Problem of the Week
Problem A and Solution
Birthday Predictions

Problem
Anika plans to do a survey in her school. She wants to find out the date of each person’s birthday. She decides to make some predictions before she actually conducts the survey.

How should Anika answer the following questions? Justify your answers.

A) Which month is likely to have the fewest birthdays?
B) Which day (1 through 31) of the month is likely to have the fewest birthdays?
C) If there are 36 students in Anika’s class, how many students are likely to have their birthdays during the summer months of July and August?

After she completes the survey, Anika discovers that her predictions were incorrect. Give some reasons that might explain why the predictions failed.

Solution
We assume that it is equally likely that an individual’s birthday would land on any particular day of the year.

A) Since February is the month with the fewest days, then we predict that February has the fewest birthdays.

B) Since only seven months of the year have 31 days, then it is less likely to have a birthday on the 31st than any other day of the month.

C) Since there are 12 months in the year, and there are 36 students in the class, then we predict that each month has $36 \div 12 = 3$ birthdays. So in July and August, we predict there will be $2 \times 3 = 6$ birthdays.

Here are a couple of reasons that the predictions might fail. The sample size is very small. It is possible with only 36 students that there are days in the month, or even a month in the year where there are no birthdays. Also, although we expect that the birthdays will be evenly distributed, it is possible that multiple students in the class share the day of the month of their birthday, or there are some months that are more popular than others, or that some students even share the same birthday.
Teacher’s Notes
How many people would we have to gather in order to have two of them share a birthday? The simple answer is that if we have at least 367 people, then there is a guarantee that two people must have the same birthday since there are only 366 possible different birthdays in a calendar year that includes a leap day.

It turns out that if you check the birthdays of 23 people, it is more likely than not that you will find that at least two of them share a birthday. This is a classic statistics problem known as the *birthday paradox*. There is no guarantee of course, but it can be proven with statistics that there is just over a 50% chance that in a group of 23 people there are two with the same birthday.