Problem of the Week

Problem E

Down to One

The *digit sum* of a positive integer is found by summing its digits. The *digital root* is found by repeatedly calculating the digit sum until a single digit is achieved.

The digit sum of 413 is 8, since $4 + 1 + 3 = 8$ and 8 is a single-digit number. Note that the digital root is also 8, and this is calculated in one step.

The digit sum of 642 is $6 + 4 + 2 = 12$, which is not a single-digit number. The digit sum of 12 is $1 + 2 = 3$, which is a single-digit number. Therefore, the digital root of 642 is 3. This is calculated in two steps.

The digital root of 4 is 4. This is calculated in zero steps.

How many three-digit numbers have a digital root of 5 that is calculated in three or fewer steps?

$$4 + 1 + 3 = 8$$