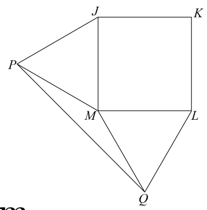


Problem of the Week

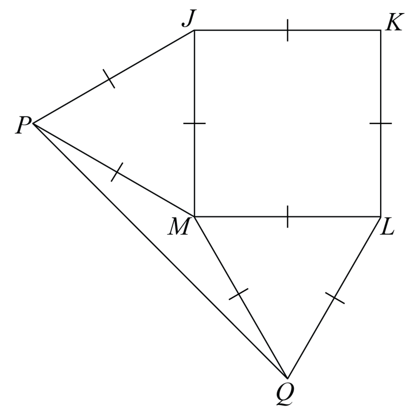
Problem C and Solution

All Squared Up

Problem

Ziibi drew a square. Starting at one corner and moving around the square, he labelled the vertices J, K, L, and M, in order. He drew points P and Q outside the square so that both $\triangle JMP$ and $\triangle MLQ$ are equilateral. Determine the measure, in degrees, of $\angle MPQ$.

Solution



Since $JKLM$ is a square, $JK = KL = LM = MJ$.

Since $\triangle JMP$ is equilateral, $MJ = JP = MP$.

Since $\triangle MLQ$ is equilateral, $LM = LQ = QM$.

It follows that

$$JK = KL = LM = MJ = JP = MP = LQ = QM.$$

Each angle in a square is 90°. Therefore, $\angle JML = 90^\circ$.

Each angle in an equilateral triangle is 60°. Therefore, $\angle JMP = 60^\circ$ and $\angle LMQ = 60^\circ$.

A complete revolution is 360°. Since $\angle PMQ$, $\angle JMP$, $\angle JML$, and $\angle LMQ$ form a complete revolution, then

$$\begin{aligned}\angle PMQ &= 360^\circ - \angle JMP - \angle JML - \angle LMQ \\ &= 360^\circ - 60^\circ - 90^\circ - 60^\circ \\ &= 150^\circ\end{aligned}$$

In $\triangle MPQ$, $MP = QM$ and the triangle is isosceles. It follows that $\angle MPQ = \angle MQP$.

In a triangle, the sum of the three angles is 180°. Since $\angle PMQ = 150^\circ$, then the sum of the two remaining equal angles must be 30°. Therefore, each of the remaining two angles must equal 15° and it follows that $\angle MPQ = 15^\circ$.