Problem of the Week
Problem B and Solution
It’s Hip to be Square

Problem
The whole number factors of any whole number $N$ are the whole numbers which divide evenly into $N$.

For example, 12 has six whole number factors. They are 1, 2, 3, 4, 6, and 12. The number 17 has two whole number factors. They are 1 and 17.

Notice that both 12 and 17 have an even number of whole number factors.

a) List all the numbers from 1 to 10 which have an odd number of whole number factors.

b) Why is there an odd number of factors for some of the numbers?

c) Figure out which numbers between 10 and 50 have an odd number of whole number factors.

d) What is the special name for the numbers that have an odd number of whole number factors?

Solution

a)

<table>
<thead>
<tr>
<th>Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Number Factors</td>
<td>1</td>
<td>1,2</td>
<td>1,3</td>
<td>1,2,4</td>
<td>1,5</td>
<td>1,2,3,6</td>
<td>1,7</td>
<td>1,2,4,8</td>
<td>1,3,9</td>
<td>1,2,5,10</td>
</tr>
<tr>
<td>Number of Factors</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

We see that the numbers 1, 4, and 9 have an odd number of factors.

b) For the numbers with an even number of whole number factors, when we place the factors in order from smallest to largest, the product of the middle pair of factors gives the number (e.g., for the number 8: $2 \times 4 = 8$). But in the numbers with an odd number of factors, the middle factor multiplied by itself gives the number. For example, for the number 9: $3 \times 3 = 9$, or $3^2 = 9$.

c) If we use the idea that an odd number of factors occurs when the number equals the product of one factor and itself, then between 10 and 50, the numbers with an odd number of factors are

$16 = 4 \times 4$, $25 = 5 \times 5$, $36 = 6 \times 6$, and $49 = 7 \times 7$.

Note that: the factors of 16 are 1, 2, 4, 8, and 16; the factors of 25 are 1, 5, and 25; the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36; and the factors of 49 are 1, 7, and 49.

d) Such numbers are called perfect squares, or square numbers.