Problem of the Week
Problem B and Solution
## Let's Divvy These Up

Problem

(a) At Yore Elementary School, each class has between 15 and 35 students. If there are 78 students in Grade 6, and all Grade 6 classes have the same number of students, how many Grade 6 classes are there?

(b) At Hyz Elementary School, there is the same number of Grade 6 classes as at Yore, but there are 84 Grade 6 students. If all Grade 6 classes have the same number of students, how many students are in each Grade 6 class?

(c) For each of the numbers in the first column of the table given below, first determine if it could be the total number of students in three very large classes of equal size, and then calculate the digit sum of the number. Do you notice any connection between these two things?

NOTE: The *digit sum* of a number is the sum of its digits. For example, the digit sum of 63 is $6 + 3 = 9$.

Number	Could be total?	Digit Sum
1008		
1023		
1741		
2238		
1759		
1902		
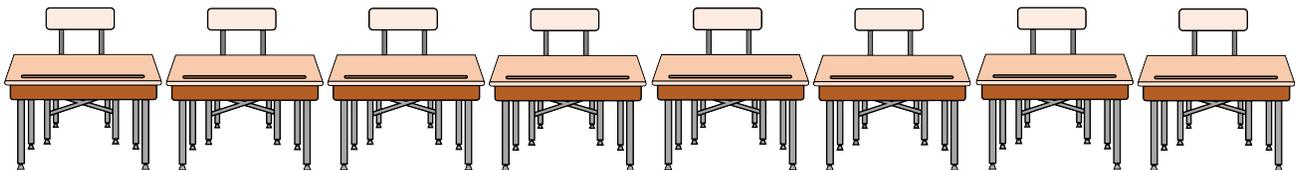
Solution

(a) Since the classes are of equal size, the number of classes must divide evenly into 78. The numbers which do so are $1, 2, 3, 6, 13, 26, 39$, and 78. The only number that is between 15 and 35 is 26. Since $3 \times 26 = 78$, that means there are 3 classes of 26 students.

(b) If there are 84 students in 3 classes, then there are $84 \div 3 = 28$ students in each class.

Alternatively, since $84 - 78 = 6$, that means there are $6 \div 3 = 2$ more students in each class at Hyz Elementary than at Yore. So there are $26 + 2 = 28$ students in each class.

(c) We can try dividing each number by 3.

$$1008 \div 3 = 336$$
$$1023 \div 3 = 341$$
$$1741 \div 3 = 580.\bar{3}$$
$$2238 \div 3 = 746$$
$$1759 \div 3 = 586.\bar{3}$$
$$1902 \div 3 = 634$$

Since 3 divides evenly into $1008, 1023, 2238$, and 1902, these four numbers could be the total number of students in 3 classes of equal size. Since 3 does not divide evenly into 1741 or 1759, these two numbers could not be the total number of students in 3 classes of equal size.

We add this information, along with the digit sum of each number, to the table.

Number	Could be total?	Digit Sum
1008	Yes	9
1023	Yes	6
1741	No	13
2238	Yes	15
1759	No	22
1902	Yes	12

The digit sums for the numbers $1008, 1023, 2238$, and 1902 are all multiples of 3. These are the numbers that can be divided into 3 equal groups. The digit sums for the other two numbers are not multiples of 3. In fact, a number is a multiple of 3 exactly when its digit sum is also a multiple of 3.