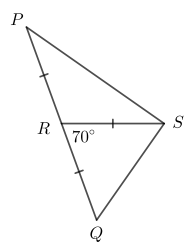
Problem of the Week

Problem C and Solution

Angled

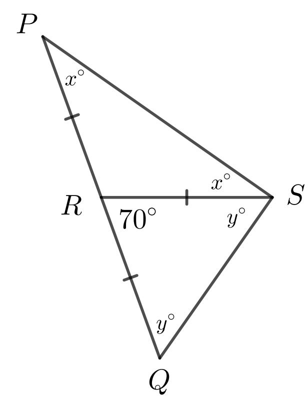


Problem

In $\triangle PQS$ above, R lies on PQ such that $PR = RQ = RS$ and $\angle QRS = 70^\circ$.
Determine the measure of $\angle PSQ$.

Solution

Solution 1



In $\triangle PRS$, since $PR = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x^\circ$.

Similarly, in $\triangle QRS$, since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y^\circ$.

Since PRQ is a straight line, $\angle PRS + \angle QRS = 180^\circ$. Since $\angle QRS = 70^\circ$, we have $\angle PRS = 110^\circ$.

The angles in a triangle sum to 180°, so in $\triangle PRS$

$$\begin{aligned} \angle RPS + \angle RSP + \angle PRS &= 180^\circ \\ x^\circ + x^\circ + 110^\circ &= 180^\circ \\ 2x &= 70 \\ x &= 35 \end{aligned}$$

The angles in a triangle sum to 180°, so in $\triangle QRS$

$$\begin{aligned} \angle RQS + \angle RSQ + \angle QRS &= 180^\circ \\ y^\circ + y^\circ + 70^\circ &= 180^\circ \\ 2y &= 110 \\ y &= 55 \end{aligned}$$

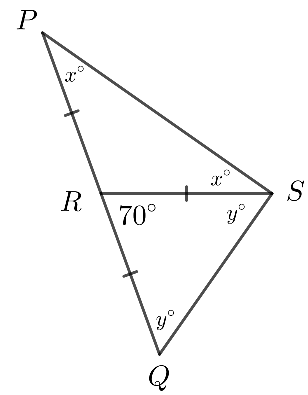
Then $\angle PSQ = \angle RSP + \angle RSQ = x^\circ + y^\circ = 35^\circ + 55^\circ = 90^\circ$.

Therefore, the measure of $\angle PSQ$ is 90°.

See Solution 2 for a more general approach to the solution of this problem.

It turns out that it is not necessary to determine the values of x and y to solve this problem.

Solution 2



In $\triangle PRS$, since $PR = RS$, $\triangle PRS$ is isosceles and $\angle RPS = \angle RSP = x^\circ$.

Similarly, in $\triangle QRS$, since $RQ = RS$, $\triangle QRS$ is isosceles and $\angle RQS = \angle RSQ = y^\circ$.

The angles in a triangle sum to 180°, so in $\triangle PQS$

$$\begin{aligned} \angle QPS + \angle PSQ + \angle PQS &= 180^\circ \\ x^\circ + (x^\circ + y^\circ) + y^\circ &= 180^\circ \\ (x^\circ + y^\circ) + (x^\circ + y^\circ) &= 180^\circ \\ 2(x^\circ + y^\circ) &= 180^\circ \\ x^\circ + y^\circ &= 90^\circ \end{aligned}$$

But $\angle PSQ = \angle RSP + \angle RSQ = x^\circ + y^\circ = 90^\circ$.

Therefore, the measure of $\angle PSQ$ is 90°.