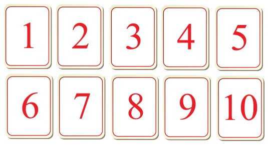

Problem of the Week

Problem E and Solution

Is it Odd?

Problem

Amna creates a game for her school carnival. She uses ten cards, each with a different integer from 1 to 10 on it, and places them face down on a table. To play the game, players randomly turn over three cards and look at the numbers. They win if the smallest number is odd and the next smallest number is even.

What is the probability that a player wins the game on their first try?

Solution

To calculate the probability we need to determine two things: the number of possible selections of three cards, and the number of these that would result in a win.

First, we will determine the number of possible selections of three cards. Since each number is distinct, then there are 10 choices for the first card, 9 choices for the second card, and 8 choices for the third card. This produces $10 \times 9 \times 8 = 720$ ordered selections. But this total includes 6 orderings for each possible selection of three numbers. For example, the three numbers 1, 2, and 3 would be included 6 times: $(1, 2, 3)$, $(1, 3, 2)$, $(2, 1, 3)$, $(2, 3, 1)$, $(3, 1, 2)$, and $(3, 2, 1)$. As we see with this example, each combination of three cards is counted six times. Therefore, there are $720 \div 6 = 120$ possible selections of three cards.

Next, we will determine the number of selections of three cards that result in a win. That is, the number of selections in which the smallest number is odd and the next smallest number is even. We can use the following table.

Smallest Number	Next Smallest Number	Possible Value(s) for Largest Number	Number of Selections of Three Cards
1	2	3, 4, 5, 6, 7, 8, 9, 10	8
	4	5, 6, 7, 8, 9, 10	6
	6	7, 8, 9, 10	4
	8	9, 10	2
3	4	5, 6, 7, 8, 9, 10	6
	6	7, 8, 9, 10	4
	8	9, 10	2
5	6	7, 8, 9, 10	4
	8	9, 10	2
7	8	9, 10	2

The total number of selections of three cards that result in a win is

$$(8 + 6 + 4 + 2) + (6 + 4 + 2) + (4 + 2) + (2) = 20 + 12 + 6 + 2 = 40.$$

Therefore, the probability that a player wins the game on their first try is $\dfrac{40}{120} = \dfrac{1}{3}$.