



Problem of the Week

Problem C and Solution

Just Sum Dice

Problem

Ahmik created a game for his school's carnival where players roll two dice and find the sum of the two numbers on the top faces. If this sum is a perfect square or a prime number, they win a prize. To make it more interesting, Ahmik made the two dice using a 3D printer so that they each have the numbers 1, 2, 3, 5, 7, and 9 on their faces. One of the dice is purple and the other is green.

What is the probability that a player will win a prize after rolling the dice once?

NOTE:

A square of any integer is called a *perfect square*. The number 25 is a perfect square since it can be expressed as 5^2 or 5×5 .

A *prime number* is an integer greater than 1 that has only two positive divisors; 1 and itself. The number 17 is prime because its only positive divisors are 1 and 17.

Solution

To solve this problem, we will create a table where the columns show the possible rolls of the green die, the rows show the possible rolls of the purple die, and each cell in the body of the table gives the sum for the corresponding pair of rolls.

		Green Die					
		1	2	3	5	7	9
Purple Die	1	2	3	4	6	8	10
	2	3	4	5	7	9	11
	3	4	5	6	8	10	12
	5	6	7	8	10	12	14
	7	8	9	10	12	14	16
	9	10	11	12	14	16	18

From the table, we see that there are 36 possible outcomes. We also see that the perfect squares 4, 9, and 16 appear as sums in the table seven times in total.

The smallest sum in the table is 2, and the largest sum in the table is 18. The prime numbers in this range appearing as sums in the table are 2, 3, 5, 7, and 11. These sums appear nine times in total.

Thus, there are 7 sums that are perfect squares and 9 sums that are prime numbers in the table. Since a number cannot be both a prime number and a perfect square, we can conclude that there are $7 + 9 = 16$ sums that are prime numbers or perfect squares.

To determine the probability of a specific outcome, we divide the number of times the specific outcome occurs by the total number of possible outcomes. Thus, the probability of a player rolling a sum that is either a prime number or a perfect square is $16 \div 36 = \frac{4}{9} \approx 44\%$.

Therefore, a player has approximately a 44% chance of winning a prize after rolling the dice once.

EXTENSION: A game is considered fair if the chance of winning is 50%. How could you change the rules of this game to make it fair?