Problem of the Week
Problem A and Solution
Area Issues

Problem
The employees of Sew Inspired need to make a quilt for a special puzzle project. They draw a
pattern for the four pieces of cloth they will cut for each section. The pattern includes two
square pieces (AKFE and GHCL), and two rectangular pieces (KBHF and EGLD).

The area of the piece AKFE is 4 square units. The area of the piece GHCL is 9 square units.
The points E, F, G, and H are on the same straight line, and the line segment FG is 5 units
long.

What is the area of the rectangular section ABCD?

Solution
One way to calculate the area of rectangle ABCD, is to determine the lengths of its sides. We
know the area of the square AKFE is 4 square units, and we know that the lengths of the
sides of a square must be the same. So if the length of one side of a square is $n$, then the area
of the square must be $n \times n$. By trial and error we can see that $2 \times 2 = 4$, so each side of the
square AKFE must be 2 units. Similarly, since the area of the square GHCL is
9 square units, we can see that $3 \times 3 = 9$. So each side of the square GHCL must be 3 units.

Another way to determine the lengths of the sides of square AKFE, would be to start with
4 unit squares (using blocks or cut out of paper for example) and determine how to arrange
them into a larger square. The only possible arrangement is a $2 \times 2$ square.

The opposite sides of a rectangle must be the same length, and the bottom of the
square AKFE is on the same line as the top of square GHCL. This means the length
side AD is equal to the sum of the lengths of the sides of the two squares. So the length of
side AD is $2 + 3 = 5$ units. Also, since the length of line segment FG is 5 units, the length of
line EH must be $2 + 5 + 3 = 10$ units. The length of this line is the same as the length of the
side AB or the side DC.

Now we can calculate the area of rectangle ABCD. The area of this rectangle is the product of
the length of side AD and the length of side AB. So the area of rectangle ABCD is
$5 \times 10 = 50$ square units.
**Teacher’s Notes**

Exploring the areas of rectangles or squares is a good way to practice multiplication of positive numbers. The result of calculating $a \times b$ is equivalent to calculating the area of a rectangle with side lengths $a$ and $b$.

Similarly, exploring the relationship between the area of a square and the lengths of its sides is a good way to learn about *square roots*. However, this exploration only produces half of the answer.

A square root of a number is defined as follows:

If $y$ is a square root of $x$, then $y \cdot y = x$.

Since the product of two negative numbers is positive, the value of $y$ could be positive or negative. So the square root of 9 is either 3 or $-3$. When we use the radical symbol, $\sqrt{}$, we are looking for the principal (or positive) square root. Although $-3$ and 3 are both square roots of 9, the $\sqrt{9} = 3$.

If $x$ is a *perfect square* (i.e. it is the result of multiplying a rational number by itself), then the result of calculating $\sqrt{x}$ is also a rational number. A *rational number* is any number that can be represented as a fraction $\frac{a}{b}$, where $a$ and $b$ are integers, and $b$ is not 0.

For example, $\sqrt{144}$ is exactly 12. There are also non-integers values that have rational square roots. For example, $\sqrt{\frac{9}{16}}$ is exactly $\frac{3}{4}$.

There are many numbers, however, that are not perfect squares. Square roots of these numbers are known as *irrational numbers*. An irrational number cannot be represented precisely as a fraction. If we try to write an irrational number in decimal form, we end up with a result where the digits after the decimal place never end, and never repeat a pattern. So, we cannot compute the square root of an irrational number on a calculator exactly. The result will appear as a limited number of digits after the decimal place. The calculator rounds the answer to a fixed number of decimal places.