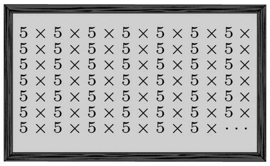

Problem of the Week

Problem C and Solution

More Power to You

Problem

In mathematics we like to write expressions concisely. For example, we will often write the expression $5 \times 5 \times 5 \times 5$ as 5^4. The lower number 5 is called the base, the raised 4 is called the exponent, and the whole expression 5^4 is called a power. So 5^3 means $5 \times 5 \times 5$ and is equal to 125. What are the last three digits in the integer equal to 5^{2020}?

Solution

Let's start by examining the last three digits of various powers of 5.

$$\begin{aligned}
5^1 &= \mathbf{005} \\
5^2 &= \mathbf{025} \\
5^3 &= \mathbf{125} \\
5^4 &= \mathbf{625} \\
5^5 &= 3\mathbf{125} \\
5^6 &= 15\,\mathbf{625} \\
5^7 &= 78\,\mathbf{125} \\
5^8 &= 390\,\mathbf{625}
\end{aligned}$$

Notice that there is a pattern for the last three digits after the first two powers of 5. For every odd integer exponent greater than 2, the last three digits are "125". For every even integer exponent greater than 2, the last three digits are "625". If the pattern continues, then 5^9 will end "125" since the exponent 9 is odd and 5^{10} will end "625" since the exponent 10 is even. This is easily verified since $5^9 = 1\,953\,\mathbf{125}$ and $5^{10} = 9\,765\,\mathbf{625}$.

We can easily justify why this pattern continues. If a power ends in "125", then the last 3 digits of the next power are the same as the last three digits of the product $125 \times 5 = 625$. That is, the last three digits of the next power are "625". If a power ends in "625", then the last 3 digits of the next power are the same as the last three digits of the product $625 \times 5 = 3125$. That is, the last three digits of the next power are "125".

For 5^{2020}, the exponent 2020 is greater than 2 and is an even number.

Therefore, the last three digits of 5^{2020} are 625.