

# Problem of the Week Problem C and Solution <br> Meal Deal 

## Problem

Jessica and Callista go the local burger joint. They both want to buy the meal deal. Jessica has $\frac{3}{4}$ of the money needed to buy the meal deal and Callista has half of the money needed to buy the meal deal. If the meal deal was $\$ 3$ cheaper, then together they would have exactly enough money to buy two of the meal deals.
What is the original price of the meal deal?

## Solution

## Solution 1:

Suppose that the cost of the meal deal, in dollars, is $C$. Then Jessica has $\frac{3}{4} C$ and Callista has $\frac{1}{2} C$. Combining their money, together Jessica and Callista have

$$
\frac{3}{4} C+\frac{1}{2} C=\frac{3}{4} C+\frac{2}{4} C=\frac{5}{4} C
$$

If the meal deal was $\$ 3$ cheaper, then the cost to buy one meal deal would be $C-3$. If the cost of one meal deal was $C-3$, then the cost to buy two meal deals at this price would be $2(C-3)=(C-3)+(C-3)=2 C-6$.
Combined, Jessica and Callista would have enough money to buy exactly two meal deals at this reduced price. Thus, $2 C-6=\frac{5}{4} C$.
Solving for $C$,

$$
\begin{aligned}
2 C-6 & =\frac{5}{4} C \\
2 C-\frac{5}{4} C & =6 \\
\frac{8}{4} C-\frac{5}{4} C & =6 \\
\frac{3}{4} C & =6 \\
3 C & =24 \\
C & =8
\end{aligned}
$$

Therefore, the original price of the meal deal is $\$ 8$.

## Solution 2:

Since the new price of the meal deal is $\$ 3$ cheaper than the original price, then the original price must be more than $\$ 3$. We will use systematic trial and error to figure out the original price.
Suppose the original price of the meal deal was $\$ 6$. Then the reduced price would be $\$ 3$. Also, Jessica has $\frac{3}{4} \times \$ 6=\$ 4.50$ and Callista has $\frac{1}{2} \times \$ 6=\$ 3$, and in total they have $\$ 4.50+\$ 3=\$ 7.50$. With $\$ 7.50$, they could buy exactly $7.50 \div 3=2.5$ meal deals at a price of $\$ 3$ each.

Suppose the original price of the meal deal was $\$ 12$. Then the reduced price would be $\$ 9$. Also, Jessica has $\frac{3}{4} \times \$ 12=\$ 9$ and Callista has $\frac{1}{2} \times \$ 12=\$ 6$, and in total they have $\$ 9+\$ 6=\$ 15$. With $\$ 15$, they could buy $15 \div 9 \approx 1.67$ meal deals at a price of $\$ 9$ each.
We can see that the original price of the meal deal lies somewhere between $\$ 6$ and $\$ 12$.

Let's suppose the original price of the meal deal was $\$ 8$. Then the reduced price would be $\$ 5$. Also, Jessica has $\frac{3}{4} \times \$ 8=\$ 6$ and Callista has $\frac{1}{2} \times \$ 8=\$ 4$, and in total they have $\$ 6+\$ 4=\$ 10$. With $\$ 10$, they could buy exactly $10 \div 5=2$ meal deals at a price of $\$ 5$ each.
Thus, we can see that the original price of the meal deal is $\$ 8$.

