Problem of the Week
Problem D and Solution
It’s a New Year

Problem

$5^3$ is a power with base 5 and exponent 3. $5^3$ means $5 \times 5 \times 5$ and equals 125 when expressed as an integer. When $8^{672} \times 5^{2019}$ is expressed as an integer, how many digits are in the product?

Solution

An immediate temptation might be to reach for a calculator. In this case, basic calculator technology will let you down. We will look at the problem using our knowledge of powers and corresponding power laws.

\[
8^{672} \times 5^{2019} = (2^3)^{672} \times 5^{2019} = 2^{3 \times 672} \times 5^{2019} = 2^{2016} \times 5^{2019} = 2^{2016} \times 5^{2016} \times 5^3 = (2 \times 5)^{2016} \times 125 = 10^{2016} \times 125
\]

But $10^{2016}$ is the number 1 followed by 2016 zeroes. When we multiply this number by the three-digit number 125, we obtain the number 125 followed by 2016 zeroes. Therefore, $8^{672} \times 5^{2019}$ has $2016 + 3 = 2019$ digits. Happy New Year again!