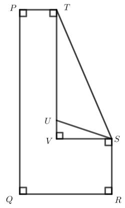


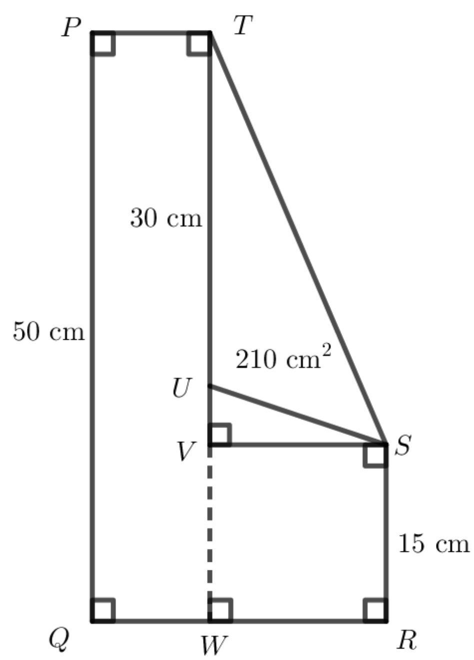
Problem of the Week
Problem C and Solution
Partitioned Pentagon

Problem

Consider pentagon $PQRST$. Starting at P and moving around the pentagon, the vertices are labelled P, Q, R, S, and T, in order. The pentagon has right angles at P, Q, and R, obtuse angles at S and T, and an area of 1000 cm^2. Point V lies inside the pentagon such that $\angle PTV$, $\angle TVS$, and $\angle VSR$ are right angles. Point U lies on TV such that $\triangle STU$ has an area of 210 cm^2. Also, it is known that $PQ = 50$ cm, $SR = 15$ cm, and $TU = 30$ cm. Determine the length of PT.

Solution

Extend TV to meet QR at W. We mark this and all of the given information on the diagram.



To find the area of a triangle, multiply the length of the base by the height and divide by 2. In $\triangle STU$, the base TU has length 30 cm. The corresponding height of $\triangle STU$ is the perpendicular distance from TU (extended) to vertex S, namely SV.

Since the area of $\triangle STU$ is given to be 210 cm^2,

$$\begin{aligned}210 &= \frac{30 \times SV}{2}\\ 210 &= 15 \times SV\\ 14 &= SV\end{aligned}$$

We know that $TW = PQ = 50$, $VW = SR = 15$, and $TW = TU + UV + VW$.

It follows that $50 = 30 + UV + 15$ and $UV = 5$ cm.

Now we can relate the total area of the pentagon to the areas of the shapes inside.

$$\begin{aligned}\text{Area } PQRST &= \text{Area } PQWT + \text{Area } RSVW + \text{Area } \triangle SUV + \text{Area } \triangle STU\\ 1000 &= PQ \times PT + SV \times SR + \frac{UV \times SV}{2} + 210\\ 1000 &= 50 \times PT + 14 \times 15 + \frac{5 \times 14}{2} + 210\\ 1000 &= 50 \times PT + 210 + 35 + 210\\ 1000 &= 50 \times PT + 455\\ 1000 - 455 &= 50 \times PT\\ 545 &= 50 \times PT\\ \frac{545}{50} &= PT\end{aligned}$$

Therefore, $PT = 10.9$ cm.