Problem of the Week
Problem E and Solution
A Tale of Two Cities

Problem

Two cities, Mytown and Yourtown, had the same population at the end 2015. The population of Mytown decreased by 2.5% from the end of 2015 to the end of 2016. Then, the population increased by 8.4% from the end of 2016 to the end of 2017. The population of Yourtown increased by \( r \%), \( r > 0 \), from the end of 2015 to the end of 2016. Then, the population of Yourtown increased by \( (r + 2)\% \) from the end of 2016 to the end of 2017. Surprisingly, the populations of both cities were the same again at the end of 2017. Determine the value of \( r \) correct to one decimal place.

Solution

Let \( p \) be the population of Mytown at the end of 2015. Since Mytown and Yourtown have the same population size, then \( p \) is also the population of Yourtown at the end of 2015.

The population of Mytown decreased by 2.5% in 2016, so the population at the end of 2016 is
\[
p - \frac{2.5}{100}p = \left(1 - \frac{2.5}{100}\right)p = 0.975p.
\]

The population of Mytown then increased by 8.4% during 2017, so the population at the end of 2017 is
\[
0.975p + \left(\frac{8.4}{100}\right)(0.975p) = \left(1 + \frac{8.4}{100}\right)(0.975p) = 1.084(0.975p) = 1.0569p.
\]

The population of Yourtown increased by \( r\% \) in 2016, so the population at the end of 2016 is
\[
p + \frac{r}{100}p = \left(1 + \frac{r}{100}\right)p.
\]

The population of Yourtown then increased by \( (r + 2)\% \) during 2017, so the population at the end of 2017 is
\[
\left(1 + \frac{r}{100}\right)p + \frac{r + 2}{100}\left(1 + \frac{r}{100}\right)p = \left(1 + \frac{r}{100}\right)p\left(1 + \frac{r + 2}{100}\right).
\]

Since the populations of Mytown and Yourtown are equal at the end of 2017, we have
\[
\left(1 + \frac{r}{100}\right)\left(1 + \frac{r + 2}{100}\right)p = 1.0569p
\]
\[
\left(\frac{100 + r}{100}\right)\left(\frac{100 + r + 2}{100}\right) = 1.0569, \quad \text{dividing both sides by} \ p, \ \text{since} \ p > 0
\]
\[
(100 + r)(102 + r) = 10569, \quad \text{multiplying both sides by} \ 10000 \ \text{to clear fractions}
\]
\[
10200 + 202r + r^2 = 10569
\]
\[
r^2 + 202r - 369 = 0
\]

After using the quadratic formula and ruling out an inadmissible \( r \) value, we obtain \( r = 1.8\% \), correct to one decimal place.