



Problem of the Week

Problem C and Solution

A Homemade Expression of Love

Problem

Maggie wants to make a special card for Valentine's Day. She starts with a square piece of red paper with a side length of 12 cm. She then pastes two white semi-circles, each with radius 3 cm, and a white triangle onto the square sheet of red paper, as shown below. (The dashed line and the right angle symbols will not actually be on the finished card.). She is going to write her valentine a message in red ink on the white region of the card. Determine the total amount of area available in the white region for Maggie's special valentine greeting.

Solution

The given information is shown on the diagram to the right.

The total area for writing the message is the area of the two semi-circles plus the area of the white triangle.

Since there are two semi-circles of radius 3 cm, the total area of the two semi-circles is equal to the area of a full circle of radius 3 cm. Therefore, the area of the two semi-circles is

$$\pi r^2 = \pi(3)^2 = 9\pi \text{ cm}^2.$$

The height of the triangle is the length of the square minus the radius of a semi-circle. Therefore the height of the triangle is $12 - 3 = 9$ cm. The base of the triangle is 12 cm, the width of the square. The area of the triangle is

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}(12)(9) = 54 \text{ cm}^2.$$

The total available area for Maggie's message is $(9\pi + 54) \text{ cm}^2$. This area is approximately 82.3 cm^2 . Happy Valentine's Day.

