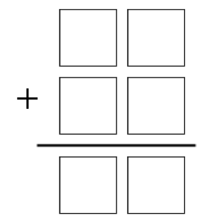
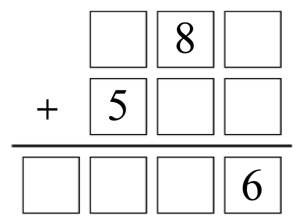
Problem of the Week
Problem B and Solution
That's About Right

Problem

(a) Place the digits 1, 3, 6, 7, 8, and 9 in the boxes shown so that each box contains a different digit, and the sum is as close as possible to 99.



(b) The digits 5, 6, and 8 have been placed in three of the boxes shown. Place the digits 0, 1, 2, 3, 4, 7, and 9 in the remaining boxes so that each box contains a different digit, and the sum is as close as possible to 1000.



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Solution

(a) Notice that 98 is the closest number to 99 that could possibly be formed using the digits 1, 3, 6, 7, 8, and 9. Let's see if we can arrange the remaining digits, 1, 3, 6, and 7, to get a sum of 98.

Using the digits 1, 3, 6, and 7, to get a sum with a ones digit of 8, we must place the 1 and the 7 in the two boxes in the ones column. If we place the remaining digits, 3 and 6, in the tens column, we will get a sum with a tens digit of 9. Therefore, it is possible to arrange the digits to get a sum of 98, which is the closest possible sum to 99. We also see that there are four possible ways to arrange the digits in the boxes to produce this sum.

$$\begin{array}{r} 61 \\ +\,37 \\ \hline 98 \end{array} \qquad \begin{array}{r} 67 \\ +\,31 \\ \hline 98 \end{array} \qquad \begin{array}{r} 31 \\ +\,67 \\ \hline 98 \end{array} \qquad \begin{array}{r} 37 \\ +\,61 \\ \hline 98 \end{array}$$

(b) Given the digits 0, 1, 2, 3, 4, 7, and 9, along with the placement of the 6, the closest number to 1000 that could be formed is 0976. The next closest number is 1026.

We will first see if we can place the digits to get a sum of 0976. If the sum is 0976, then the digits 0, 9, and 7 have been placed, and the remaining boxes will be filled with the digits 1, 2, 3, and 4. Of these digits, the only two that have a sum with ones digit 6 are 2 and 4. Therefore, the 2 and the 4 would need to go in the ones column. This leaves 1 and 3 to be placed. Looking at the tens column of the sum, we need to place one of these numbers in the tens column so that the sum of that number with 8 has a ones digit of 7. This is not possible. Therefore, we see that it is not possible to place the numbers so that the sum is 0976.

Next, we try to place the digits to get a sum of 1026. If the sum is 1026, then the digits 0, 1, and 2 have been placed, and the remaining boxes must be filled with the digits 3, 4, 7, and 9. Of these digits, the only two that have a sum with ones digit 6 are 7 and 9. Therefore, the 7 and the 9 would need to go in the ones column. This leaves 3 and 4 to be placed. Looking at the tens column of the sum, we need to place one of these numbers in the tens column so that the sum of that number with 8 and 1 (the carry from the ones column) has a ones digit of 2. This is possible if we place the 3 in this box. That leaves the 4 to go in the empty box in the hundreds column. Indeed, we see that the sum of 4 with 5 and 1 (the carry from the tens column) is 10, as required.

Thus, it is possible to arrange the digits to get a sum of 1026, and this is the closest we can get to a sum of 1000. We see that there are two possible ways to arrange the digits in the boxes to produce this sum.

$$\begin{array}{r} \boxed{4}\,\boxed{8}\,\boxed{9} \\ +\ \boxed{5}\,\boxed{3}\,\boxed{7} \\ \hline \boxed{1}\,\boxed{0}\,\boxed{2}\,\boxed{6} \end{array} \qquad \begin{array}{r} \boxed{4}\,\boxed{8}\,\boxed{7} \\ +\ \boxed{5}\,\boxed{3}\,\boxed{9} \\ \hline \boxed{1}\,\boxed{0}\,\boxed{2}\,\boxed{6} \end{array}$$

Extension: Can you find a better solution for (b) by placing the 5, 6, and 8 elsewhere?