

Problem of the Week

Problem D and Solution

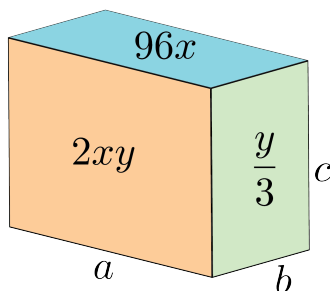
I Want Some Volume

Problem

The areas of the front, side, and top faces of a rectangular prism are $2xy$, $\frac{y}{3}$, and $96x$ cm², respectively. Calculate the volume of the rectangular prism in terms of x and y .

Solution

Since $\frac{y}{3}$ and $96x$ are areas, then x and y must be positive. Let the length, width, and height of the rectangular prism be a , b , and c , respectively.



The volume is equal to the product abc .

By multiplying side lengths, we can write the following three equations using the given areas.

$$ac = 2xy$$

$$bc = \frac{y}{3}$$

$$ab = 96x$$

Multiplying the left sides and multiplying the right sides of each of the three equations gives us the following.

$$(ac)(bc)(ab) = (2xy) \left(\frac{y}{3}\right) (96x)$$

$$a^2b^2c^2 = 64x^2y^2$$

$$(abc)^2 = (8xy)^2$$

$$\sqrt{(abc)^2} = \pm \sqrt{(8xy)^2}$$

$$abc = \pm 8xy$$

Since all quantities are positive, we can conclude that $abc = 8xy$.

Therefore, the volume of the rectangular prism is $8xy$ cm³.