



Problem of the Week

Problem B and Solution

Let the Game Begin!

Problem

Parvinder and Carlos are each rolling a standard six-sided die, with numbers 1 to 6 on its sides, to see who gets to go first in their game. The person who rolls the lower number gets to go first. If they tie, they roll again.

- What is the theoretical probability that Parvinder will roll a number lower than Carlos on their first roll? HINT: What are the possibilities for Carlos' roll if Parvinder rolls a 1? a 2? a 3?
- What is the theoretical probability that Parvinder will roll a number greater than Carlos on their first roll?
- Is this a fair way to determine who goes first?

Solution

The following table shows all the possible rolls. For each possibility, there is a T if Parvinder and Carlos roll the same number, a P if Parvinder rolls a number lower than Carlos, and a C if Carlos rolls a number lower than Parvinder.

		Carlos' Roll					
		1	2	3	4	5	6
Parvinder's Roll	1	T	P	P	P	P	P
	2	C	T	P	P	P	P
	3	C	C	T	P	P	P
	4	C	C	C	T	P	P
	5	C	C	C	C	T	P
	6	C	C	C	C	C	T

The table shows that the total number of possible rolls is $6 \times 6 = 36$.

- There are 15 Ps in the table. These occur when Parvinder rolls a number lower than Carlos. Thus, the theoretical probability that Parvinder rolls a number lower than Carlos on their first roll is $\frac{15}{36} = \frac{5}{12}$.
- There are 15 Cs in the table. These occur when Parvinder rolls a number greater than Carlos. Thus, the theoretical probability that Parvinder rolls a number greater than Carlos on their first roll is $\frac{15}{36} = \frac{5}{12}$.
- From (a) and (b), we see that the probabilities of each person rolling the lower number are the same. Therefore, this is a fair way to determine who goes first.