Problem of the Week
Problem C and Solution
Just My Two Cents

Problem

In Canada, pennies are 1 cent coins that were used up until 2012. Antonio and Bjorn are playing a game using two pennies and a game board consisting of a row of 6 squares. To start the game, the pennies are placed in the two leftmost squares, as shown.

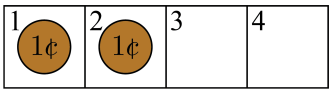
The rules of the game are as follows:

- On a player's turn, the player must move one penny one or more squares to the right.
- The penny may not pass over any other penny or land on a square that is occupied by another penny.
- The game ends when the pennies are in the two rightmost squares. The last player to move a penny wins the game.

Bjorn knows that if he goes second he can always win the game, regardless of where Antonio moves the pennies on his turns. Describe Bjorn's winning strategy.

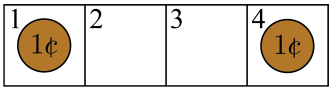
Solution

First, consider playing the game with just four squares. We will number the squares from 1 to 4, starting on the left. The two pennies would start in squares 1 and 2.

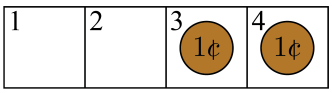


Player 1 has two options for their first turn. They can move the penny in square 2 to either square 4 or square 3.

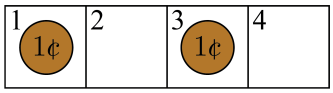
- *Option 1:* Player 1 moves the penny in square 2 to square 4. Then the pennies would be in squares 1 and 4.



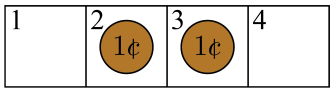
If Player 2 moves the penny in square 1 to square 3, then they would win the game because the pennies would be in squares 3 and 4.



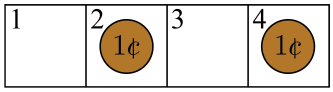
- *Option 2:* Player 1 moves the penny in square 2 to square 3. Then the pennies would be in squares 1 and 3.



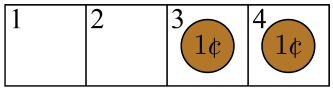
Then Player 2 has two options for their turn. They can either move the penny in square 3 or move the penny in square 1. However, if Player 2 wants to win the game, they should not move the penny in square 3 to square 4. If they do, then the pennies would be in squares 1 and 4, and then Player 1 could move the penny in square 1 to square 3 and win the game. So, Player 2 should move the penny in square 1 to square 2. Then the pennies would be in squares 2 and 3.



Player 1 would be forced to move the penny in square 3 to square 4. Then the pennies would be in squares 2 and 4.



Player 2 would then move the penny in square 2 to square 3, and win the game because the pennies would be in squares 3 and 4.



In the game with just two pennies and four squares, Player 2 is always able to win, regardless of what Player 1 does on their turn. If you look closely, you will see that the winning strategy for Player 2 is to copy whatever Player 1 did with the other penny. The two pennies start together. Player 1 must move the rightmost penny, creating a gap between the two pennies. On the following turn, Player 2 can move the other penny in such a way that there is no longer a gap between the two pennies. Doing this ensures that Player 1 must always move the rightmost penny, creating a gap between the pennies which allows Player 2 to always be able to move the leftmost penny and close the gap. Doing this also ensures that Player 2 wins the game.

In fact, the number of squares really does not matter. Bjorn can use this same strategy to win our game with six squares. Whatever Antonio does with the penny on the right, Bjorn "mimics" with the penny on the left. This strategy will guarantee that Bjorn will win the game.