

Problem of the Week

Problem C and Solution

One Dot at a Time

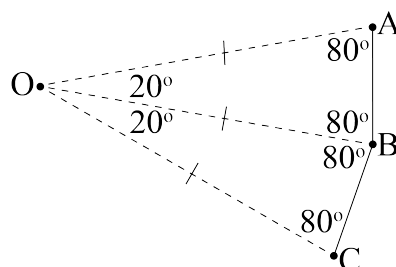
Problem

Priya is drawing a polygon on a piece of wood. First she hammers a nail into the piece of wood, calling this point O . Then she attaches one end of a piece of string to the nail, and the other end to a pencil. She pulls the string tight and makes a dot on the wood, calling this point A . Keeping the string tight, she rotates it 20° clockwise and makes another dot, calling this point B . She then connects points A and B with a straight line. She repeats this process, rotating the string 20° clockwise, making a dot, and connecting this point to the previous point with a straight line each time, until she has gone all the way around the circle and completed the polygon.

- How many sides does Priya's completed polygon have?
- What is the sum of all the interior angles in the polygon?

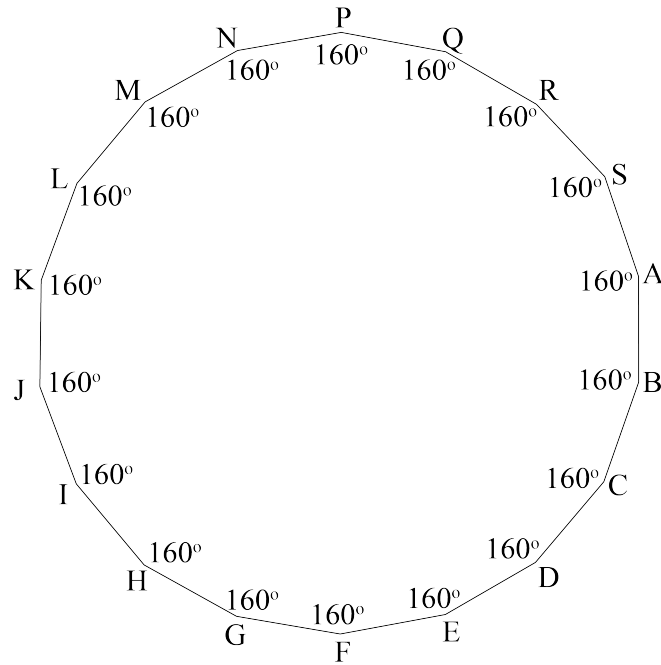
Solution

- Each time the process is repeated, another congruent triangle is created. Each of these triangles has a 20° angle at O , the centre of the circle. Since a complete rotation at the centre is 360° , that means there are $360 \div 20 = 18$ triangles formed. Since each triangle has one edge on the side of the polygon, it follows that the polygon has 18 sides. An 18-sided polygon is called an *octadecagon*, from *octa* meaning 8 and *deca* meaning 10.
- Since the distance between each dot and point O (the nail) is always the same, it follows that the two sides of each congruent triangle that connect to point O are equal. Therefore, the congruent triangles are all isosceles, and the angles that are not at point O are all equal. The angles in a triangle sum to 180° , so after the 20° angle is removed, there is 160° remaining for the other two angles. It follows that each of the other two angles in each triangle measures $160^\circ \div 2 = 80^\circ$. The following diagram illustrates this information for the two adjacent triangles AOB and BOC .





Each interior angle in the polygon is formed by an 80° angle from one triangle and the adjacent 80° angle from the next triangle. It follows that each interior angle measures $80^\circ + 80^\circ = 160^\circ$. Thus, there are 18 interior angles in the octadecagon and each angle measures 160° .



Therefore, the sum of all the interior angles in the octadecagon is $18 \times 160^\circ = 2880^\circ$.