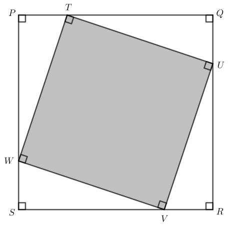


Problem of the Week
Problem C and Solution
A Square in a Square

Problem

In the diagram, $PQRS$ is a square. Points T, U, V, and W are on sides PQ, QR, RS, and ST, respectively, forming square $TUVW$.

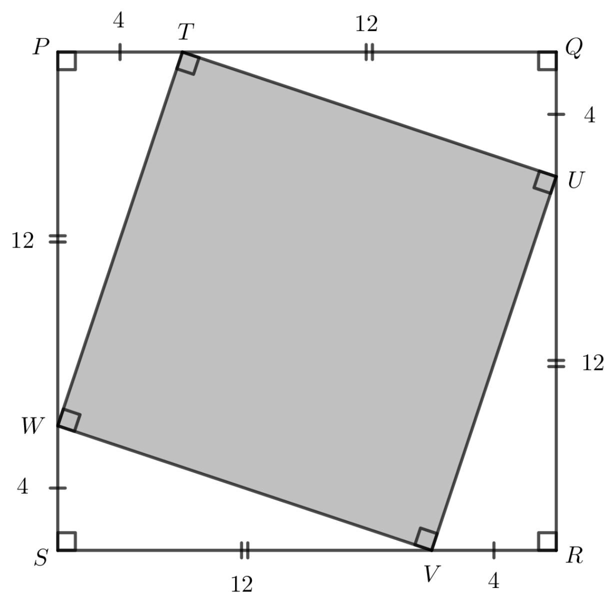
If $PT = QU = RV = SW = 4$ m and $PQRS$ has area 256 m^2, determine the area of $TUVW$.

Solution

The area of square $PQRS$ is 256 m^2. Therefore, square $PQRS$ has side length equal to 16 m, since $16 \times 16 = 256$ and the area of a square is the product of its length and width.

We are given that $PT = QU = RV = SW = 4$ m. Since $16 - 4 = 12$, we know that $TQ = UR = VS = WP = 12$ m.

We add this information to the diagram.



From this point, we will present two different solutions that calculate the area of square $TUVW$.

Solution 1

In $\triangle WPT$, $PT = 4$ and $WP = 12$. Also, this triangle is right-angled, so we can use one of PT and WP as the base and the other as the height in the calculation of the area of the triangle, since they are perpendicular to each other. Therefore, the area of $\triangle WPT$ is equal to $\frac{PT\times WP}{2} = \frac{4\times 12}{2} = 24$ m^2. Since the triangles $\triangle WPT$, $\triangle TQU$, $\triangle URV$, and $\triangle VSW$ each have the same base length and height, their areas are equal. Therefore, the total area of the four triangles is $4 \times 24 = 96$ m^2.

The area of square $TUVW$ can be determined by subtracting the area of the four triangles from the area of square $PQRS$.

Therefore, the area of square $TUVW$ is $256 - 96 = 160$ m^2.

Solution 2

Some students may be familiar with the Pythagorean Theorem. This theorem states that in a right-angled triangle, the square of the length of the hypotenuse (the longest side) is equal to the sum of the squares of the other two sides. The longest side is located opposite the right angle.

$\triangle WPT$ is a right-angled triangle with $PT = 4$, $WP = 12$, and TW is the hypotenuse. Therefore,

$$\begin{aligned} TW^2 &= PT^2 + WP^2 \\ &= 4^2 + 12^2 \\ &= 16 + 144 \\ &= 160 \end{aligned}$$

Taking the square root, we have $TW = \sqrt{160}$ m, since $TW > 0$.

Now $TUVW$ is a square. Therefore, all of its side lengths are equal to $\sqrt{160}$. The area of $TUVW$ is calculated by multiplying its length by its width.

Therefore, the area of $TUVW$ is equal to $\sqrt{160} \times \sqrt{160} = 160$ m^2.

Note: Alternatively, we could have found the area of square $TUVW$ by noticing that the area of a square is s^2, where s is the side length of the square. For square $TUVW$, $s = TW$, and therefore the area is $s^2 = TW^2$. Now, from the Pythagorean Theorem above, we see $TW^2 = 160$ m^2. Therefore, the area is 160 m^2.