



## Problem of the Week

### Problem A and Solution

#### Animal Ages

#### Problem

Robbie has five pets: two dogs named Bonnie and Espo, two cats named Walt and Ginny, and a ferret named Lucky. He makes the following true statements about the ages of his pets, in years:

1. Bonnie is 5 years older than Espo.
2. Ginny is half the age of Walt.
3. The sum of Espo's age and Lucky's age is equal to Walt's age.
4. Bonnie is 1 year older than Walt.
5. Ginny is 6 years old.

How many years old are each of Robbie's pets?

#### Solution

From statement 5, we know Ginny is 6 years old.

From statement 2, Ginny is half the age of Walt. That means Walt is twice as old as Ginny. So Walt is  $6 \times 2 = 12$  years old.

From statement 4, Bonnie is 1 year older than Walt. So Bonnie is  $12 + 1 = 13$  years old.

From statement 1, Bonnie is 5 years older than Espo. That means Espo is 5 years younger than Bonnie. So Espo is  $13 - 5 = 8$  years old.

From statement 3, the sum of Espo's age and Lucky's age is equal to Walt's age. That means the difference of Walt's age and Espo's age is equal to Lucky's age. So Lucky is  $12 - 8 = 4$  years old.

In summary:

- Ginny is 6 years old.
- Walt is 12 years old.
- Bonnie is 13 years old.
- Espo is 8 years old.
- Lucky is 4 years old.



## Teacher's Notes

We can describe the statements in this problem algebraically and write a system of equations. Suppose each animal's age is represented by a variable that is the first letter of its name. Then based on the statements in the problem we know:

$$B = E + 5$$

$$G = W \div 2$$

$$E + L = W$$

$$B = W + 1$$

$$G = 6$$

Now, we could use algebraic techniques to find the values of these five variables.

We expect that there is a single solution to these types of problems, and it turns out that there is for this problem. However, for some problems we are not given enough information to determine specific values for each variable. For a problem with five variables, a minimum requirement to guarantee a single solution is that we need at least five equations. However, if an equation in the problem can be written as a *linear combination* of one or more of the other equations in the problem, it turns out that five equations will not guarantee a single solution. This is something that students will learn more about in higher level math courses.