Problem
Samantha has chickens and sheep on her farm. Each chicken has two legs and each sheep has four legs. Each chicken has one head and each sheep has one head. She looked out one day and counted 48 animal heads and 134 legs. How many of each animal live on the farm?

Solution
One way to determine the answer is to make a table that keeps track of chickens, sheep, heads, and legs. The number of chickens plus the number of sheep must always total 48. So we could start with 0 chickens and 48 sheep. If there were 48 sheep, then there would be \(48 \times 4 = 192\) legs. Then in each row of the table that follows, we increase the number of chicken heads by 1, and decrease the number of sheep heads by 1. We also increase the number of chicken legs by 2, and decrease the number of sheep legs by 4.

<table>
<thead>
<tr>
<th>Chicken Heads</th>
<th>Sheep Heads</th>
<th>Chicken Legs</th>
<th>Sheep Legs</th>
<th>Total Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>48 – 0 = 48</td>
<td>0 × 2 = 0</td>
<td>48 × 4 = 192</td>
<td>192</td>
</tr>
<tr>
<td>1</td>
<td>48 – 1 = 47</td>
<td>1 × 2 = 2</td>
<td>47 × 4 = 188</td>
<td>190</td>
</tr>
<tr>
<td>2</td>
<td>48 – 2 = 46</td>
<td>2 × 2 = 4</td>
<td>46 × 4 = 184</td>
<td>188</td>
</tr>
<tr>
<td>3</td>
<td>48 – 3 = 45</td>
<td>3 × 2 = 6</td>
<td>45 × 4 = 180</td>
<td>186</td>
</tr>
<tr>
<td>4</td>
<td>48 – 4 = 44</td>
<td>4 × 2 = 8</td>
<td>44 × 4 = 176</td>
<td>184</td>
</tr>
</tbody>
</table>

From the first few rows in this table, we see there is a pattern forming for the total number of legs. Each time we increase the number of chickens by 1, the total number of legs decreases by 2. We could continue filling in the table until we get a row where the total number of legs is 134. However, this would mean filling in many more entries in the table. We need to get the total legs from 184 to 134, which is a difference of \(184 – 134 = 50\). Since the total number of legs decreases by 2 for each additional chicken, then we would reach the correct total number of legs if we filled in \(50 ÷ 2 = 25\) more rows. This means we have \(4 + 25 = 29\) chickens. That row in the table would look like:

<table>
<thead>
<tr>
<th>Chicken Heads</th>
<th>Sheep Heads</th>
<th>Chicken Legs</th>
<th>Sheep Legs</th>
<th>Total Legs</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>48 – 29 = 19</td>
<td>29 × 2 = 58</td>
<td>19 × 4 = 76</td>
<td>134</td>
</tr>
</tbody>
</table>

We see that there are 29 chickens and 19 sheep on the farm.
Teacher’s Notes

Here is an alternative approach to solving the problem.

We know all of the animals have at least 2 legs. Since we count 48 heads, then we can account for $2 \times 48 = 96$ legs, no matter what type of animals are on the farm. However, we have counted $134 - 96 = 38$ more legs than if all of the animals only had 2 legs. Those extra 38 legs must be on sheep. Each sheep contributes 2 more legs to the count. So we need to have $38 \div 2 = 19$ sheep on the farm. This means we have $48 - 19 = 29$ chickens.

We could also solve this problem algebraically.

Let $s$ represent the number of sheep.
Let $c$ represent the number of chickens.
Knowing that each animal has 1 head, that chickens have 2 legs, and that sheep have 4 legs, we can write the following equations:

\[
\begin{align*}
    s + c &= 48 \quad (equation \ 1) \\
    4s + 2c &= 134 \quad (equation \ 2)
\end{align*}
\]

Then we multiply both sides of equation 1 by 2:

\[
\begin{align*}
    2 \cdot (s + c) &= 2 \cdot 48 \\
    2s + 2c &= 96 \quad (equation \ 3)
\end{align*}
\]

Then we subtract equation $2 - equation \ 3$ and solve for $s$:

\[
\begin{align*}
    (4s - 2s) + (2c - 2c) &= 134 - 96 \\
    2s &= 38 \\
    s &= \frac{38}{2} \\
    s &= 19
\end{align*}
\]

Finally, we can substitute the value of $s$ into equation 1 and solve for $c$:

\[
\begin{align*}
    (19) + c &= 48 \\
    19 - 19 + c &= 48 - 19 \\
    c &= 29
\end{align*}
\]

This means we have 19 sheep and 29 chickens.