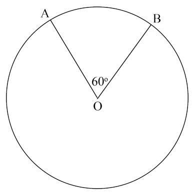


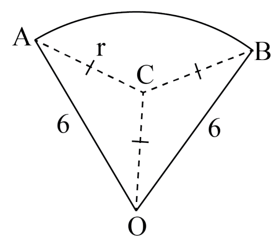
Problem of the Week
Problem E and Solution
Another Circle

Problem

Points A and B are on a circle with centre O and radius 6 cm, such that $\angle AOB = 60°$. Determine the radius of the circle which passes through points $A,\ B$, and O.

Solution

Let C be the centre of the circle that passes through $A,\ B$, and O. Then $CA,\ CB$, and CO are radii. Therefore, $CA = CB = CO = r$, where r is the radius of the circle.



In $\triangle CAO$ and $\triangle CBO$, $CA = CB$, CO is common, and $OA = OB$. Therefore, $\triangle CAO \cong \triangle CBO$ and it follows that $\angle COA = \angle COB$. But $\angle AOB = 60°$. Therefore, $\angle COA = \angle COB = 30°$.

In $\triangle CAO$, $CA = CO = r$ and $\triangle CAO$ is isosceles. Therefore, $\angle CAO = \angle COA = 30°$ and $\angle ACO = 180° - 30° - 30° = 120°$.

From here we can find the length of r using either the sine law or the cosine law.

Method 1: Using the sine law,

$$\begin{aligned}
\frac{CA}{\sin(\angle COA)} &= \frac{OA}{\sin(\angle ACO)} \\
\frac{r}{\sin 30°} &= \frac{6}{\sin 120°} \\
r &= \frac{6}{\sin 120°} \times \sin 30° \\
r &= \frac{6}{\frac{\sqrt{3}}{2}} \times \frac{1}{2} \\
r &= 6 \times \frac{2}{\sqrt{3}} \times \frac{1}{2} \\
r &= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
r &= 2\sqrt{3} \text{ cm}
\end{aligned}$$

Therefore, the radius of the circle that passes through $A,\ B$, and O is $2\sqrt{3}$ cm.

Method 2: Using the cosine law,

$$\begin{aligned}
CA^2 &= CO^2 + AO^2 - 2 \times CO \times AO \times \cos(\angle COA) \\
r^2 &= r^2 + 6^2 - 2(6)(r)\cos 30^\circ \\
12r\cos 30^\circ &= 36 \\
r\cos 30^\circ &= 3 \\
r \times \frac{\sqrt{3}}{2} &= 3 \\
r \times \sqrt{3} &= 6 \\
r \times \sqrt{3} \times \sqrt{3} &= 6 \times \sqrt{3} \\
3r &= 6\sqrt{3} \\
r &= 2\sqrt{3}\text{ cm}
\end{aligned}$$

Therefore, the radius of the circle that passes through A, B, and O is $2\sqrt{3}$ cm.