Problem of the Week
Problem E and Solution
Average Again

Problem

Erin has written three tests for her math class. She calculates the average of her marks on the first two tests. This average is then averaged with her third test mark to get 80%. She then calculates the average of her marks on the last two tests. This average is then averaged with her first test mark to get 83.5%. Finally, she calculates the average of her marks on the first and third tests. This average is then averaged with her second test mark to get 84.5%. Her fourth test is coming up and after writing this test she wants her overall average for the four tests to be exactly 86%. If all four tests are out of 100 marks, what mark does Erin need to get on her fourth test in order for her overall average to be exactly 86%?

Solution

Let $a$ represent Erin’s first test mark, $b$ represent Erin’s second test mark, $c$ represent Erin’s third test mark and $d$ represent Erin’s fourth test mark.

When the average of her first and second marks is averaged with her third test mark, the new average is 80 so $\frac{a+b+c}{2} = 80$. Multiplying by 2 gives $\frac{a+b}{2} + c = 160$. Multiplying by 2 again yields $a + b + 2c = 320$. (1)

When the average of her second and third marks is averaged with her first test mark, the new average is 83.5 so $\frac{b+c+a}{2} = 83.5$. Multiplying by 2 gives $\frac{b+c}{2} + a = 167$. Multiplying by 2 again yields $b + c + 2a = 334$. (2)

When the average of her first and third marks is averaged with her second test mark, the new average is 84.5 so $\frac{a+c+b}{2} = 84.5$. Multiplying by 2 gives $\frac{a+c}{2} + b = 169$. Multiplying by 2 again yields $a + c + 2b = 338$. (3)

By adding equations (1), (2) and (3) we obtain $4a + 4b + 4c = 992$, which simplifies to $a + b + c = 248$ after dividing by 4. This means that the sum of her first three marks is 248.

To obtain an average of 86% over the four tests, Erin needs a total of $86 \times 4 = 344$ marks. In other words, $a + b + c + d = 344$. We know that $a + b + c = 248$ so $248 + d = 344$, and $d = 96$ follows.

Therefore, to obtain an average of 86%, Erin needs 96% on her fourth test.

It would be a straight forward process to determine Erin’s first three test marks but our question did not ask us to do this. However, for the curious, on her first test Erin got 86, on her second test Erin got 90, and on her third test Erin got 72.