Problem

For Amanda’s birthday, Rhett made an amazing, cylindrical, chocolate cream cheesecake. The radius and height of the cake were the same. Rhett cut the cake into 8 congruent slices and ate the first slice for quality control purposes. After removing Rhett’s slice, is there more or less total surface area (top, bottom and all exposed sides) in the remaining cake, and by what percentage, to 1 decimal place, has the remaining surface area increased or decreased?

Solution

A net illustrating the 3 parts that make up the total surface area is shown to the right.

The total surface area includes the areas of two circles with radius $r$ and a rectangle with length equal to the circumference of the circle and width equal to the height of the cake, $h = r$.

\[
\text{Total Surface Area} = 2\left(\pi r^2\right) + (2\pi r)(r)
\]
\[
= 2\pi r^2 + 2\pi r^2
\]
\[
= 4\pi r^2
\]

A net illustrating the 3 parts removed from the total surface area is shown to the right.

The surface area removed includes $\frac{1}{8}$ of the area of each of two circles with radius $r$ and a rectangle $\frac{1}{8}$ of the area of the original rectangle area. Therefore, the surface area removed is $\frac{1}{8}$ of the total surface area.

\[
\text{Surface Area Removed} = \frac{1}{8} (4\pi r^2)
\]
\[
= \frac{1}{2} \pi r^2
\]

But there are two areas that are added as a result of removing the slice.
A diagram illustrating the 2 parts added to the total surface area is shown to the right.

The surface area added includes 2 rectangles, each with length \( r \) and width \( h = r \).

Surface Area Added \[= 2(r)(r)\]
\[= 2r^2\]

We can now calculate the new total surface area.

Surface Area \[= \text{Original Surface Area} - \text{Surface Area Removed} + \text{Surface Area Added}\]
\[= 4\pi r^2 - \frac{1}{2} (\pi r^2) + 2r^2\]
\[= 4\pi r^2 + r^2 \left(\frac{1}{2} - \pi + 2\right)\]

Now \( \frac{1}{2} \pi < 2 \), so \( -\frac{1}{2} \pi + 2 > 0 \), and the surface area actually increases as a result of removing the slice.

To calculate the percentage that the area has increased, divide the increase by the original area. The increase is \( r^2 \left(\frac{1}{2} - \pi + 2\right)\).

Percentage Increase in Area \[= \frac{r^2 \left(\frac{1}{2} - \pi + 2\right)}{4\pi r^2} \times 100\%\]
\[= \frac{\left(\frac{1}{2} - \pi + 2\right)}{4\pi} \times 100\%\]
\[= \left(-\frac{1}{8} + \frac{1}{2\pi}\right) \times 100\%\]
\[= \left(-\frac{\pi}{8\pi} + 4\right) \times 100\%\]
\[\approx 3.4\%\]

The surface area of the cake increases by approximately 3.4% after the slice has been removed.