Problem of the Week
Problem E
Taking Grapes

Quinn and Birgitta are playing a game using a bowl of grapes. The rules of the game are as follows:

- Write the numbers 1, 2, 3, 4, and 5 on a whiteboard.
- Players take turns choosing a number from the whiteboard, removing that number of grapes from the bowl, and then erasing that number from the whiteboard.
- The game continues until all the numbers are erased or a player is not able to take any of the remaining numbers of grapes from the bowl.
- The last player who erased a number from the whiteboard is the winner.

The game starts with 8 grapes and Quinn goes first. Quinn has developed a winning strategy so that she is guaranteed to win this game, regardless of how many grapes Birgitta takes on her turns. Find all the possible first moves in Quinn’s winning strategy. Justify your answer.