Problem of the Week
Problem C and Solution
Cookie Shares

Problem
Sheetal has some cookies. She gives one-third of her cookies to Gil. She then eats 4 cookies, after which she gives one-half of her remaining cookies to Anna. If Sheetal then has 16 cookies left, how many cookies did she have to begin?

Solution
Solution 1
We work backwards from the last piece of information given.
Sheetal has 16 cookies left after giving one-half of her remaining cookies to Anna. This means that she had \(2 \times 16 = 32\) cookies immediately before giving cookies to Anna.
Immediately before giving cookies to Anna, she ate 4 cookies, which means that she had \(32 + 4 = 36\) cookies immediately before eating the 4 cookies.
Immediately before eating these cookies, she gave one-third of her cookies to Gil, which would have left her with two-thirds of her original amount.
Since two-thirds of her original amount equals 36 cookies, then one-third equals one half of 36 or \(\frac{36}{2} = 18\) cookies.
Thus, she gave 18 cookies to Gil, and so Sheetal started with \(36 + 18 = 54\) cookies.

Solution 2
Suppose Sheetal started with \(x\) cookies.
She gives \(\frac{1}{3}x\) cookies to Gil, leaving her with \(1 - \frac{1}{3}x = \frac{2}{3}x\) cookies.
She then eats 4 cookies, leaving her with \(\frac{2}{3}x - 4\) cookies.
Finally, she gives away one-half of what she has left to Anna, which means that she keeps one-half of what she has left, and so she keeps \(\frac{1}{2}(\frac{2}{3}x - 4)\) cookies.
Simplifying this expression, we obtain \(\frac{2}{6}x - \frac{4}{2} = \frac{1}{3}x - 2\) cookies.
Since she has 16 cookies left, then \(\frac{1}{3}x - 2 = 16\) and so \(\frac{1}{3}x = 18\) or \(x = 54\).
Therefore, Sheetal began with 54 cookies.