Problem of the Week
Problem E and Solution
Can You Relate?

Problem
Points $A$, $B$ and $D$ lie on the circumference of a circle with centre $C$. $\angle CAD = p^\circ$ and $\angle CBD = q^\circ$. Determine the measure of $\angle ACB$ and the measure of $\angle ADB$. What is the relationship between $\angle ACB$ and $\angle ADB$?

Solution
We start by constructing radius $CD$.

$CA$ and $CD$ are both radii of the circle, so $CA = CD$. Then $\triangle CAD$ is isosceles and $\angle CDA = \angle CAD = p^\circ$. Since the angles in a triangle add to $180^\circ$, $\angle ACD = (180 - 2p)^\circ$.

$CB$ and $CD$ are both radii of the circle, so $CB = CD$. Then $\triangle CBD$ is isosceles and $\angle CDB = \angle CBD = q^\circ$. Since the angles in a triangle add to $180^\circ$, $\angle BCD = (180 - 2q)^\circ$.

We will now find the measure of $\angle ADB$ and of $\angle ACB$ in order to determine the relationship.

\[
\angle ADB = \angle CDB - \angle CDA \\
= (q - p)^\circ \\
\angle ACB = \angle ACD - \angle BCD \\
= (180 - 2p)^\circ - (180 - 2q)^\circ \\
= (2q - 2p)^\circ \\
= 2 \times (q - p)^\circ \\
= 2 \times \angle ADB
\]

$\therefore \angle ACB$ is double the size of $\angle ADB$.

In general, the angle inscribed at the centre of a circle is twice the size of the angle inscribed at the circumference by the same chord. In the following diagram, $\angle ACB$ is inscribed at the centre of the circle by chord $AB$ and $\angle ADB$ is inscribed at the circumference by the same chord. Therefore, $\angle ACB = 2\angle ADB$. 