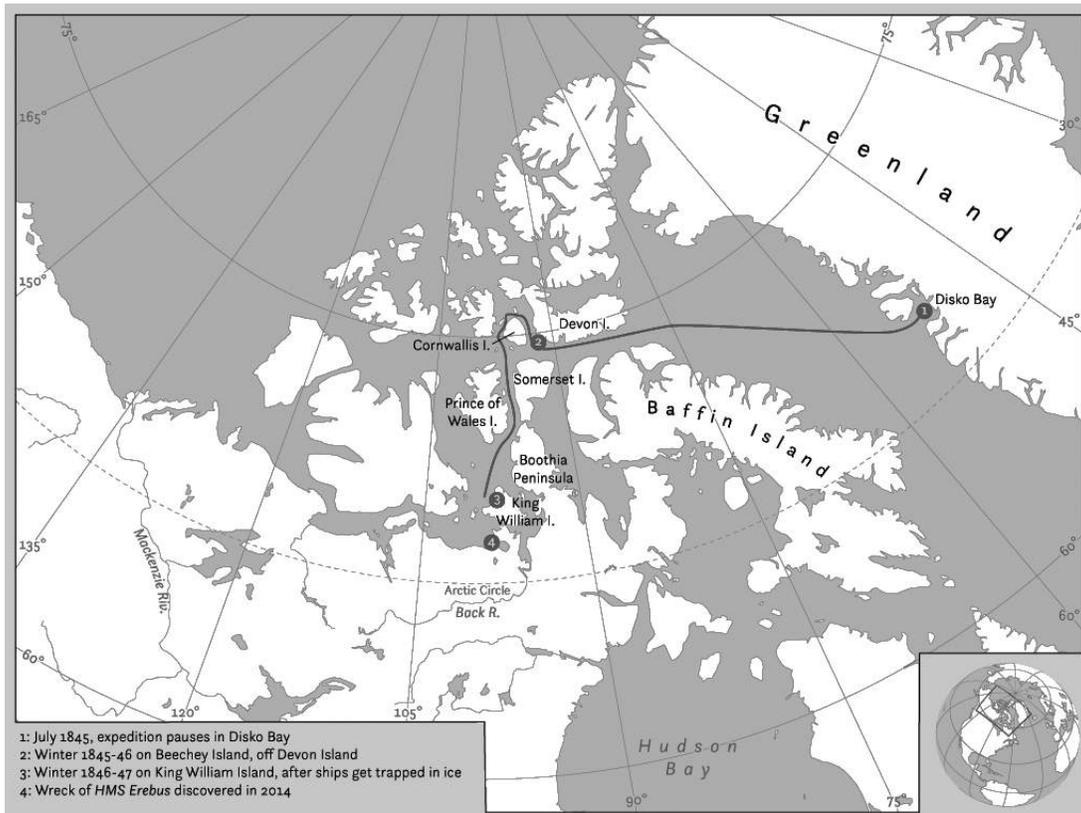
In 1845, Sir John Franklin left England on HMS Erebus, along with HMS Terror, in search of a northwest passage to India. He traveled approximately 12 000 km before getting stuck in the ice near King William Island, coming within about 300 km of Umingmaktok.

- What fraction of the distance he sailed in 1845 was the remaining distance to Umingmaktok (which would have revealed a complete northwest passage)?
- According to a scrawled note, the HMS Erebus was last seen by a European on April 22, 1848. The wreck of the ship was located on September 1, 2014.

How many years, months and days passed between these two sightings?

- The HMS Terror was originally launched on June 19, 1813, and its wreck was discovered in 2016. It participated in the War of 1812, and participated in a further Antarctic Expedition, from 1839 to 1843.

How old, in years, is the HMS Terror now (June 19, 2019)?



(wikimedia.commons)



Problem of the Week

Problem B and Solution

A Terror on Ice

Problem

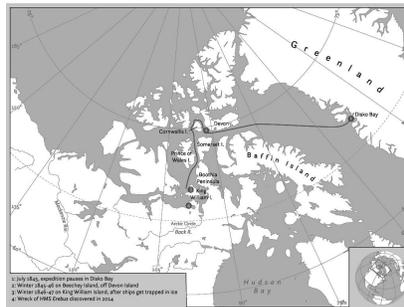
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(wikimedia.commons)

Solution

- Since he sailed 12 000 km in 1845 and still had 300 km to go to Umingmaktok, the remaining distance was $\frac{300}{12000} = \frac{1}{40}$ of his trip.
- The number of years is $2014 - 1848 = 166$ years. The number of months from May to August is 4 months. The number of days is $8 + 1 = 9$ days. Thus a total of 166 years, 4 months, and 9 days passed between the two sightings.
- The age of the terror, in years as of June 19, 2019, is $2019 - 1813 = 206$ years.



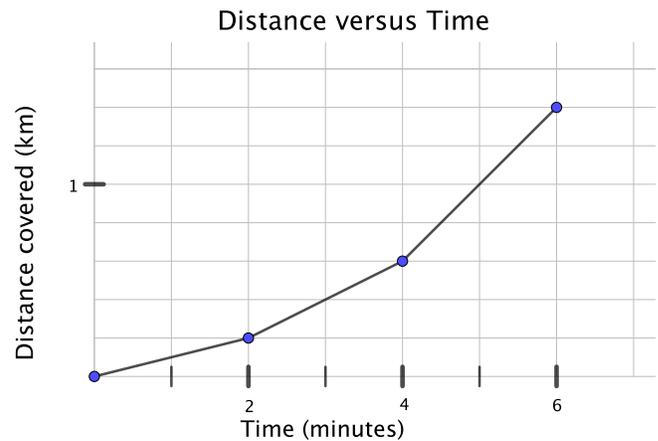
Problem of the Week

Problem B

Ambulate and Calculate

Sierra goes for a quick bit of exercise before breakfast. She walks for 2 minutes, then jogs for 2 minutes, and then runs for 2 minutes. The graph on the right below shows her distance covered, in kilometres, as she exercises for 6 minutes.

- How much distance does Sierra cover in the first 2 minutes?
- What is Sierra's walking speed, in metres per minute?
- What is her jogging speed, in metres per minute?
- What is her running speed, in metres per minute?
- What is the total distance that Sierra covers, in kilometres?
- What is her average speed for the trip, in metres per minute?





Problem of the Week

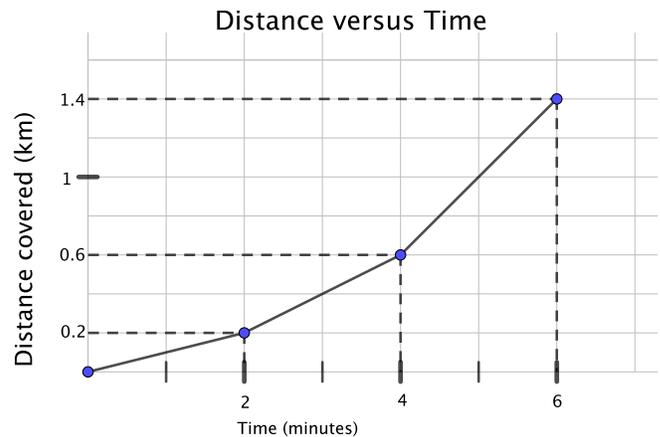
Problem B and Solution

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Solution

- We first note that $1 \div 5 = 0.2$. Therefore, from the graph, after 2 minutes Sierra has covered 0.2 km, or 200 m.
- Sierra's walking speed is 0.2 km per 2 min, or 200 m per 2 min, which is equivalent to 100 m/min.
- Sierra jogs from 2 min to 4 min, where she covers $0.6 - 0.2 = 0.4$ km. Therefore, she covers 0.4 km in 2 min, and her jogging speed is 0.4 km per 2 min, or 400 m per 2 min, which is equivalent to 200 m/min.
- Since she runs from 4 min to 6 min, her running speed is $(1.4 - 0.6) \text{ km} \div 2 \text{ min} = 0.8 \text{ km} \div 2 \text{ min} = 800 \text{ m} \div 2 \text{ min} = 400 \text{ m/min}$.
- Sierra covers a total of 1.4 km.
- Her average speed is $1.4 \text{ km} \div 6 \text{ min} = 1400 \text{ m} \div 6 \text{ min} = 233 \frac{1}{3} \text{ m/min}$.



Problem of the Week

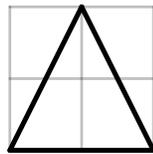
Problem B

Triangular Targets

This is a game for two players. Each player needs a copy of the gameboard (the 8×8 grid on the following page), and a pencil.

Here are the rules of the game, which are similar to BattleshipTM rules.

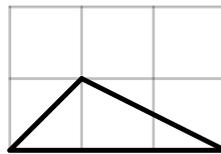
1. Draw one of each of the four types of triangles shown below on your gameboard. The triangles can be rotated (turned) or reflected (flipped) from the given orientations. Each vertex (corner) must be on an intersection of two gridlines. Do not overlap your triangles or let them have any points in common



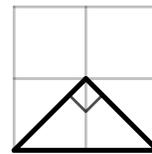
Acute Angle
Isosceles



Right Angle
Scalene



Obtuse Angle
Scalene



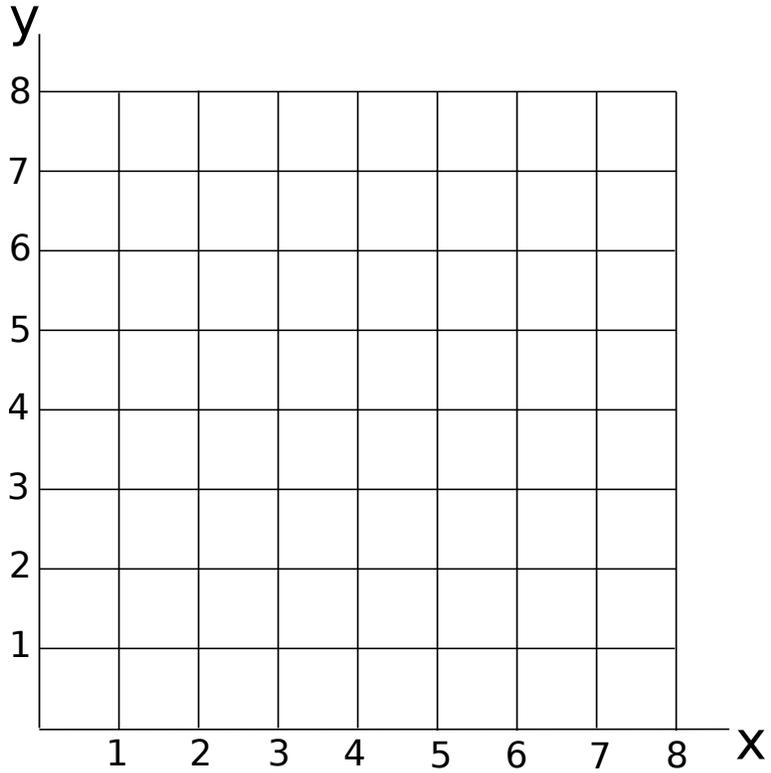
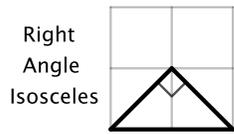
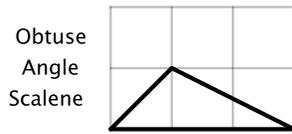
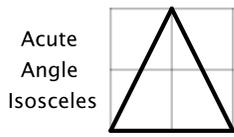
Right Angle
Isosceles

2. Decide who will go first, and then take turns calling out coordinates.
3. If your guess is on a vertex of one of your opponent's triangles, then your opponent must tell you it's a vertex hit, and both of you mark that point with a small x.
4. If your guess is on an edge (side) of one of your opponent's triangles, then your opponent must tell you it was an edge hit, and both of you mark that point with a ✓.
5. A triangle is destroyed when all three vertices have been hit.
6. If a player guesses the grid point inside the Acute Angle Isosceles triangle, then that triangle is destroyed instantly.
7. If your guess is not a vertex, edge, or inside any of your opponent's triangles, then your opponent must tell you it's a miss, and both of you mark that point on the grid with a small o.
8. If you destroy one of your opponent's triangles, then you get another turn.
9. The first player to destroy all of the other player's triangles wins the game.

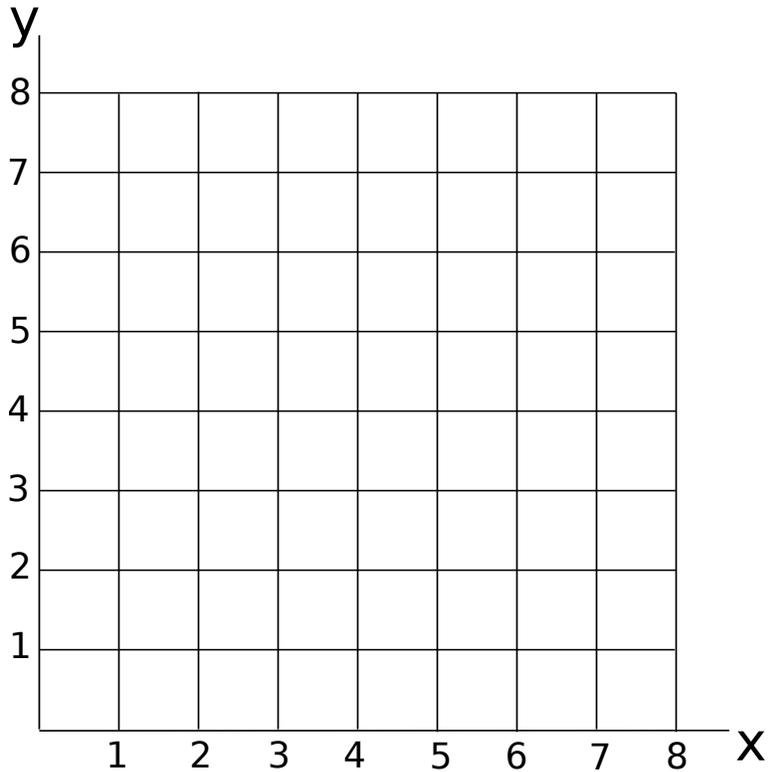
Keep track of both your guesses and your opponent's, using o for misses, x for vertex hits, and ✓ for edge hits.



My Gameboard (Opponent's Guesses)



Opponent's Gameboard (My Guesses)





Problem of the Week

Problem B and Solution

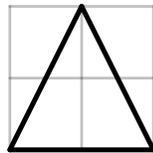
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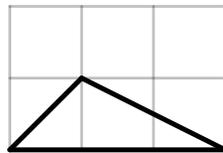
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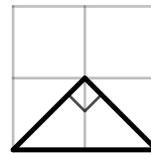
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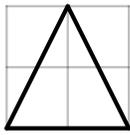
2. Decide who will go first, and then take turns calling out coordinates.
3. If your guess is on a vertex of one of your opponent's triangles, then your opponent must tell you it's a vertex hit, and both of you mark that point with a small x .
4. If your guess is on an edge (side) of one of your opponent's triangles, then your opponent must tell you it was an edge hit, and both of you mark that point with a \checkmark .
5. A triangle is destroyed when all three vertices have been hit.
6. If a player guesses the grid point inside the Acute Angle Isosceles triangle, then that triangle is destroyed instantly.
7. If your guess is not a vertex, edge, or inside any of your opponent's triangles, then your opponent must tell you it's a miss, and both of you mark that point on the grid with a small \circ .
8. If you destroy one of your opponent's triangles, then you get another turn.
9. The first player to destroy all of the other player's triangles wins the game.

Keep track of both your guesses and your opponent's, using \circ for misses, x for vertex hits, and \checkmark for edge hits.



My Gameboard (Opponent's Guesses)

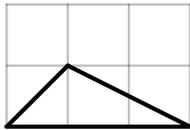
Acute
Angle
Isosceles



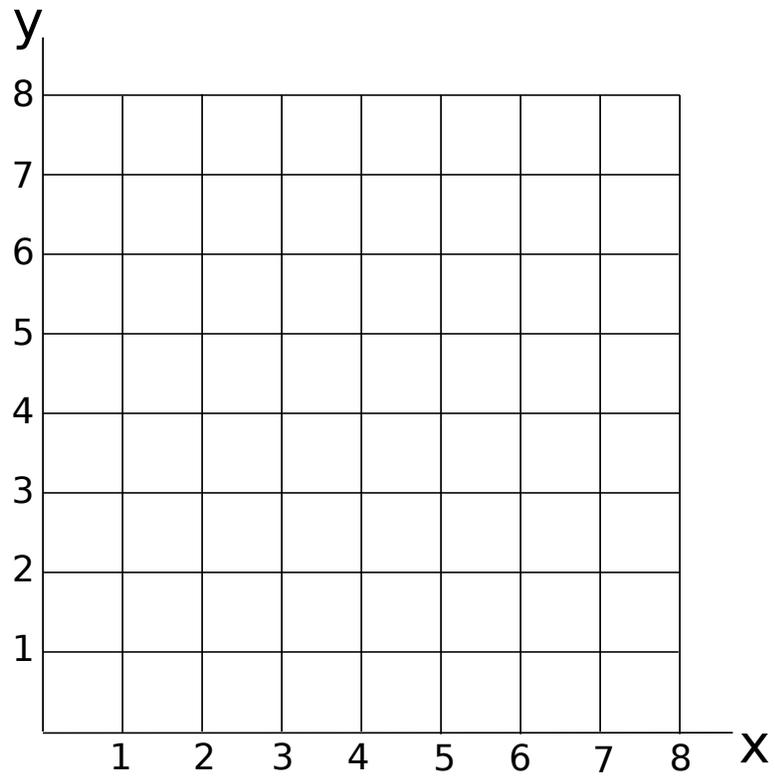
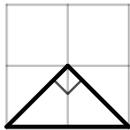
Right
Angle
Scalene



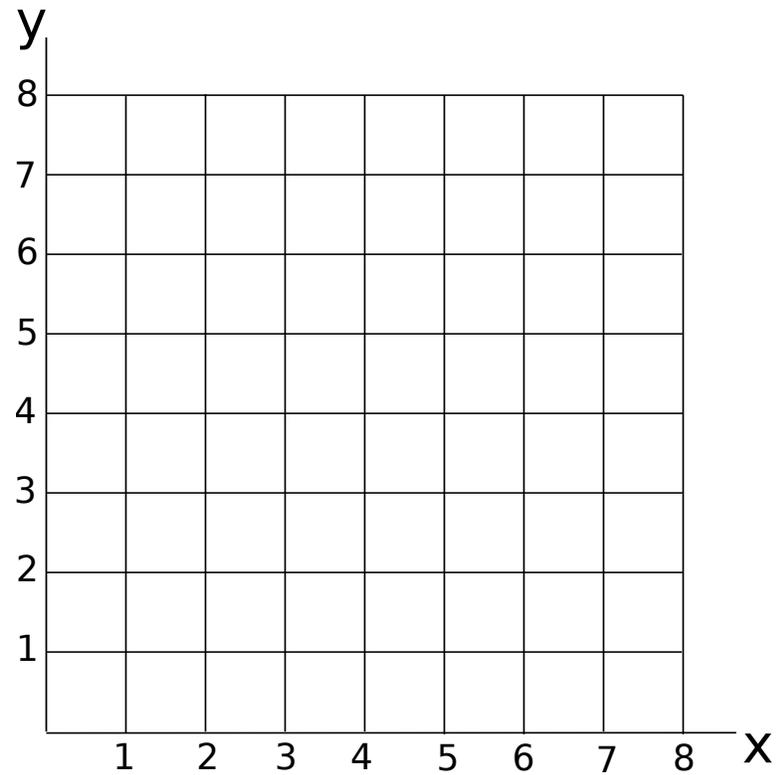
Obtuse
Angle
Scalene



Right
Angle
Isosceles



Opponent's Gameboard (My Guesses)





Solution

The game of Triangular Targets is intended to reinforce important features of Cartesian coordinates and plotting points in the first quadrant:

- for the first coordinate (x -coordinate), move from the origin to the right along the x -axis;
- for the second coordinate (y -coordinate), move up from the position on the x -axis parallel to the y -axis;
- points along the y -axis will have 0 as the x -coordinate;
- points along the x -axis will have 0 as the y -coordinate;
- points on any horizontal side of a triangle will have the same y -coordinate as two of its vertices.
- points on any vertical side of a triangle will have the same x -coordinate as two of its vertices.

In addition, we hope that learning about Cartesian coordinates via Triangular Targets has been fun.



Problem of the Week

Problem B

Round and Round with Robbie

Cool Cole has been building his robot Robbie for the big 4 metre drag race. He has learned that his robot moves 75 mm in a straight line every time its tires do a full 360 degree rotation. He programs Robbie to do 50 tire rotations for the race.

- a) Robbie is 25 cm long. If the race starts with the nose of the robot touching the starting line, will the nose of the robot get to the end line of the drag race after 50 rotations?
- b) How many complete rotations will his robot's tires need to do in order for the entire robot to completely cross the finish line?





Problem of the Week

Problem B and Solution

Round and Round with Robbie

Problem

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- a) Robbie is 25 cm long. If the race starts with the nose of the robot touching the starting line, will the nose of the robot get to the end line of the drag race after 50 rotations?
- b) How many complete rotations will his robot's tires need to do in order for the entire robot to completely cross the finish line?



Solution

- a) In 50 rotations, Robbie will move

$$50 \times 75 = 3750 \text{ mm, or } 3.75 \text{ m.}$$

Since the race is 4 m, this is not enough for Robbie's nose to cross the finish line.

- b) To completely cross the finish line, Robbie must go the 4 m of the race, plus 25 cm, the length of the robot. Thus it must travel 4.25 m, or 4250 mm, which is $4250 \div 75 = 56\frac{2}{3}$ rotations.
So Robbie's back end will cross the finish line during the 57th rotation.



Problem of the Week

Problem B

Cubic Competition

Duha and Mamdouh play a game with different sized cubes. Duha gets a point for each square cm of surface area, and Mamdouh gets a point for each cubic cm of volume.

They start with a $1 \times 1 \times 1$ cm cube for round one. Duha gets 6 points and Mamdouh gets 1 point, so Duha wins this round.

They continue playing the game by increasing the cube dimensions by 1 cm each round.

- a) For which cube will Duha's points for that round be twice Mamdouh's?
- b) For which cube will they have a tie for the number of points for that round?
- c) For which cube will Mamdouh finally win a round?
- d) Explain the reasons for your answers to b) and c) to a classmate.

Cube	Surface Area (Duha)	Volume (Mamdouh)
1. 	6	1
2. 		
3. 		
4.		
5.		
6.		
7.		



Problem of the Week

Problem B and Solution

Cubic Competition

Problem

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They continue playing the game by increasing the cube dimensions by 1 cm each round.

	Cube	Surface Area (Duha)	Volume (Mamdouh)
a) For which cube will Duha's points for that round be twice Mamdouh's?	1. 	6	1
b) For which cube will they have a tie for the number of points for that round?	2. 	24	8
c) For which cube will Mamdouh finally win a round?	3. 	54	27
	4.	96	64
d) Explain the reasons for your answers to b) and c) to a classmate.	5.	150	125
	6.	216	216
	7.	294	343

Solution

The results for each cube have been added to the table until the number of points for Mamdouh exceeds the number of points for Duha for the first time.

- a) For the cube of side length 3, Duha's 54 points is twice Mamdouh's 27 points.
- b) For the cube of side length 6, they have a tie for the number of points at 216.
- c) For the cube of side length 7, Mamdouh finally wins with 343 points to Duha's 294 points.
- d) For b), when the side length of the cube is 6 cm the surface area is $6 \times 36 \text{ cm}^2 = 216 \text{ cm}^2$ and the volume is $6 \times 6 \times 6 = 216 \text{ cm}^3$.
For c), once the side length is greater than the number of sides, the number of cubic centimetres for the volume will be greater than the number of square centimetres for the surface area. This is because the volume = $L \times L \times L$, but surface area = $6 \times L \times L$. Since the side length L is now greater than 6, then the volume is greater than the surface area.



Problem of the Week

Problem B

Tri-Rect-angulation

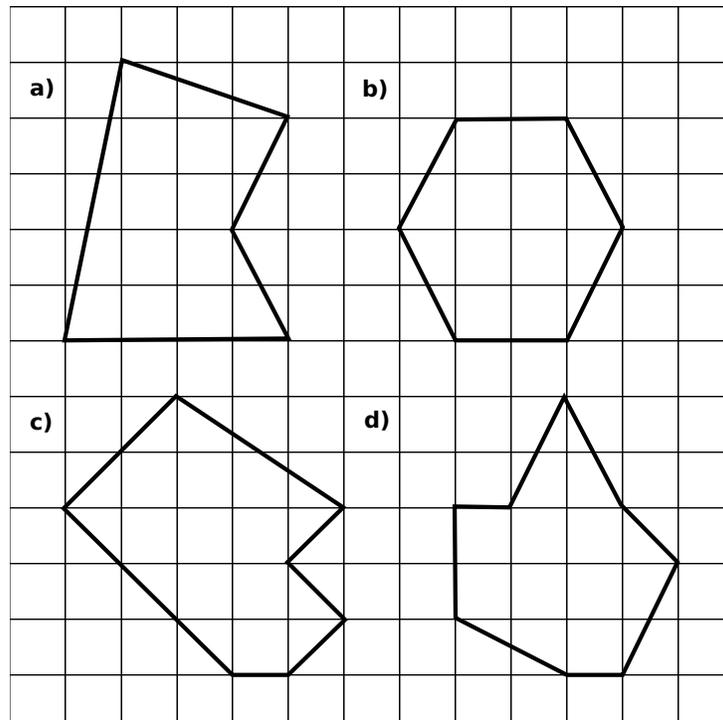
The area of a rectangle can be calculated using the formula

$$area = length \times width$$

The area of a right-angled triangle can be calculated using the formula

$$area = (base \times height) \div 2$$

- Determine a way to divide each polygon below into right-angled triangles and rectangles. (The sides of each triangle and rectangle must lie on grid lines or on a boundary line of the figure.)
- Compare your answers to part (a) with your classmates.
- Find the total area of each of the given polygons using your answers from part (a) and the formulas for the area of a rectangle and for the area of a right-angled triangle.





Problem of the Week

Problem B and Solution

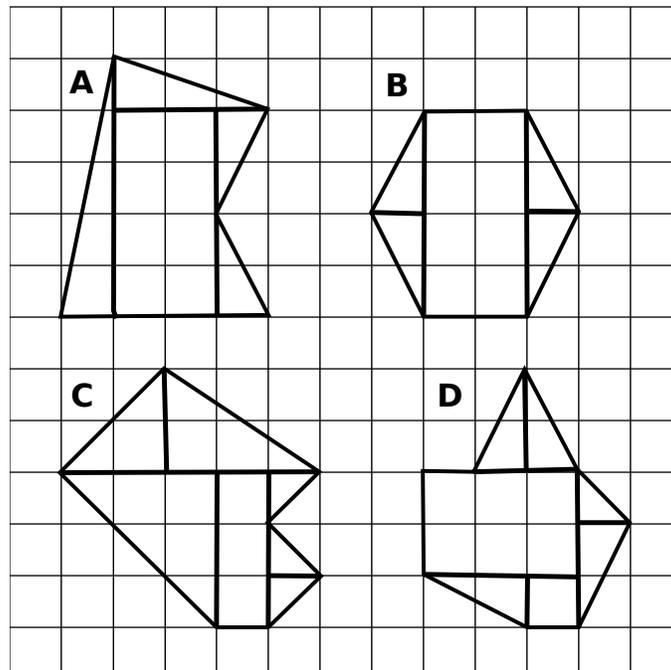
Tri-Rect-angulation

Problem

The area of a rectangle can be calculated using the formula $area = length \times width$.

A right-angled triangle's area can be calculated using the formula $area = (base \times height) \div 2$.

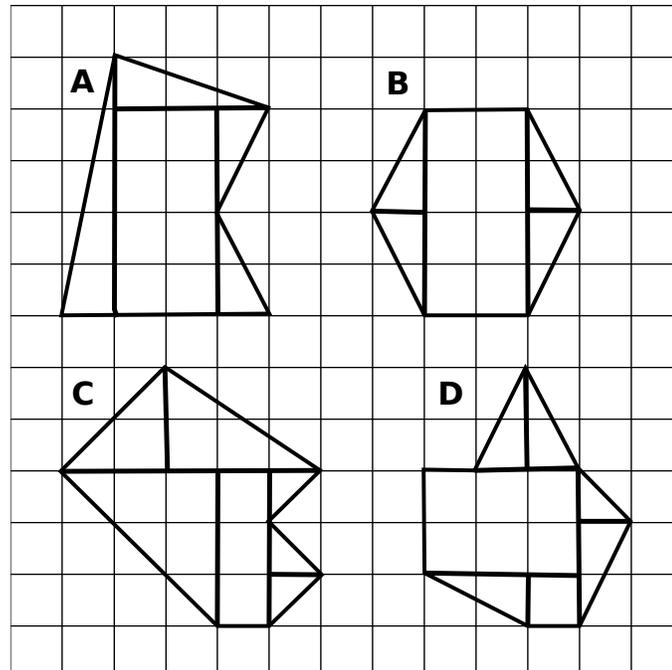
- Determine a way to divide each polygon below into right-angled triangles and rectangles. (The sides of each triangle and rectangle must lie on grid lines or on a boundary line of the figure.)
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- Find the total area of each of the given polygons using your answers from part (a) and the formulas for the area of a rectangle and for the area of a right-angled triangle.





Solution

- a) One possible division of each figure is shown in the diagram above.
Figures A and B: 1 rectangle and 4 triangles;
Figure C: 1 rectangle and 6 triangles;
Figure D: 2 rectangles and 5 triangles.
- b) Answers will vary.



c) The areas of the figures are as follows:

A: The rectangle has length 4 and width 2, for an area of $2 \times 4 = 8$.

The triangle with base 1 and height 5 has area $1 \times 5 \div 2 = 2.5$. The triangle with base 3 and height 1 has area $3 \times 1 \div 2 = 1.5$. The two triangles with base 1 and height 2 each have area $1 \times 2 \div 2 = 1$.

The total area of figure A is $8 + 2.5 + 1.5 + 1 + 1 = 14$ square units.

B: The rectangle has length 4 and width 2, for an area of $2 \times 4 = 8$. The four triangles with base 1 and height 2 each have area $1 \times 2 \div 2 = 1$.

The total area of figure B is $8 + 1 + 1 + 1 + 1 = 12$ square units.

C: The rectangle has length 3 and width 1, for an area of $1 \times 3 = 3$.

The triangle with base 2 and height 2 has area $2 \times 2 \div 2 = 2$. One triangle with base 3 has a height of 2 and an area of $3 \times 2 \div 2 = 3$. The other triangle with base 3 has a height of 3 has an area of $3 \times 3 \div 2 = 4.5$. The three triangles with base 1 and height 1 each have area $1 \times 1 \div 2 = 0.5$.

The total area of figure C is $3 + 2 + 3 + 4.5 + 0.5 + 0.5 + 0.5 = 14$ square units.

D: The rectangle with length 3 and width 2 has area $3 \times 2 = 6$. The rectangle with length 1 and width 1 has area $1 \times 1 = 1$.

The four triangles with base 1 and height 2 each have area $1 \times 2 \div 2 = 1$. The triangle with base 1 and height 1 has area $1 \times 1 \div 2 = 0.5$.

The total area of figure D is $6 + 1 + 1 + 1 + 1 + 1 + 1 + 0.5 = 11.5$ square units.



Problem of the Week

Problem B

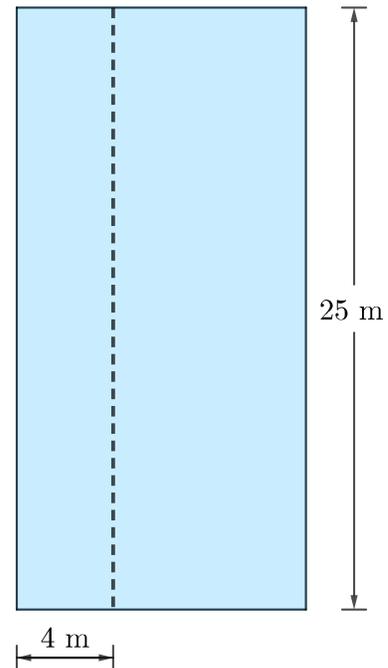
Laps and Lengths

A swimming pool is 25 m long and 12 m wide. A single 4 m wide lane is roped off.

Ismaeel swims in the 4 m wide lane. Swimming two lengths of the pool counts as one lap for Ismaeel.

Kosha swims on the other side of the roped off lane, in the larger rectangular portion of the pool. A lap for Kosha consists of swimming once around a circuit that is $\frac{1}{2}$ m away from each edge and $\frac{1}{2}$ m away from the rope divider.

- How much longer is Kosha's lap than Ismaeel's lap?
- Ismaeel and Kosha each complete ten laps. Determine how much farther than Ismaeel that Kosha swam.





Problem of the Week

Problem B and Solution

Laps and Lengths

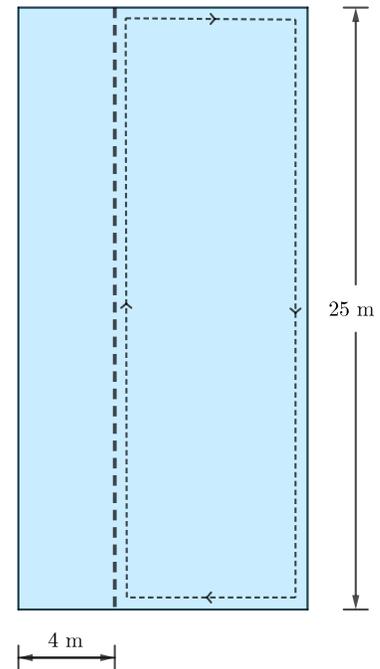
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- How much longer is Kosha's lap than Ismaeel's lap?
- Ismaeel and Kosha each complete ten laps. Determine how much farther than Ismaeel that Kosha swam.



Solution

- The large rectangular part of the pool is $12 - 4 = 8$ m wide. Since Kosha swims $\frac{1}{2}$ m from the 'edges', Kosha swims around a rectangle that is $8 - \frac{1}{2} - \frac{1}{2} = 7$ m wide and $25 - \frac{1}{2} - \frac{1}{2} = 24$ m long. The total distance Kosha swims in one lap is $24 + 7 + 24 + 7 = 62$ m. Ismaeel swims $25 + 25 = 50$ m in two lengths, or one lap. Thus Kosha's lap is $62 - 50 = 12$ m longer than Ismaeel's lap.
- In ten laps, Kosha swims $10 \times 62 = 620$ m, while Ismaeel swims $10 \times 50 = 500$ m. So Kosha swims $620 - 500 = 120$ m further.
ALTERNATIVELY: Since in each lap Kosha swims 12 m further, in 10 laps she swims $10 \times 12 = 120$ m further.

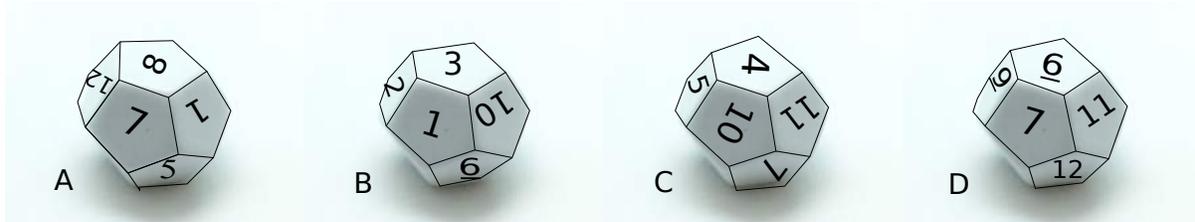


Problem of the Week

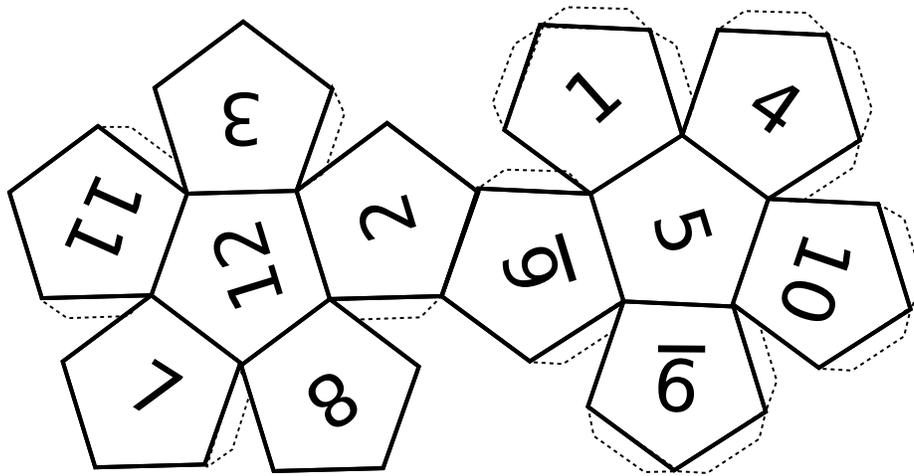
Problem B

Deese Dice are D12s

Dice A, B, C, and D shown below are also known as D12s. A D12 is a 12-sided die with a distinct number from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ on each side.



One of the four dice above can be made from the net shown below. Which of A, B, C, or D corresponds to the given net?





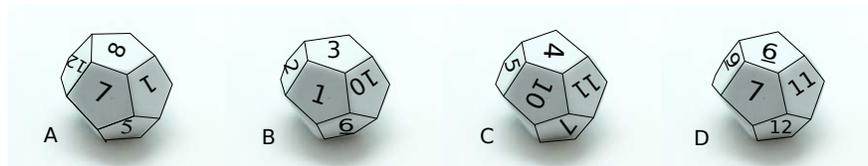
Problem of the Week

Problem B and Solution

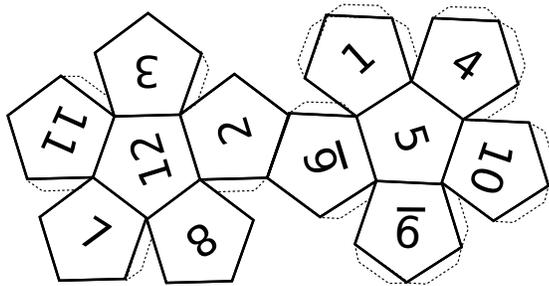
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One of the four dice above can be made from the net shown below. Which of A, B, C, or D corresponds to the given net?



Solution

One way to solve the problem would be to use the net to construct the 12-sided die. A construction of the 12-sided die from the given net would reveal that it corresponds to die C. A photo of the die somewhat roughly constructed from the original net is shown above.

Alternatively, we can reject each of the other three die as follows:

- The 1 is adjacent to 3, 2, 6, 5, and 4 in the net. The 8 is adjacent to the 1 on the first die. So we can rule out die A.
- The 1 is adjacent to 3, 2, 6, 5, and 4 in the net. The 10 is adjacent to the 1 on the second die. So we can rule out die B.
- The 6 is adjacent to 1, 2, 8, 9, and 5 in the net. The 11 is adjacent to the 6 on the fourth die. So we can rule out die D.

Since one die corresponds to the net and we have ruled out the first, second and fourth die, we can conclude that the third die, die C, can be made from the given net.

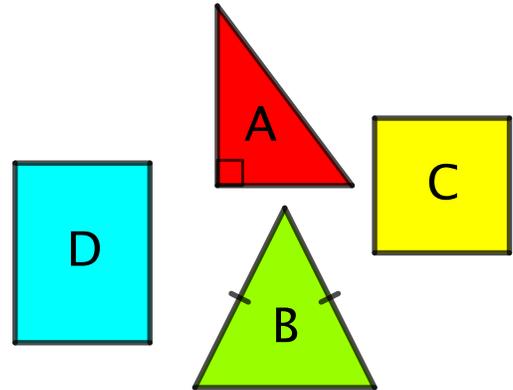


Problem of the Week

Problem B

Alternate Dimensions

The four shapes to the right are each drawn with a horizontal base and a vertical height. Figure A is a right-angled triangle, Figure B is an isosceles triangle, Figure C is a square, and Figure D is a rectangle. The figures are not drawn to scale.



Using the following clues, determine the measure of the (horizontal) base and the measure of the (vertical) height of each figure.

1. The measure of the base of Figure A is the same as the measure of the base of Figure D.
2. The measure of the base of Figure A is one unit less than the measure of the base of Figure B.
3. The side length of Figure C is the same as the measure of the base of Figure A.
4. The measure of the height of Figure B is the same as the measure of the height of Figure A and also the same as the measure of the base of Figure B.
5. The area of Figure C is 9 square units.
6. The total area of all four figures is 38 square units.



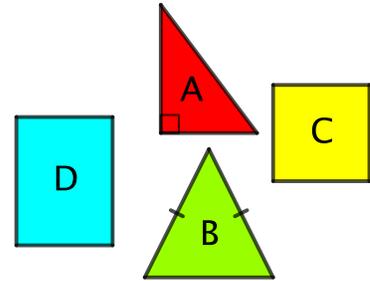
Problem of the Week

Problem B and Solution

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3. The side length of Figure C is the same as the measure of the base of Figure A.
4. The measure of the height of Figure B is the same as the measure of the height of Figure A and also the same as the measure of the base of Figure B.
5. The area of Figure C is 9 square units.
6. The total area of all four figures is 38 square units.

Solution

From 5., since C is a square with an area of 9 square units, its side length is 3 units.

From 3., A must have a base that is 3 units long.

From 1., D has a base that is 3 units long as well.

From 2., the measure of the base of B is 1 unit greater than that of A, so B has a base that is 4 units long.

From 4., the heights of A and B thus both have a measure of 4 units.

We can now calculate that the area of triangle A is $\frac{1}{2} \times 3 \times 4 = 6$ square units and the area of triangle B is $\frac{1}{2} \times 4 \times 4 = 8$ square units. We also know that the area of square C is 9 square units. Summing, the total area of figures A, B, and C is $6 + 8 + 9 = 23$ square units.

From 6., the total area of all figures is 38 square units, so the area of D is $38 - 23 = 15$ square units. Since D is a rectangle with a base of measure 3, it thus has a height of measure $15 \div 3 = 5$ units.

Therefore,

Figure A has a base of measure 3 units and a height of measure 4 units;

Figure B has a base of measure 4 units and a height of measure 4 units;

Figure C is 3 units by 3 units; and

Figure D has a base of measure 3 units and a height of measure 5 units.

Algebra (A)





Problem of the Week

Problem B

‘Wrecked-Angles’

In this problem, you will be carefully counting rectangles. Enter your answers in the given table, where **NSR** is the **N**umber of **S**mall **R**ectangles, and **TNR** is the **T**otal **N**umber of **R**ectangles.

- a) In the first diagram (**Dgm**) below, there is one rectangle. In the second diagram below, there are three rectangles, two smaller ones within one larger one.



How many rectangles are there in the third diagram?

- b) How many rectangles are there in the fourth diagram?



Dgm	NSR	TNR
1	1	1
2	2	3
3		
4		
5		
6		
7		
8		
9		
10		

- c) If the pattern continues, how many rectangles will there be in the fifth diagram?
- d) Describe how the total number of rectangles can be predicted. In particular, how would you determine the number of rectangles in a diagram which had 10 small rectangles in a row?



Problem of the Week

Problem B and Solution

‘Wrecked-Angles’

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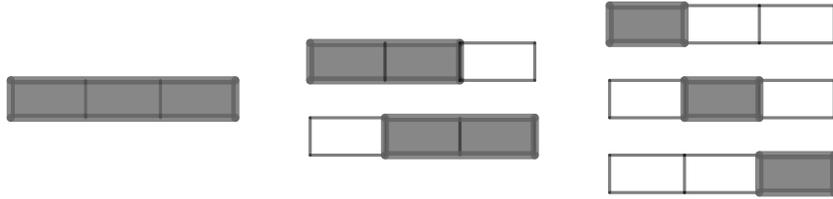
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- d) Describe how the total number of rectangles can be predicted. In particular, how would you determine the number of rectangles in a diagram which had 10 small rectangles in a row?



Solution

- a) In the third diagram, there are 6 rectangles: 1 with all three together, 2 with two smaller rectangles, and 3 individual small rectangles. These are illustrated below.





- b) In a similar manner to part a), we see that there are 10 rectangles in the fourth diagram: 1 outer rectangle, 2 containing three small rectangles, 3 containing two small rectangles, and 4 individual small rectangles.
- c) The pattern appears to be that the number of small rectangles is added to the previous total to get the new total. Next, there are $10 + 5 = 15$ total rectangles in the fifth diagram.

- d) Assuming that the pattern continues, the total will be the sum of the integers up to and including the number of small rectangles. So for 10, it will be (as shown in the table)

$$1 + 2 + 3 + 4 + \cdots + 9 + 10 = 55$$

For your information, there is a general formula for this sum. If n is the number of small rectangles, then the total number of rectangles is

$$S_n = \frac{1}{2} \times n \times (n + 1)$$

For example, $n = 10$ gives $n + 1 = 11$ and

$$S_{10} = \frac{1}{2} \times 10 \times 11 = 5 \times 11 = 55, \text{ as above.}$$

Dgm	NSR	TNR
1	1	1
2	2	3
3	3	6
4	4	10
5	5	15
6	6	21
7	7	28
8	8	36
9	9	45
10	10	55

If we had 1000 small rectangles then $n = 1000$ and $n + 1 = 1001$, and there would be

$$S_{1000} = \frac{1}{2} \times 1000 \times 1001 = 500 \times 1001 = 500\,500 \text{ rectangles.}$$



Problem of the Week

Problem B

Years and Years of ‘Threepcats’

Find all years between 1000 BCE and 2020 CE, inclusive, which have exactly three digits the same. For example, the years 1011 and 1222 each have exactly three digits the same, but 1111 and 1123 do not.





Problem of the Week

Problem B and Solution

Years and Years of ‘Threeppeats’

Problem

Find all years between 1000 BCE and 2020 CE, inclusive, which have exactly three digits the same. For example, the years 1011 and 1222 each have exactly three digits the same, but 1111 and 1123 do not.



Solution

Between 1000 BCE and 2020, there are several different types of year labels which have exactly 3 digits the same. The table below enumerates and totals them.

Years	Years with triple digits	Total
1000 BCE to 1 BCE	1000, 999, 888, 777, 666, 555, 444, 333, 222, 111	10
1 CE to 1000 CE	111, 222, 333, 444, 555, 666, 777, 888, 999, 1000 (reverse order of 1000 BCE to 1 BCE)	10
1001 to 2020 CE	1011, 1211, 1311, 1411, 1511, 1611, 1711, 1811, 1911,	9
	1101, 1121, 1131, 1141, 1151, 1161, 1171, 1181, 1191,	9
	1110, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119,	9
	1222, 1333, 1444, 1555, 1666, 1777, 1888, 1999, 2000	9
Total		56

Data Management (D)





Problem of the Week

Problem B

Hoops Data

At the time of writing, Kyle Lowry plays basketball with the NBA Champions, the Toronto Raptors. At one point he had played 805 career games. Here are some questions related to his performance in those games.

- If he started in 569 games, in what fraction of the games did he start?
- In a typical 48 minute game, he averaged playing 31.1 minutes. What fraction of each game did he play?
- He scored an average of 14.5 points per game. How many points, on average, has he scored per minute he played in a game? Express your answer as a decimal to the nearest hundredth (i.e., two decimal places).
- Over his career, he made, on average, 43 of 100 shots he took. Based on his past performance, if he took 11 shots in a particular game, how many of these 11 shots would you expect him to make?

For Further Thought:

Try comparing his performance to that of another NBA player by researching their data at the website <https://www.basketball-reference.com/>.





Problem of the Week

Problem B and Solution

Hoops Data

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Solution

- He started in 569 of the 805 games, that is $\frac{569}{805}$ of the games. Expressed as a decimal to two decimal places, $\frac{569}{805} \approx 0.71$, so this represents 71% of the games he played.
- He played 31.1 minutes out of 48 minutes each game, that is $\frac{31.1}{48} = \frac{311}{480}$ of each game. Expressed as a decimal to two decimal places, $\frac{311}{480} \approx 0.65$, so this represents 65% of each game.
- He scored an average of 14.5 points per game played and played an average of 31.1 minutes in a game. Therefore, he scored $14.5 \div 31.1 \approx 0.47$ points per minute played.
- He made, on average, 43 of the 100 shots he took. If he took 11 shots, we would expect him to make $\frac{43}{100} \times 11 = 4.73$ shots (that is, we would expect him to make 4 or 5 shots).

For Further Thought: Answers will vary.



Problem of the Week

Problem B

Go Outside and Play!

The Sony PlayStation2™ is the best selling video game console of all time.

Annual sales data (in millions of units) are displayed in the second column of the table below for the years 2000 to 2011.

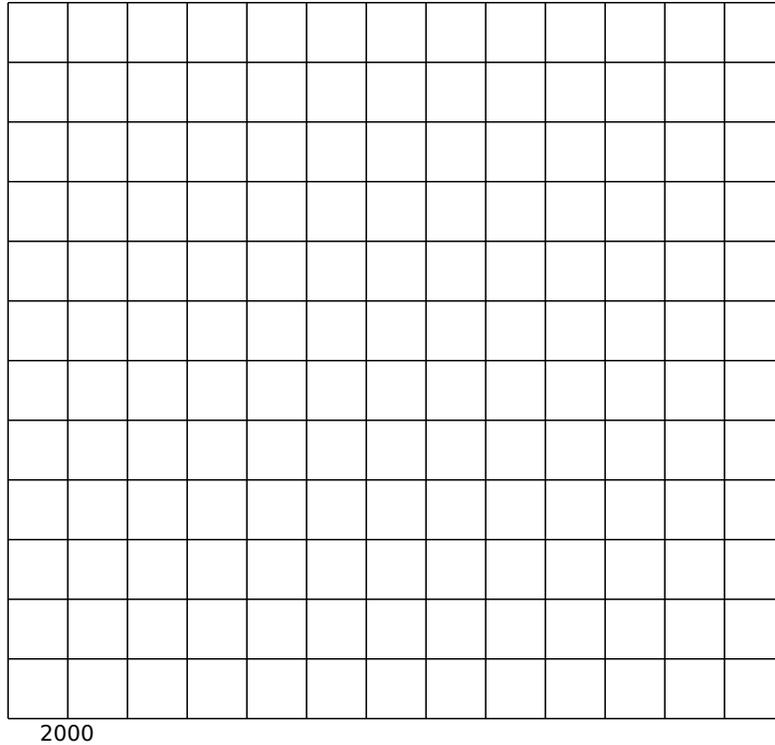


- a) Round each year's sales to the nearest million, and enter your answers in the table.
- b) Using the rounded numbers, approximately how many PS2s were sold from 2000 to 2011?
- c) Plot the rounded sales on a broken line graph. Include labels and units for each axis, and a title for your graph. A grid is provided for your use on the next page.
- d) Why is a broken line graph a good choice to display this data?
- e) Make up some questions for a friend to answer based on your graph.

Date of fiscal year end	Annual sales	Annual sales (to nearest million)
March 31, 2000	1.41	
March 31, 2001	9.20	
March 31, 2002	18.07	
March 31, 2003	22.52	
March 31, 2004	20.10	
March 31, 2005	16.17	
March 31, 2006	16.22	
March 31, 2007	14.20	
March 31, 2008	13.7	
March 31, 2009	7.90	
March 31, 2010	7.30	
March 31, 2011	6.4	

vgsales.fandom.com/wiki/PlayStation_2





- Remember that your graph needs a label for each axis, and units if necessary.
- Your graph also needs a title. A good title will contain something about each axis.
- Remember that the numbers on the vertical axis need to go up by the same amount, and they should be on the lines, not the spaces.



Problem of the Week

Problem B and Solution

Go Outside and Play!

Problem

The Sony PlayStation2™ is the best selling video game console of all time.

Annual sales data (in millions of units) are displayed in the second column of the table below for the years 2000 to 2011.

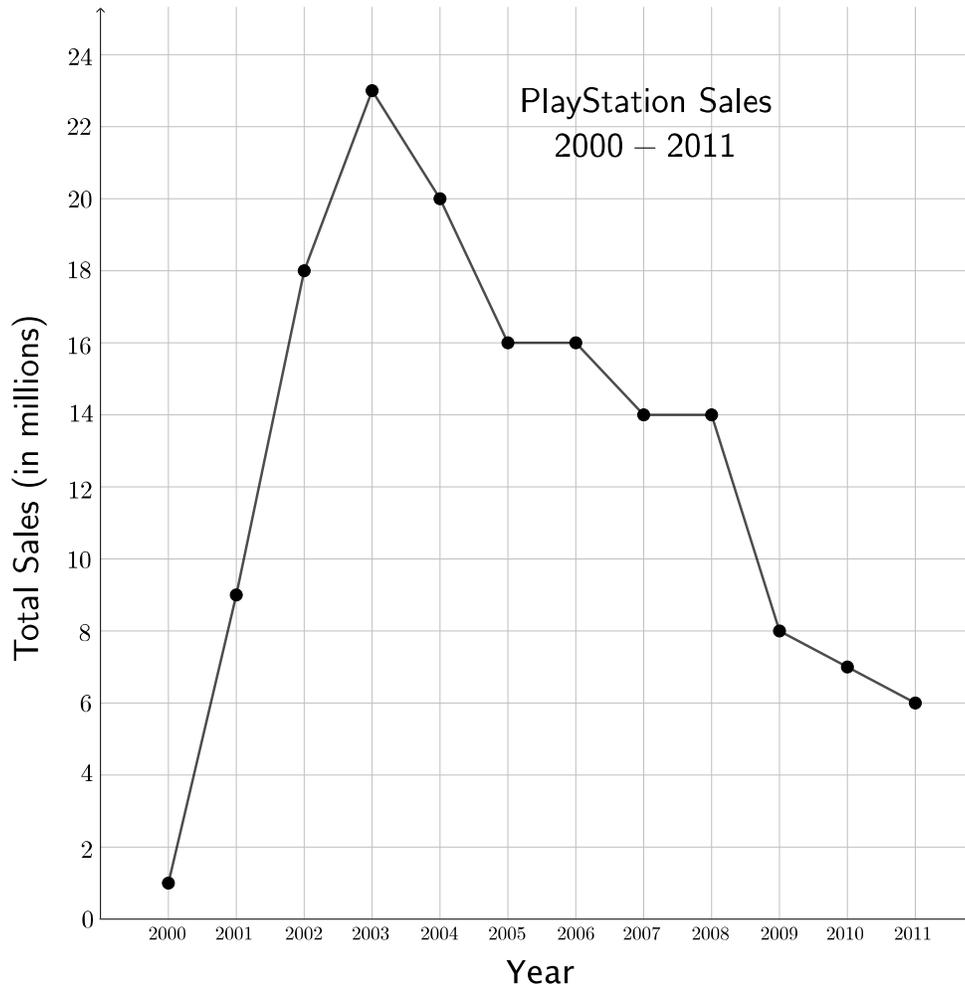


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- d) Why is a broken line graph a good choice to display this data?
- e) Make up some questions for a friend to answer based on your graph.

Date of fiscal year end	Annual sales	Annual sales (to nearest million)
March 31, 2000	1.41	1
March 31, 2001	9.20	9
March 31, 2002	18.07	18
March 31, 2003	22.52	23
March 31, 2004	20.10	20
March 31, 2005	16.17	16
March 31, 2006	16.22	16
March 31, 2007	14.20	14
March 31, 2008	13.7	14
March 31, 2009	7.90	8
March 31, 2010	7.30	7
March 31, 2011	6.4	6

vgsales.fandom.com/wiki/PlayStation_2

- Remember that your graph needs a label for each axis, and units if necessary.
- Your graph also needs a title. A good title will contain something about each axis.
- Remember that the numbers on the vertical axis need to go up by the same amount, and they should be on the lines, not the spaces.



Solution

- a) The sales rounded to the nearest million are shown in the revised table within the question.
- b) The sum of the rounded sales from 2000 to 2011 is 152 million.
- c) See the graph above.
- d) A broken line graph is a good choice because it shows the general trend of the data over time.
- e) Answers will vary.



Problem of the Week

Problem B

Raise a Buck at Starstruck's

Last year, the ten employees at Starstruck's had an average annual income of \$30 000 each. This year, three of them received equal raises in pay, raising the total annual income for all ten employees to \$303 000.

- How much was the raise received by each of the three employees?
- What is the new average annual income of each of the ten employees this year?
- If the word "equal" is deleted from the given information, could you find the answer to part a)? Explain your reasoning.





Problem of the Week

Problem B and Solution

Raise a Buck at Starstruck's

Problem

Last year, the ten employees at Starstruck's had an average annual income of \$30 000 each. This year, three of them received equal raises in pay, raising the total annual income for all ten employees to \$303 000.

- How much was the raise received by each of the three employees?
- What is the new average annual income of each of the ten employees this year?
- If the word "equal" is deleted from the given information, could you find the answer to part a)? Explain your reasoning.



Solution

- Last year, the total income for all 10 employees was $\$30\,000 \times 10 = \$300\,000$. Thus the raises totaled $\$303\,000 - \$300\,000 = \$3\,000$. So each of the three employees received a raise of $\$3\,000 \div 3 = \$1\,000$.
- After the new raises, the new average annual income was $\$303\,000 \div 10 = \$30\,300$.
- If the word 'equal' is deleted from part a), all we would know is the total of the raises, \$3 000. So we could not find individual raises, as any three amounts that sum to \$3 000 would be possible.



Problem of the Week

Problem B

Luck of the Draw

A deck of cards without jokers has 52 cards, with 13 cards in each of the four suits: spades, hearts, diamonds, and clubs. The 13 cards are 2, 3, . . . , 9, 10, Jack, Queen, King, and Ace.

- a) What is the theoretical probability, P , of drawing an Ace (of any suit) from a full deck?
- b) After drawing the first Ace, if that card is not replaced in the deck, what is the probability, R , of drawing a second Ace from the remaining cards?
- c) How would your answer to part b) change if the first card had been replaced in the deck before the second card was drawn?
- d) The probability of two desired cards being drawn one after the other is the product of the two individual probabilities. For example, the probability of drawing two Aces when the first card is replaced in the deck is $P \times P$, with P as in part a). The probability of drawing two Aces when the first card is not replaced in the deck is $P \times R$, with P as in part a) and R as in part b). What is the probability of drawing the Ace of hearts, and then a 6 of any suit, if the first card is NOT replaced in the deck before the second card is drawn?

			
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
J	J	J	J
Q	Q	Q	Q
K	K	K	K
A	A	A	A



Problem of the Week

Problem B and Solution

Luck of the Draw

Problem

A deck of cards without jokers has 52 cards, with 13 cards in each of the four suits: spades, hearts, diamonds, and clubs. The 13 cards are 2, 3, ..., 9, 10, Jack, Queen, King, and Ace.

- a) What is the theoretical probability, P , of drawing an Ace (of any suit) from a full deck?
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2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10
J	J	J	J
Q	Q	Q	Q
K	K	K	K
A	A	A	A

What is the probability of drawing the Ace of hearts, and then a 6 of any suit, if the first card is NOT replaced in the deck before the second card is drawn?

Solution

- a) Since there are 4 Aces among the 52 cards, the probability of drawing an Ace is $P = 4 \div 52 = \frac{1}{13}$.
- b) After one Ace is removed, there remain 51 cards with 3 Aces among them. Thus the probability of drawing a second Ace is $R = 3 \div 51 = \frac{1}{17}$.
- c) If the first card had been replaced, there would once again have been 52 cards, 4 of which are Aces. So, the probability of drawing the second Ace would again be $P = \frac{1}{13}$.
- d) Since there is one Ace of hearts, the probability of that Ace first being drawn is $\frac{1}{52}$. Since there are four 6's in the remaining 51 cards, the probability of then drawing any 6 is $\frac{4}{51}$. Thus the probability of both draws being successful is $\frac{1}{52} \times \frac{4}{51} = \frac{1}{663}$.



Problem of the Week

Problem B

Fishing for Thermoclines

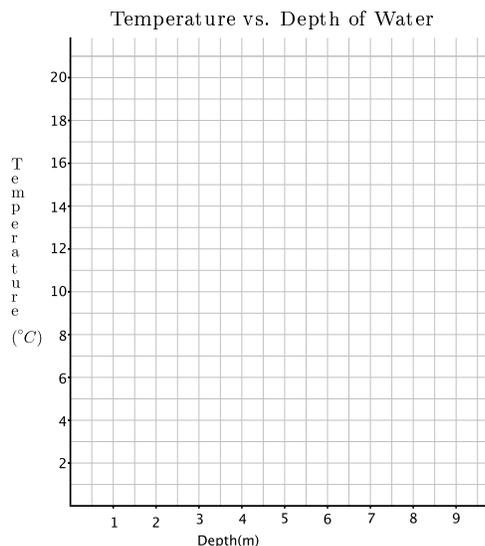
Paige is headed to Lac Nilgault to do some fishing for lake trout and pike. She knows that small pike generally like 18°C water, large pike like 12°C water, and lake trout like 10°C water.

She measures the water temperature in the lake at different depths, and collects the following data.

Depth m	Temp $^{\circ}\text{C}$	Depth m	Temp $^{\circ}\text{C}$
0.5	20	5	15
1	19	5.5	15
1.5	18	6	15
2	18	6.5	12
2.5	18	7	11
3	17	7.5	10
3.5	17	8	10
4	16	8.5	10
4.5	16	9	9

Plot points to make a broken-line graph to illustrate this data.

- a) At what depths should Paige be fishing for small pike?
- b) At what depths should Paige be fishing for lake trout?
- c) Paige knows that most lakes have a thermocline where the water will rapidly change temperature. At what depth is the top of the thermocline in Lac Nilgault?





Problem of the Week

Problem B and Solution

Fishing for Thermoclines

Problem

Paige is headed to Lac Nilgault to do some fishing for lake trout and pike. She knows that small pike generally like 18°C water, large pike like 12°C water, and lake trout like 10°C water. She measures the water temperature in the lake at different depths, and collects the following data.

Depth m	Temp °C	Depth m	Temp °C
0.5	20	5	15
1	19	5.5	15
1.5	18	6	15
2	18	6.5	12
2.5	18	7	11
3	17	7.5	10
3.5	17	8	10
4	16	8.5	10
4.5	16	9	9

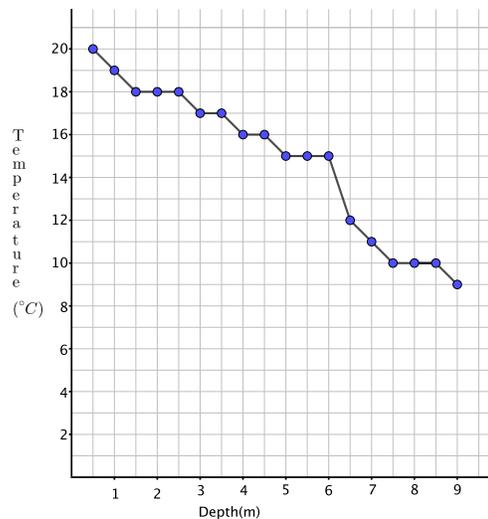
Plot points to make a broken-line graph to illustrate this data.

- At what depths should Paige be fishing for small pike?
- At what depths should Paige be fishing for lake trout?
- Paige knows that most lakes have a thermocline where the water will rapidly change temperature. At what depth is the top of the thermocline in Lac Nilgault?

Solution

- According to the data, Paige should fish for small pike at a depth between 1.5 m and 2.5 m.
- She should fish for lake trout at a depth between 7.5 m and 8.5 m.
- We can see from the graph that the temperature starts to quickly drop at a depth of about 6 m. Therefore, the top of the thermocline is at a depth of about 6 m.

Temperature vs. Depth of Water



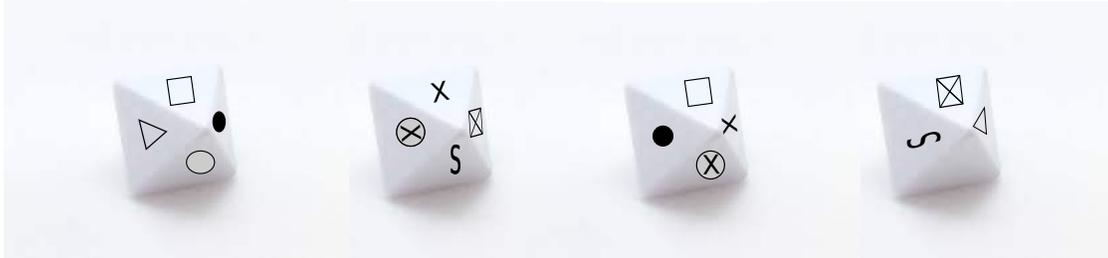
Computational Thinking (C)



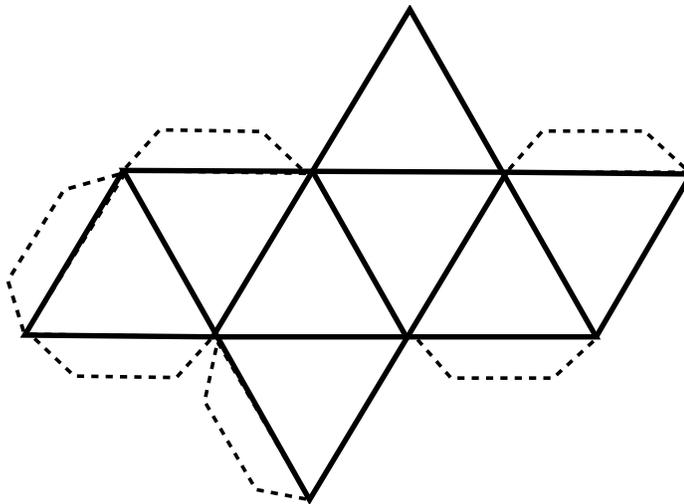


Problem of the Week Problem B The Die is Cast

An 8-sided die contains a different symbol on each side. The diagrams below are pictures of the die from different angles.



- a) Given the first three views of this die, determine what symbol would appear on the blank side shown in the fourth diagram. (You may find the net given below helpful.)



- b) Describe to a classmate how you solved the problem.



Problem of the Week

Problem B and Solution

The Die is Cast

Problem

An 8-sided die contains a different symbol on each side. The diagrams below are pictures of the die from different angles.



- Given the first three views of this die, determine what symbol would appear on the blank side shown in the fourth diagram.
- Describe to a classmate how you solved the problem.



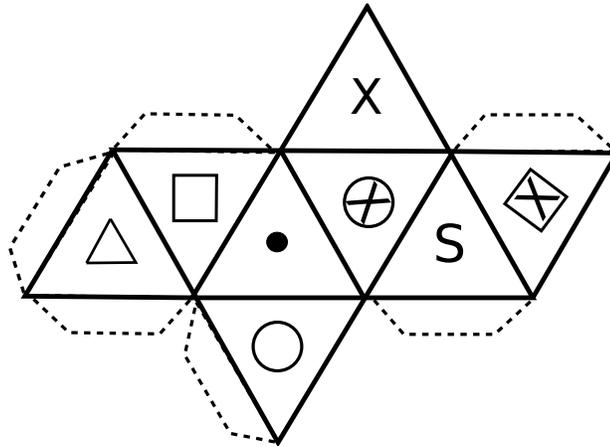
Solution

- a) A circle \bigcirc will appear in the blank space on the fourth die.

Reasoning:

Examining the first two views of the die, we see that the eight possible symbols on the octahedron are $\triangle \bigcirc \bullet \square \times \otimes S \boxtimes$. On the last two views, seven of those eight symbols appear, but on different 'halves' of the die than the first two views. Since the pair $\square \bullet$ is with the pair $\otimes \times$ on the third view, the fourth view must show the pairs $S \boxtimes$ and $\triangle \bigcirc$.

A completed version of the net for the die is shown below.



- b) Explanations will vary. The teacher may wish to choose students with several different versions to present to the class.



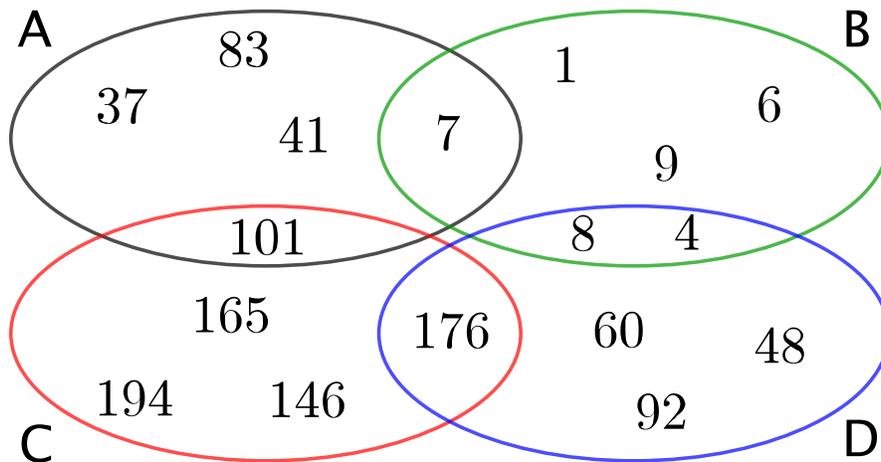
Problem of the Week

Problem B

Now and Venn

A *Venn Diagram* consists of overlapping shapes and is used to show what two or more sets have in common and do not have in common.

Consider the Venn diagram shown below.



- Think of possible reasons for putting the numbers in each section of this Venn diagram.
- Where would the number 144 belong? What about 2?
- Is there a number that would fit in an overlap of any three of these sections, according to your criteria from part a)?



Problem of the Week

Problem B and Solution

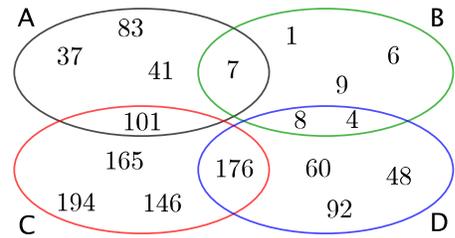
Now and Venn

Problem

A *Venn Diagram* consists of overlapping shapes and is used to show what two or more sets have in common and do not have in common.

Consider the Venn diagram shown to the right.

- Think of possible reasons for putting the numbers in each section of this Venn diagram.
- Where would the number 144 belong? What about 2?
- Is there a number that would fit in an overlap of any three of these sections, according to your criteria from part a)?



Solution

- Possible characteristics of the numbers in each section are:
 - A - prime numbers;
 - B - single digit numbers;
 - C - three-digit numbers (or numbers in the hundreds);
 - D - numbers which are multiples of 4.
- By the above definitions, the number 144 could go in the section shared by C and D, and the number 2 could go in the section shared by A and B.
- To fit in an overlap of three of these four conditions, a number would have to satisfy at least three of the four conditions.

To show that this is not possible, first notice that a number cannot be both a single digit number and three-digit number. That is, no overlap between sections B and C is possible.

Is it possible to have an overlap of the three sections labeled A, B and D? That is, can a number be a single digit number, prime, and a multiple of 4? The single digit numbers that are prime are 2, 3, 5, and 7, none of which are a multiple of 4. Therefore, a number cannot be a single digit and satisfy three of the four conditions.

Is it possible to have an overlap of the three sections labeled A, C and D? That is, can a number be a three-digit number, prime, and a multiple of 4? If a three-digit number is a multiple of 4, then 4 is a factor of that three-digit number, so the number is not prime. Therefore, a number cannot be a three-digit number and satisfy three of the four conditions.



Problem of the Week

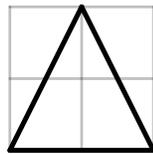
Problem B

Triangular Targets

This is a game for two players. Each player needs a copy of the gameboard (the 8×8 grid on the following page), and a pencil.

Here are the rules of the game, which are similar to BattleshipTM rules.

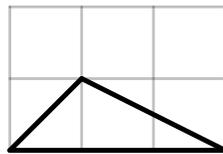
1. Draw one of each of the four types of triangles shown below on your gameboard. The triangles can be rotated (turned) or reflected (flipped) from the given orientations. Each vertex (corner) must be on an intersection of two gridlines. Do not overlap your triangles or let them have any points in common



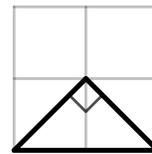
Acute Angle
Isosceles



Right Angle
Scalene



Obtuse Angle
Scalene



Right Angle
Isosceles

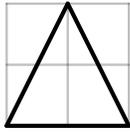
2. Decide who will go first, and then take turns calling out coordinates.
3. If your guess is on a vertex of one of your opponent's triangles, then your opponent must tell you it's a vertex hit, and both of you mark that point with a small x.
4. If your guess is on an edge (side) of one of your opponent's triangles, then your opponent must tell you it was an edge hit, and both of you mark that point with a ✓.
5. A triangle is destroyed when all three vertices have been hit.
6. If a player guesses the grid point inside the Acute Angle Isosceles triangle, then that triangle is destroyed instantly.
7. If your guess is not a vertex, edge, or inside any of your opponent's triangles, then your opponent must tell you it's a miss, and both of you mark that point on the grid with a small o.
8. If you destroy one of your opponent's triangles, then you get another turn.
9. The first player to destroy all of the other player's triangles wins the game.

Keep track of both your guesses and your opponent's, using o for misses, x for vertex hits, and ✓ for edge hits.



My Gameboard (Opponent's Guesses)

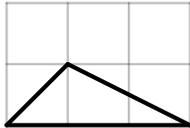
Acute
Angle
Isosceles



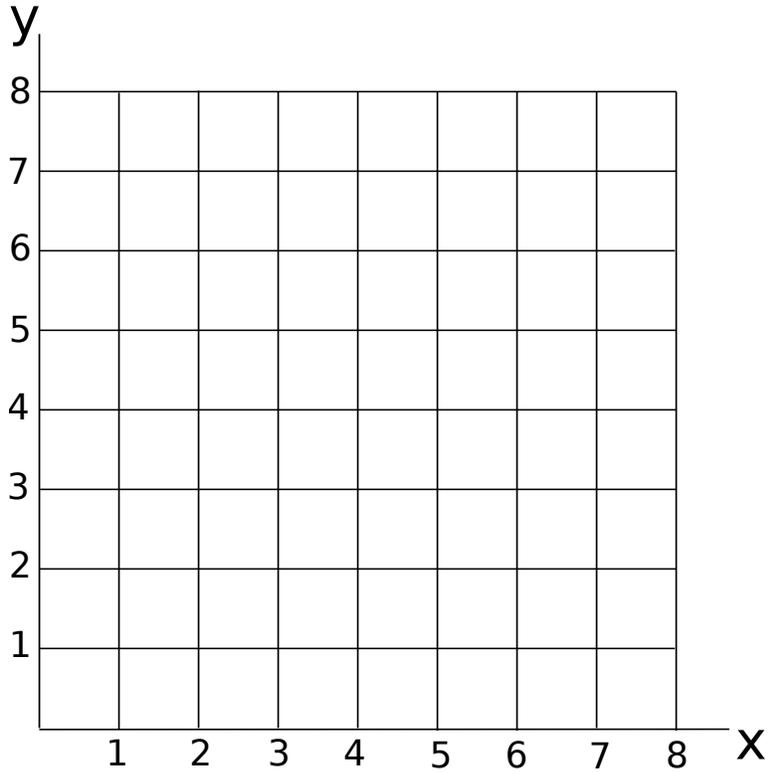
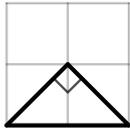
Right
Angle
Scalene



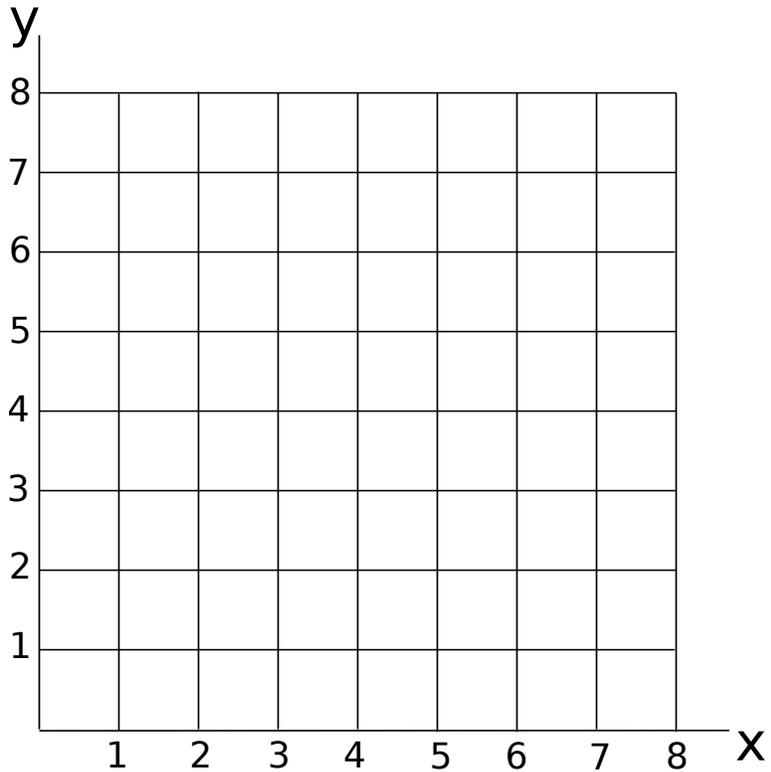
Obtuse
Angle
Scalene



Right
Angle
Isosceles



Opponent's Gameboard (My Guesses)





Problem of the Week

Problem B and Solution

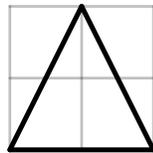
Triangular Targets

Problem

This is a game for two players. Each player needs a copy of the gameboard (the 8×8 grid on the following page), and a pencil.

Here are the rules of the game, which are similar to BattleshipTM rules.

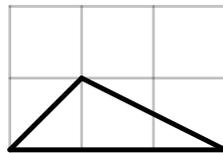
1. Draw one of each of the four types of triangles shown below on your gameboard. The triangles can be rotated (turned) or reflected (flipped) from the given orientations. Each vertex (corner) must be on an intersection of two gridlines. Do not overlap your triangles or let them have any points in common.



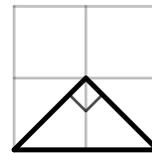
Acute Angle
Isosceles



Right Angle
Scalene



Obtuse Angle
Scalene



Right Angle
Isosceles

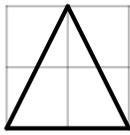
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8. If you destroy one of your opponent's triangles, then you get another turn.
9. The first player to destroy all of the other player's triangles wins the game.

Keep track of both your guesses and your opponent's, using \circ for misses, x for vertex hits, and \checkmark for edge hits.



My Gameboard (Opponent's Guesses)

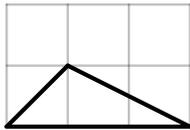
Acute
Angle
Isosceles



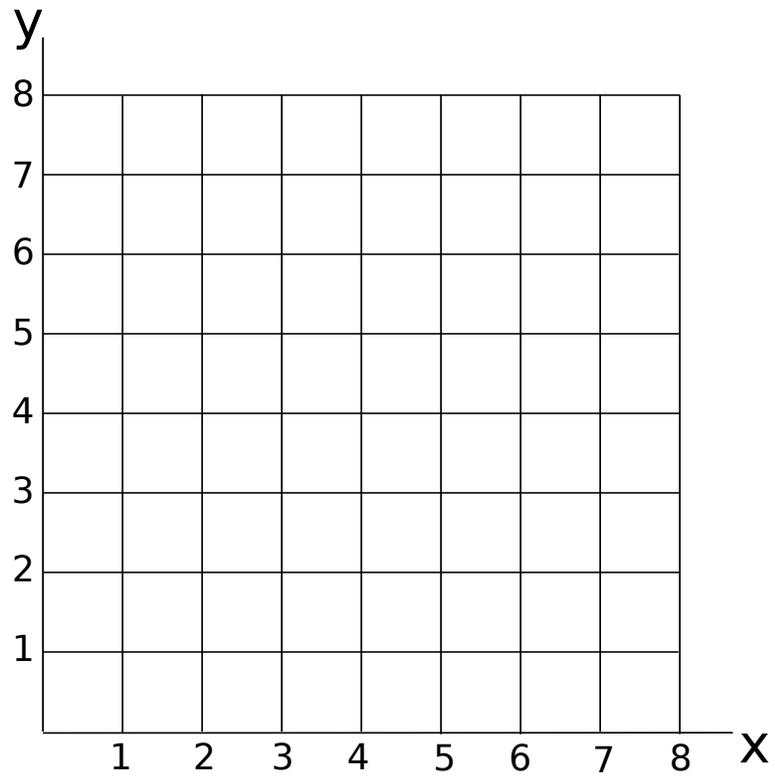
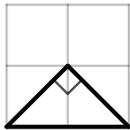
Right
Angle
Scalene



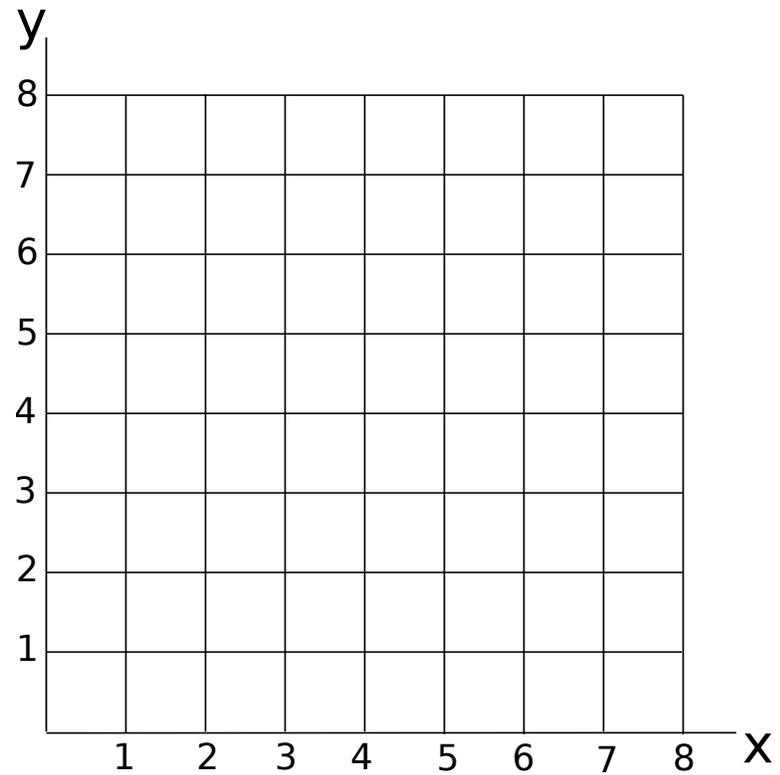
Obtuse
Angle
Scalene



Right
Angle
Isosceles



Opponent's Gameboard (My Guesses)





Solution

The game of Triangular Targets is intended to reinforce important features of Cartesian coordinates and plotting points in the first quadrant:

- for the first coordinate (x -coordinate), move from the origin to the right along the x -axis;
- for the second coordinate (y -coordinate), move up from the position on the x -axis parallel to the y -axis;
- points along the y -axis will have 0 as the x -coordinate;
- points along the x -axis will have 0 as the y -coordinate;
- points on any horizontal side of a triangle will have the same y -coordinate as two of its vertices.
- points on any vertical side of a triangle will have the same x -coordinate as two of its vertices.

In addition, we hope that learning about Cartesian coordinates via Triangular Targets has been fun.

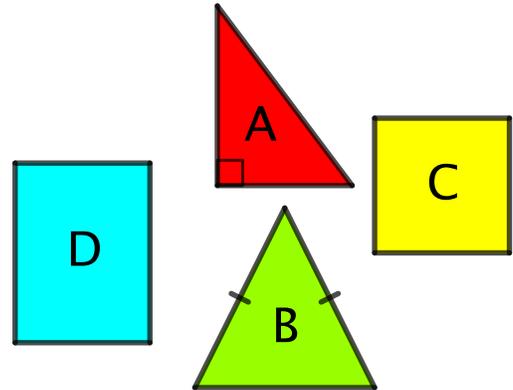


Problem of the Week

Problem B

Alternate Dimensions

The four shapes to the right are each drawn with a horizontal base and a vertical height. Figure A is a right-angled triangle, Figure B is an isosceles triangle, Figure C is a square, and Figure D is a rectangle. The figures are not drawn to scale.



Using the following clues, determine the measure of the (horizontal) base and the measure of the (vertical) height of each figure.

1. The measure of the base of Figure A is the same as the measure of the base of Figure D.
2. The measure of the base of Figure A is one unit less than the measure of the base of Figure B.
3. The side length of Figure C is the same as the measure of the base of Figure A.
4. The measure of the height of Figure B is the same as the measure of the height of Figure A and also the same as the measure of the base of Figure B.
5. The area of Figure C is 9 square units.
6. The total area of all four figures is 38 square units.



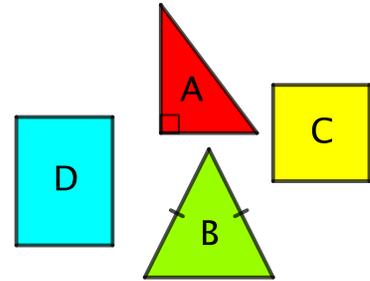
Problem of the Week

Problem B and Solution

Alternate Dimensions

Problem

The four shapes to the right are each drawn with a horizontal base and a vertical height. Figure A is a right-angled triangle, Figure B is an isosceles triangle, Figure C is a square, and Figure D is a rectangle. The figures are not drawn to scale.



Using the following clues, determine the measure of the (horizontal) base and the measure of the (vertical) height of each figure.

1. The measure of the base of Figure A is the same as the measure of the base of Figure D.
2. The measure of the base of Figure A is one unit less than the measure of the base of Figure B.
3. The side length of Figure C is the same as the measure of the base of Figure A.
4. The measure of the height of Figure B is the same as the measure of the height of Figure A and also the same as the measure of the base of Figure B.
5. The area of Figure C is 9 square units.
6. The total area of all four figures is 38 square units.

Solution

From 5., since C is a square with an area of 9 square units, its side length is 3 units.

From 3., A must have a base that is 3 units long.

From 1., D has a base that is 3 units long as well.

From 2., the measure of the base of B is 1 unit greater than that of A, so B has a base that is 4 units long.

From 4., the heights of A and B thus both have a measure of 4 units.

We can now calculate that the area of triangle A is $\frac{1}{2} \times 3 \times 4 = 6$ square units and the area of triangle B is $\frac{1}{2} \times 4 \times 4 = 8$ square units. We also know that the area of square C is 9 square units. Summing, the total area of figures A, B, and C is $6 + 8 + 9 = 23$ square units.

From 6., the total area of all figures is 38 square units, so the area of D is $38 - 23 = 15$ square units. Since D is a rectangle with a base of measure 3, it thus has a height of measure $15 \div 3 = 5$ units.

Therefore,

Figure A has a base of measure 3 units and a height of measure 4 units;

Figure B has a base of measure 4 units and a height of measure 4 units;

Figure C is 3 units by 3 units; and

Figure D has a base of measure 3 units and a height of measure 5 units.